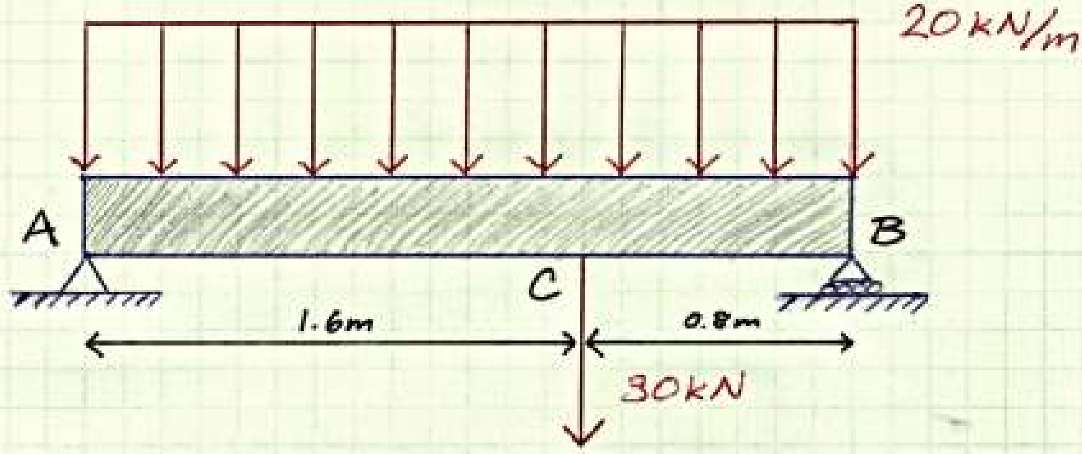
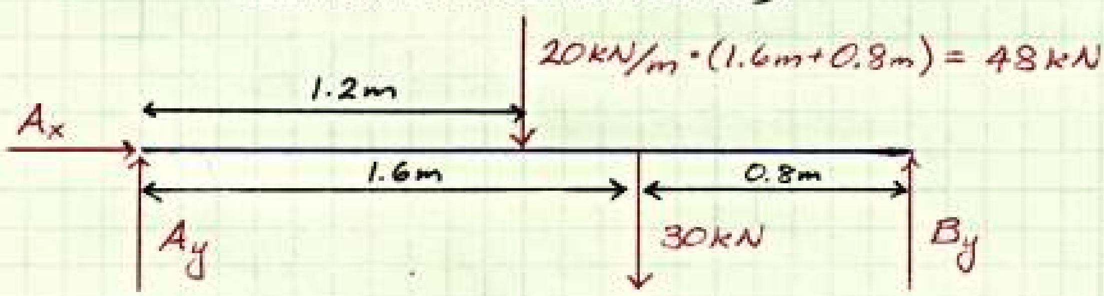
MEC85 / CEC30 ~~B18~~ Midterm 2

Problem 1 Solutions



a) Sketch the shear force and bending moment diagrams

- Start w/ FBD of the beam (concentrate the distributed load to find reaction forces)



$\Sigma F_x = 0: \quad \underline{A_x = 0}$

$\Sigma M_A = 0: \quad -48\,kN(1.2m) - 30kN(1.6m) + B_y(2.4m) = 0$

$\underline{B_y = 44\,kN} \uparrow$

$\Sigma F_y = 0: \quad A_y - 48kN - 30kN + 44kN = 0$

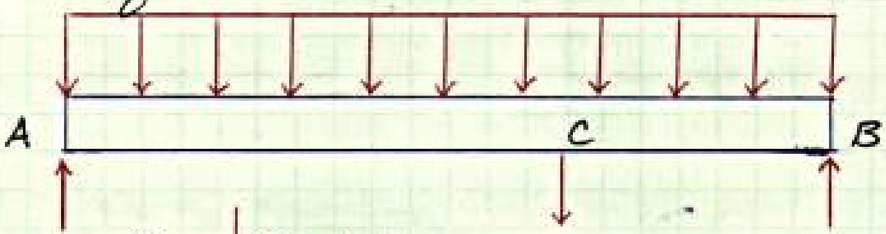
$\underline{A_y = 34\,kN} \uparrow$

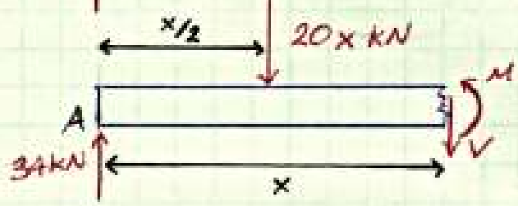
Note: Other equilibrium equations could have been used (i.e. moments about other points, moment instead of $\Sigma F_y = 0$). All calculations will given same reaction forces.

Problem 1 Solutions cont.

Note: There are many ways to construct shear force and bending moment diagrams. Two such ways will be shown here: Method of Section and Method of Calculus.

Method of Section:





$\Sigma F_y = 0$: $34kN - 20x\,kN - V = 0$

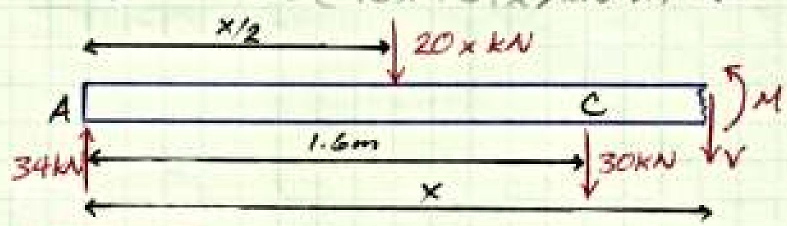
$V = (-20x + 34)\,kN$ ← $V(0) = 34kN$, $V(1.6) = 2kN$

$\Sigma M_A = 0$: $-20x\,kN(x/2\,m) - Vx + M = 0$

Concave Down

$M = 10x^2 + Vx$
$= 10x^2 - 20x^2 + 34x$
$= (-10x^2 + 34x)\,kN \cdot m$ ←

$M(0) = 0\,kN \cdot m$
$M(1.6) = 28.8\,kN \cdot m$



$\Sigma F_y = 0$: $34kN - 20x\,kN - 30kN - V = 0$

$V = (-20x + 4)\,kN$ ← $V(1.6) = -28kN$, $V(2.4) = -44kN$

$\Sigma M_A = 0$: $-20x\,kN(x/2\,m) - 30kN(1.6m) - Vx + M = 0$

Concave Down

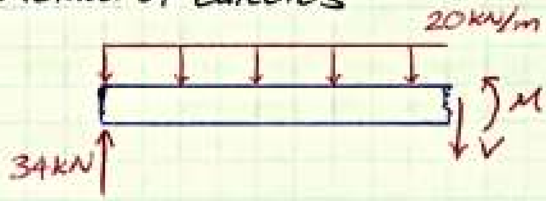
$M = 10x^2 + 48 + Vx$
$= 10x^2 + 48 - 20x^2 + 4x$
$= (-10x^2 + 4x + 48)\,kN \cdot m$ ←

$M(1.6) = 28.8\,kN \cdot m$
$M(2.4) = 0\,kN \cdot m$

Note: Negative coefficient in front of x^2 indicates the parabola is concave down, meaning it opens downward.

Problem 1 Solutions cont.

Method of Calculus



Recall:

$$-w = \frac{dV}{dx}$$

$$V = \frac{dM}{dx}$$

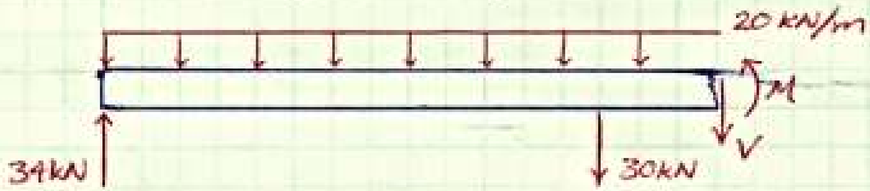
$\Sigma F_y = 0: \quad 34\,kN - V_{dist} - V = 0$

$$V_{dist} = \int_0^x -w\,dx = \int_0^x -20\,dx = -20x$$

$$V = (34 - 20x)\,kN \leftarrow$$

$$M = \int_0^x V\,dx$$

$$= (34x - 10x^2)\,kN\cdot m \leftarrow$$



$\Sigma F_y = 0: \quad 34\,kN - 30\,kN - V_{dist} - V = 0$

$$V_{dist} = \int_0^x -20\,dx = -20x$$

$$V = (4 - 20x)\,kN \leftarrow$$

$$M = \int_0^{1.6} V_{old}\,dx + \int_{1.6}^x V\,dx$$

$$= \left[34x - 10x^2\right]_0^{1.6} + \left[4x - 10x^2\right]_{1.6}^x$$

$$= (48 + 4x - 10x^2)\,kN\cdot m \leftarrow$$

Note: Moment integral has been divided since shear (V) is different in each section.

Notice: Both methods give the same answer!

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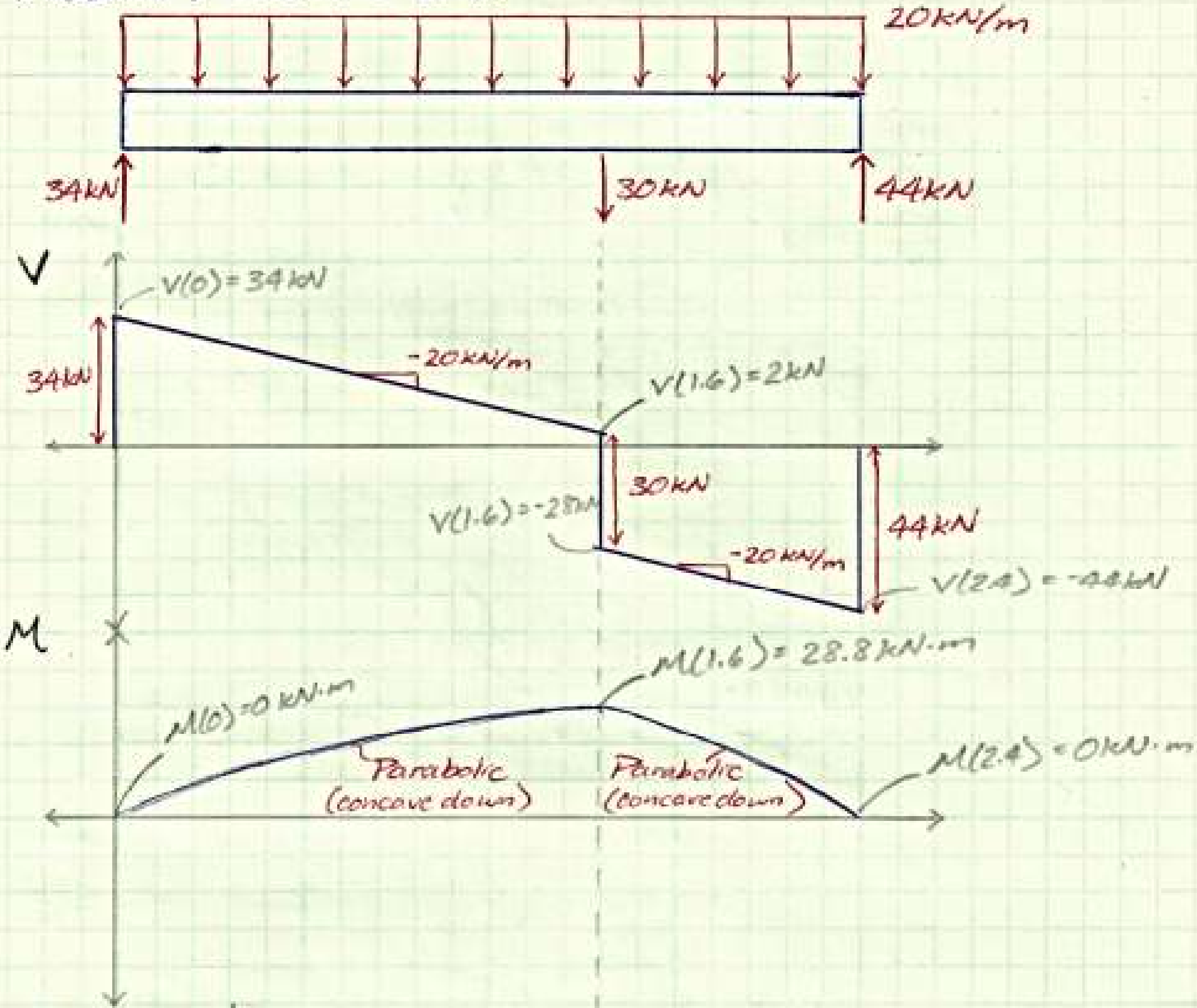
Problem 1 Solutions cont.



Remember!

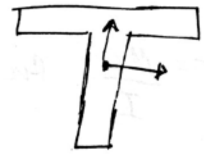
- Distributed (constant) loading => linear shear!
- Both diagrams start and end at 0
- Jumps are the result of concentrated loads (therefore jumps only in shear)

b) Calculate maximum value of bending moment.

From graph, $\boxed{M_{max} = 28.8\,kN\cdot m}$

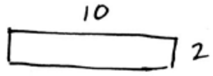
$M(1.6m) = -10(1.6)^2 + 4(1.6) + 48 = \underline{28.8\,kN\cdot m}$

② ⓐ I_z for cross section:



Break into sections and use the parallel axis theorem.

There are <u>tons</u> of ways to separate this cross-section - any will work if applied correctly.

(1) 

$$I_{1z} = \frac{1}{12}bh^3 = \frac{1}{12}(10)(2)^3 = \frac{80}{12} = 6.67$$

(2) 

$$I_{2z} = \frac{1}{12}bh^3 = \frac{1}{12}(2)(8)^3 = 85.33$$

Now, use the parallel axis theorem to shift these to calculate I around the <u>centroid</u>.

$$I_{1z'} = 6.67 + Ar^2 = 6.67 + (2)(10)(9-6.77)^2 = 106.13$$

$$I_{2z'} = 85.33 + Ar^2 = 85.33 + (2)(8)(6.77-4)^2 = 208.09$$

$$I_{TOT} = I_{1z'} + I_{2z'} = 106.13 + 208.81 = \boxed{314.23 \text{ in}^4}$$

<u>note</u>: r is the distance between the centroid of the section and the centroid of the entire cross section.

② ⓑ

$\sigma = \dfrac{Mc}{I}$ for max stress.

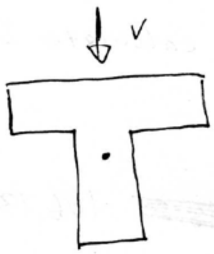
- compression: above neutral axis (centroid)

$$\sigma_{comp} = \frac{-(1000)(10-6.77)}{314.23} = \boxed{-10.3 \text{ psi}}$$

- tension: below

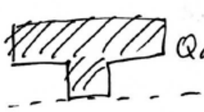
$$\sigma_{tension} = \frac{(1000)(6.77)}{314.23} = \boxed{21.5 \text{ psi}}$$

②ⓒ



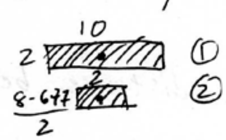
$\tau_{max} = \dfrac{VQ_{max}}{It}$

Q_{max} occurs @ the neutral axis - You can take either Q_{above} or Q_{below}, and will get the same result (slightly off b/c the centroid is given to 2 decimal places).



Q_{above}

$Q_{above} = \Sigma \bar{y}A \Rightarrow$



$\bar{y}_1 = 9-6.77 \quad A_1 = 20$

$\bar{y}_2 = \dfrac{8-6.77}{2} \quad A_2 = (2)\left(\dfrac{1.23}{2}\right)$

note $\bar{y}$ is the distance from neutral axis to the centroid of subsection.

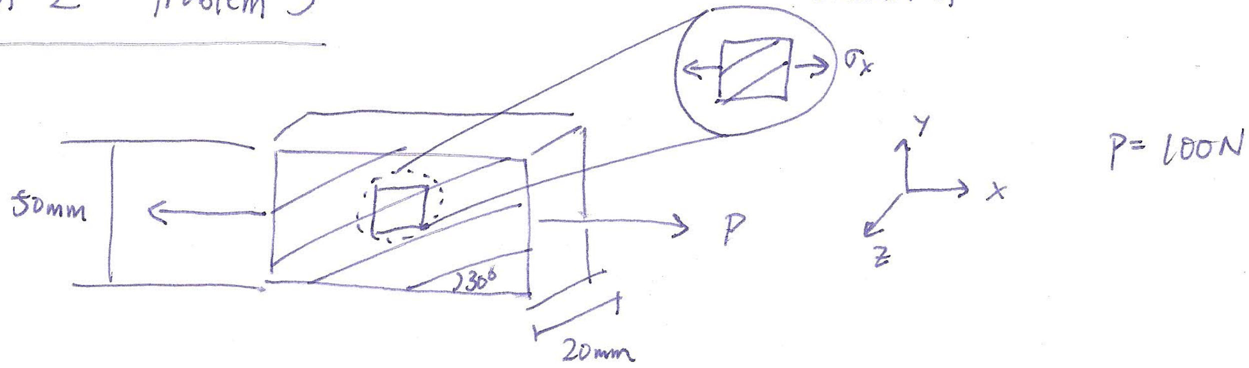
Q_{below}

$$Q_{above} = (9-6.77)(20) + \left(\frac{1.23}{2}\right)(2)\left(\frac{1.23}{2}\right) = 45.36 \text{ in}^3$$

$$Q_{below} = \bar{y}A = \left(\frac{6.77}{2}\right)(6.77)(2) = 45.83.$$

so: $\tau_{max} = \dfrac{VQ_{above}}{It} = \dfrac{(250)(45.36)}{(314.23)(2)} = \boxed{17.32 \text{ psi.}}$

Midterm 2 Problem 3



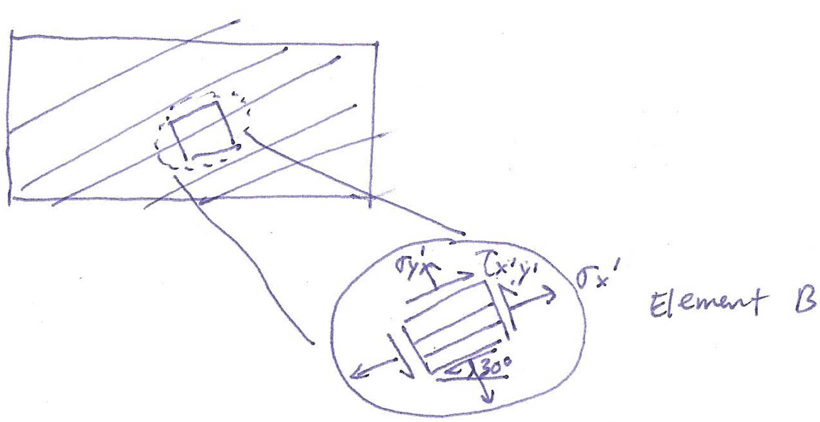
$$\sigma_x = \frac{P}{A_c} = \frac{100\ N}{(0.05m)(0.02m)} = 100000\ Pa = 0.1\ MPa$$

$\sigma_y = 0$

$\tau_{xy} = 0$

a) Normal and shear stresses along a direction parallel to the grain

1) Rotate Element A by 30 degrees so the sides of the element are either parallel or perpendicular to the grain



2) From element B, $\tau_{x'y'}$ and σ_x' are the shear and normal stresses, respectively, along a direction parallel to the grain

From the equation sheet, we get:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$\sigma_x = 0.1\ MPa$ $\quad \sigma_y = 0\ MPa$ $\quad \tau_{xy} = 0\ MPa$
(NOTE: These stresses are from the initial frame)

$\theta = 30°$

$$\sigma_{x'} = \frac{0.1\ MPa}{2} + \frac{0.1\ MPa}{2}\cos[2(30)] + 0 = 0.075\ MPa$$
or
$75000\ Pa$

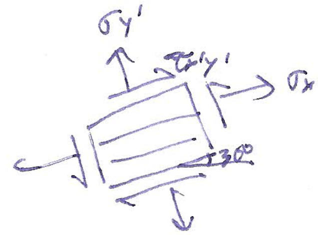
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

$$= -\frac{0.1}{2}\sin(2(30)) = -0.0433\ MPa = -43301\ Pa$$

$$\boxed{\begin{aligned}\sigma_{x'} &= 0.075\ MPa \\ \tau_{x'y'} &= 0.0433\ MPa\end{aligned}}$$

b) Normal and shear stresses along a direction perpendicular to the grain.

1) From element B (redrawn below)



we see that $\sigma_{y'}$ is the normal stress perpendicular to the grain

2) $\tau_{x'y'}$ perpendicular $=$ $\tau_{x'y'}$ parallel $= 0.0433$ MPa

The shear stresses are equivalent to maintain equilibrium.

3) From the equation sheet, we get:

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$= \frac{0.1}{2} - \frac{0.1}{2}\cos(2(30)) - 0 = 0.025 \text{ MPa}$$

or

2500 Pa

$$\boxed{\begin{aligned} \sigma_{y'} &= 0.025 \text{ MPa} \\ \tau_{x'y'} &= 0.0433 \text{ MPa} \end{aligned}}$$