EECS 20N: Structure and Interpretation of Signals and Systems — A MIDTERM 2

Department of Electrical Engineering and Computer Sciences — March 29, 2011

UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ____________________ FIRST Name ____________________

MT2.1 (5 Points) Which lab are you in?

Mon 12-3PM	Mon 3-6PM	Tue 9-12PM	Wed 12-3PM	Thu 3-6PM	Fri 9-12PM

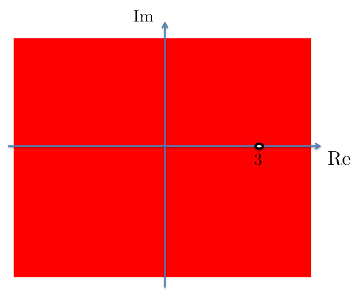
Problem	Points	Your Score
Name, Lab	5	
2	30	
3	15	
4	20	
5	15	
6	30	
Total	**115**	

You may use the blank space below for scratch work. Nothing written below this line on this page will be considered in evaluating your work.

MT2.2 30 Points For each set defined below, provide a well-labeled diagram identifying all the points on the *complex plane* that belong to it. $\mathbb{C}$ refers to the set of complex numbers, $\mathbb{R}$ refers to the set of real numbers.

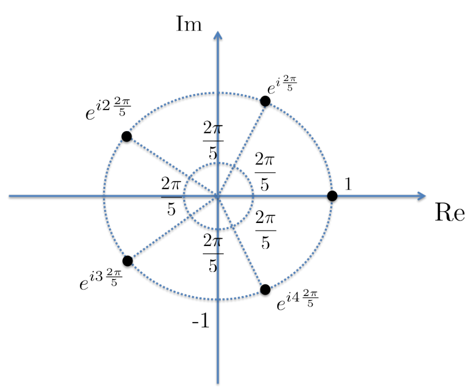
(a) $\{z \in \mathbb{C} \mid \angle((z-3)(z-3)^*) = 0\}$

Solution: We have for all $z \in \mathbb{C} - \{3\}$ that $\angle((z-3)(z-3)^*) = \angle(z-3) + \angle((z-3)^*) = \angle(z-3) - \angle(z-3) = 0$. We accepted solutions including 3 as well as solutions excluding 3 as the angle of $(z-3)(z-3)^*$ is not well defined.



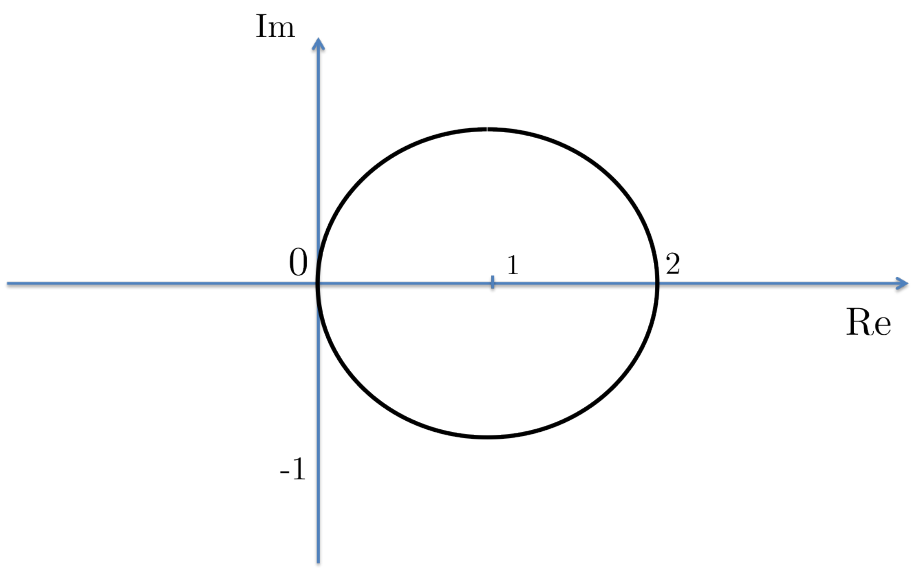
(b) $\{z \in \mathbb{C} \mid z^5 = 1\}$

Solution: $z^5 = e^{i2\pi k}\ \ \forall k \in \mathbb{Z}$ hence we have $z = e^{i\frac{2\pi}{5}k}\ \ \forall k \in \mathbb{Z}$. This gives 5 distinct solutions: $\{1, e^{i\frac{2\pi}{5}}, e^{i2\frac{2\pi}{5}}, e^{i3\frac{2\pi}{5}}, e^{i4\frac{2\pi}{5}}\}$.



(c) $\{z \in \mathbb{C} \mid z = 1 + e^{i2\pi t}, t \in \mathbb{R}\}$

Solution: $\{z \in \mathbb{C} \mid z = e^{i2\pi t}, t \in \mathbb{R}\}$ is the unit circle. The set $\{z \in \mathbb{C} \mid z = 1 + e^{i2\pi t}, t \in \mathbb{R}\}$ is the unit circle shifted by 1.



MT2.3 (15 Points) For each of the following signals, state whether they are periodic or not. If periodic, compute the period p and fundamental frequency ω_0. If not periodic, briefly justify why not.

(a) $x \in [\mathbb{Z} \to \mathbb{R}]$ with $\forall n \in \mathbb{Z}, x(n) = \cos(2\pi\frac{4}{7}n)$.

Solution: The period p is the minimal positive integer such that for all n we have $\cos(2\pi\frac{4}{7}n) = \cos(2\pi\frac{4}{7}(n+p))$. Hence we need to find the minimal positive integer p satisfying: $2\pi\frac{4}{7}n + k2\pi = 2\pi\frac{4}{7}(n+p)$ for some $k \in \mathbb{Z}$, or equivalently: $k2\pi = 2\pi\frac{4}{7}p$, or equivalently: $p = \frac{7}{4}k$. This gives $p = 7$ samples (for choice of $k = 4$).

The fundamental frequency $\omega_0 = \frac{2\pi}{p} = \frac{2\pi}{7}$ radians/sample.

(b) $x \in [\mathbb{R} \to \mathbb{R}]$ with $\forall t \in \mathbb{R}, x(t) = \cos(2\pi 220t) + \sin(2\pi 330t + \frac{\pi}{4})$.

Solution: The period p is the minimal positive real number such that for all $t \in \mathbb{R}$ we have: $\cos(2\pi 220t) + \sin(2\pi 330t + \frac{\pi}{4}) = \cos(2\pi 220(t+p)) + \sin(2\pi 330(t+p) + \frac{\pi}{4})$. Hence we need to find the minimal positive real number p satisfying: $2\pi 220t + 2\pi k = 2\pi 220(t+p)$ and $2\pi 330t + \frac{\pi}{4} + 2\pi l = 2\pi 330(t+p) + \frac{\pi}{4}$ for some integers l,k, or equivalently: $k = 220p$ and $l = 330p$. Hence we need to find integers k,l that find the minimal positive real number $p = \frac{k}{220} = \frac{l}{330}$. This gives $p = \frac{2}{220} = \frac{3}{330} = \frac{1}{110}$.

The fundamental frequency $\omega_0 = \frac{2\pi}{p} = 2\pi 110$ radians/second.

(c) $x \in [\mathbb{R} \to \mathbb{R}]$ with $\forall t \in \mathbb{R}, x(t) = \cos(t) + \sin(2\pi t)$.

Solution: The period p is the minimal positive real number such that for all $t \in \mathbb{R}$ we have: $\cos(t) + \sin(2\pi t) = \cos(t+p) + \sin(2\pi(t+p))$. Hence we need to find the minimal positive real number p satisfying: $t + 2\pi k = t + p$ and $2\pi t + 2\pi l = 2\pi(t+p)$ for some integers l,k, or equivalently: $2\pi k = p$ and $l = p$. Hence we need to find integers k,l that minimize $p = 2\pi k = l$. This means k and l have to satisfy $\frac{l}{k} = 2\pi$. This is impossible as 2π is irrational. Hence there is no period p, and the signal is not periodic.

MT2.4 20 Points (a) Determine the fundamental frequency of and the Fourier coefficients of the periodic signal $x, \forall t \in \mathbb{R}, x(t) = \cos(2\pi t) + \cos(6\pi t)$. Use the Fourier series with complex exponentials.

Recall that for a periodic signal $x \in [\mathbb{R} \to \mathbb{C}]$ with period p, the Fourier series with complex exponentials is of the form $\forall t \in \mathbb{R}, x(t) = \sum_{k=-\infty}^{+\infty} X_k e^{i\frac{2\pi}{p}kt}$.

Solution:

$$\begin{aligned} \forall t \in \mathbb{R} x(t) &= \cos(2\pi t) + \cos(6\pi t) && (1) \\ &= \frac{1}{2}e^{i2\pi t} + \frac{1}{2}e^{-i2\pi t} + \frac{1}{2}e^{i6\pi t} + \frac{1}{2}e^{-i6\pi t}. && (2) \end{aligned}$$

The fundamental frequency $\omega_0 = 2\pi$ radians/second. $X_1 = X_{-1} = X_3 = X_{-3} = \frac{1}{2}$, for all other k we have $X_k = 0$.

(b) Determine the fundamental frequency of and the Fourier coefficients of the periodic signal $x, \forall t \in \mathbb{R}, x(t) = \cos(2\pi t) + \sin(4\pi t + \frac{\pi}{2})$. Use the Fourier series with complex exponentials.

Solution:

$$\begin{aligned} \forall t \in \mathbb{R}, x(t) &= \cos(2\pi t) + \sin(4\pi t + \frac{\pi}{2}) && (3) \\ &= \frac{1}{2}e^{i2\pi t} + \frac{1}{2}e^{-i2\pi t} + \frac{1}{2i}e^{i(4\pi t + \frac{\pi}{2})} - \frac{1}{2i}e^{-i(4\pi t + \frac{\pi}{2})} && (4) \end{aligned}$$

The fundamental frequency $\omega_0 = 2\pi$ radians/second. $X_1 = X_{-1} = \frac{1}{2}$, $X_2 = \frac{e^{i\frac{\pi}{2}}}{2i} = \frac{1}{2}$, and $X_{-2} = -\frac{e^{-i\frac{\pi}{2}}}{2i} = \frac{1}{2}$. For all other k we have $X_k = 0$.

MT2.5 15 Points For a system $S : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ let the output y for an input x be given by:

$$y(n) - \frac{1}{2}y(n-1) = \frac{1}{2}x(n).$$

(a) What is the frequency response $H(\omega)$?

Solution:

$$H(\omega) = \frac{\frac{1}{2}}{1 - \frac{1}{2}e^{-i\omega}}$$

.

(b) What is $y = S(x)$ for $x \in [\mathbb{Z} \to \mathbb{C}]$ with $\forall n \in \mathbb{Z}, x(n) = e^{i\frac{\pi}{4}n} + (-1)^n$?

Solution: We have $\forall n \in \mathbb{Z}$ that

$$x(n) = e^{i\frac{\pi}{4}n} + e^{i\pi n}$$

.

Hence we have for the output y that $\forall n \in \mathbb{Z}$:

$$y(n) = H(\frac{\pi}{4})e^{i\frac{\pi}{4}n} + H(\pi)e^{i\pi n} = \frac{\frac{1}{2}}{1 - \frac{1}{2}e^{-i\frac{\pi}{4}}}e^{i\frac{\pi}{4}n} + \frac{\frac{1}{2}}{1 - \frac{1}{2}e^{-i\pi}}e^{i\pi n}$$

MT2.6 30 Points For each of the following questions, select the strongest true assertion from the list of assertions. For each part, explain your reasoning succinctly, but clearly and convincingly.

(a) A discrete-time system $S : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y,

$$y(n) = \sin(\frac{\pi}{4}n), \quad \forall n \in \mathbb{Z},$$

in response to the input signal x,

$$x(n) = e^{i\frac{\pi}{4}n}, \quad \forall n \in \mathbb{Z}.$$

(i) The system must be LTI.

(ii) The system could be LTI, but does not have to be.

(iii) The system cannot be LTI.

Solution: (iii) The system cannot be LTI.

The input signal is a complex exponential. If the system were LTI, then the output signal has to be a constant (complex) coefficient times the input signal. This is not the case, as the output is the sum of two complex exponentials, one with frequency $\frac{\pi}{4}$ and another one with frequency $\frac{-\pi}{4}$. Hence the system cannot be LTI.

(b) A continuous-time system $S : [\mathbb{R} \to \mathbb{C}] \to [\mathbb{R} \to \mathbb{C}]$ produces the output signal y,

$$y(t) = e^{i(t-10)}, \quad \forall t \in \mathbb{R},$$

in response to the input signal x,

$$x(t) = e^{it} + e^{2it}, \quad \forall t \in \mathbb{R}.$$

(i) The system must be LTI.

(ii) The system could be LTI, but does not have to be.

(iii) The system cannot be LTI.

Solution: (ii) The system could be LTI, but does not have to be.

The input signal is the sum of two complex exponentials. If the system is LTI then the output signal has to be a weighted sum of the same two complex exponentials. This is indeed the case as $e^{i(t-10)} = e^{-i10}e^{it} + 0e^{2it}$. Hence the input-output pair is consistent with the system being LTI. As we only know how the system behaves for one input signal, we cannot conclude it must be LTI.

(c) A continuous-time system $S : [\mathbb{R} \to \mathbb{C}] \to [\mathbb{R} \to \mathbb{C}]$ produces the output signal y,

$$y(t) = \sin(20\pi t + \frac{\pi}{4}) \quad \forall t \in \mathbb{R},$$

in response to the input signal x,

$$x(t) = \cos(2\pi t), \quad \forall t \in \mathbb{R}.$$

(i) The system must be LTI.

(ii) The system could be linear or time-invariant, but not both.

(iii) The system cannot be linear and cannot be time-invariant.

Solution: (ii) The system could be linear or time-invariant, but not both.

The input signal is a sum of two complex exponentials, one with frequency 2π, and one with frequency -2π. The output signal is a sum of two complex exponentials with two different frequencies, 20π and -20π. Hence the system cannot be LTI.

The system could be linear: for example a system that for any input u produces the output signal $u(0)y$ is linear.

The system could be time-invariant: for example a system that for x produces y and for all $\tau \in \mathbb{R}$ produces $D_\tau y$ for input $D_\tau x$, and for all other input signals generates an output that is always zero is time-invariant.