UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Materials Science and Engineering

Fall 2011 | MSE 113 | Professor Ritchie

Mechanical Behavior of Materials

Midterm 2 Solution

Problem 1

1a) Acceptable answers:

- Crack deflection, where the crack path is deflected by either hardened particles or weak interfaces. This occurs in composite structures like wood.
- Microcrack toughening, where microcrack in front of the crack tip acts in compression to reduce the stress intensity in front of the crack. Typical examine, bones.
- Transformation toughening, where the plastic zone material undergoes a phase transformation that results in a positive dilation. This puts the zone in compression, which reduces the stress intensity. Example: Zirconia.
- Crack bridging: when the material is infused with fibers/ligaments that does not crack and ends up behind the crack. It requires additional stress to grow the crack. Typically, bones.

1c) Plastic zone size is given as:

$$r_y = \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$
$$= \frac{1}{2\pi}\left(\frac{50}{50}\right)^2 = 0.16 \text{ in}$$

The LEFM validity criterion:

$$a, b, B, w \geq 15 r_y = 2.39 \text{ in}$$

The only relevant dimensions here are the diameter of the notched and un-notched rods, which both satisfy the LEFM validity criterion. Therefore we can apply LEFM in this case. Our answer will not be conservative unless we specifically apply a Safety Factor in the calculation.

1d) Circumferential notch has a K_I solution given in the stress intensity handout.

$$K_I = \frac{0.932 P \sqrt{D}}{\sqrt{\pi} d^2}$$

Given $D = 12$ in and $d = D/2 = 6$ in. The solution is only good for when $1.2 \leq D/d \leq 2.1$, our $D/d = 2$. Note that the critical flaw size in this case is actually not in the equation, we need to calculate it from the notched diameter.

i) Given $P = 600$ kips and $K_{Ic} = 50 \text{ ksi}\sqrt{\text{in}}$, we can solve for the notched diameter that make the rod go critical:

$$K_I = \frac{0.932P\sqrt{D}}{\sqrt{\pi}d_c^2} = K_{Ic}$$

$$\frac{0.932(600)\sqrt{12}}{\sqrt{\pi}d_c^2} = 50$$

$$d_c = \sqrt{\frac{0.932(600)\sqrt{12}}{\sqrt{\pi}50}} = \boxed{4.68\text{ in}}$$

Therefore the critical flaw size is given as the half difference:

$$a_c = (D-d)/2 = \boxed{3.66\text{ in}}$$

ii) Now the notched diameter is assumed to be $d = 6$ in. The load necessary to cause localize yielding is:

$$P_y = \frac{\pi}{4}d^2\sigma_y$$
$$= \frac{\pi}{4}6^2 50 = 1414.72\text{ kips}$$

The new critical notched diameter:

$$d_c = \sqrt{\frac{0.932(1413.72)\sqrt{12}}{\sqrt{\pi}50}} = \boxed{7.18\text{ in}}$$

The new flaw size:

$$a_c = (D-d)/2 = \boxed{2.41\text{ in}}$$

1e) The current flaw size is:

$$a = (D-d)/2 = 3\text{ in}$$

Given the operating loading configuration, the critical flaw size is larger than the current flaw size. The rod is expected to survive.

1f) The new toughness will reduce the critical notched diameter, therefore we need to recompute the value:

$$d_c = \sqrt{\frac{0.932(600)\sqrt{12}}{\sqrt{\pi}10}} = 10.45\text{ in}$$

The critical notched diameter is 10.45 inches which is larger than the current notched diameter of 6 inches. The material is expected to fail in this new condition.

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Problem 2

2a) Some acceptable creep deformation mechanisms:

- Dislocation glide
- Dislocation creep
- Diffusional creep
- Grain boundary sliding

2c) Creep exponent can be found with the following:

$$m = \left.\frac{\Delta \log_{10} \dot{\varepsilon}_{ss}}{\Delta \log_{10} \sigma_{11}}\right|_T$$

Picking the following data points, m can be easily tabulated to be $\boxed{5.68}$.

T (°F)	σ_{11} (ksi)	ε_{ss} (hr^{-1})	$\Delta \log_{10} \dot{\varepsilon}_{ss}$	$\Delta \log_{10} \sigma_{11}$	m
1300	20	3.31×10^{-7}			
1300	30	3.31×10^{-6}	5.68	1.0	5.68
1400	20	1.45×10^{-5}			
1400	30	1.45×10^{-4}	5.68	1.0	5.68
1500	20	1.8×10^{-4}			
1500	30	1.8×10^{-3}	5.68	1.0	5.68

2d) H is given below, but since the graph is not plotted in terms of inverse temperature and strain rate, we must fix the stress and tabulate our own inverse temperature strain rate data. Note that the temperature is in °F but must be converted to Rankins.

$$H = -\left.\frac{2.303k\Delta \log_{10} \dot{\varepsilon}_{ss}}{\Delta(1/T)}\right|_{\sigma_{11}}$$

T (°F)	$1/T$ (R^{-1})	$\dot{\varepsilon}_{ss}$ (hr^{-1})	H (in · lbf/R)
1300	5.68×10^{-4}	3.31×10^{-7}	7.38×10^{-18}
1400	5.38×10^{-4}	1.45×10^{-5}	7.38×10^{-18}
1500	5.10×10^{-4}	1.80×10^{-4}	7.38×10^{-18}

H can be found either by plotting inverse temperature and strain rate and find the slope, or just use the equations above, either way it is about $\boxed{7.38 \times 10^{-18} \text{ in} \cdot \text{lbf/R}}$.

2e) The only missing unknown is σ_0, this can be back solved by picking any point:

$$\sigma_0 = \sigma_{11}\left(\frac{1}{\dot{\varepsilon}_{ss}}\right)^{\frac{1}{m}} \exp\left(-\frac{H}{mkT}\right)$$
$$= 5.25 \text{ psi}$$

The service requirement is 3% for 40,000 hours. We know that creep follows the following strain behavior:

$$\varepsilon_{total} = \varepsilon_p + \varepsilon_e + \varepsilon_{ss} + \varepsilon_t$$

Primary creep is given:

$$\varepsilon_p = 0.005$$

Elastic creep can be calculated:

$$\varepsilon_e = \frac{\sigma_{11}}{E} = \frac{50}{27000} = 0.002$$

Elastic strain is small, it can be ignored, but for safety sake, it will be included.

$$\varepsilon_{ss} = \dot{\varepsilon}_{ss} t$$
$$\varepsilon_t = 0$$

Thus, we need to solve for a maximum allowed steady state creep rate given the design:

$$\dot{\varepsilon}_{ss} = \frac{\varepsilon_{total} - \varepsilon_p - \varepsilon_e}{t} = \frac{0.03 - 0.005 - 0.002}{40000} = 5.79 \times 10^{-7} \text{ hr}^{-1}$$

Now to compute the actual steady state strain rate given the operating conditions:

$$\dot{\varepsilon}_{ss} = \left(\frac{50000}{5.25}\right)^{5.68} \exp\left[-\frac{7.38 \times 10^{-18}}{6.79 \times 10^{-23}(1350 + 460)}\right] = 3.32 \times 10^{-4}$$

The actual steady state strain rate is way too high. The <u>blade will not survive</u>.

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Problem 3

3a) using Frank's rule, the only energetically favorable dislocation disassociation is the ones that:

$$\boldsymbol{b}^2 \geq \boldsymbol{b}_1^2 + \boldsymbol{b}_2^2$$

First check to see if this disassociation even make sense:

$$\boldsymbol{b}_1 + \boldsymbol{b}_2 = \frac{a}{6}[121] + \frac{a}{6}[21\bar{1}]$$
$$= \frac{a}{6}[330] = \frac{a}{2}[110] = \boldsymbol{b}$$

Now for Frank's rule:

$$\boldsymbol{b}^2 = \frac{a^2}{2^2}(1^2 + 1^2) = \frac{a^2}{2}$$

$$\boldsymbol{b}_1^2 + \boldsymbol{b}_2^2 = \frac{a^2}{6^2}(1^2 + 2^2 + 1^2) + \frac{a^2}{6^2}(2^2 + 1^2 + 1^2) = \frac{a^2}{6} + \frac{a^2}{6} = \frac{a^2}{3}$$

$$\boldsymbol{b}^2 \geq \boldsymbol{b}_1^2 + \boldsymbol{b}_2^2$$
$$\frac{a^2}{2} \geq \frac{a^2}{3}$$

So yes, the above disassociation is not only possible but energetically favorable.

3b) In between the two partial dislocations is stacking flaw. This occurs when the stacking order between nearest atomic planes is disrupted. The perfect dislocation keeps the stacking order intact, however the first partial dislocation interrupts the order, but the second one restores the order. In between is material that has normal stacking order again.

3c) Burger's vector for the two partials are $a/\sqrt{6}$, so:

$$w = \frac{45 \times 10^9 \cdot (3.68 \times 10^{-10})^2}{6 \cdot 2\pi(14 \times 10^{-3})} = \boxed{11.5\text{ nm}}$$

The width is 11.5 nanometers, which is about some 30 times larger than the lattice spacing. This is acceptable of a figure.

3d) in the formula width is inversely related to the stacking fault energy. If the energy is high, the stacking fault width is small. Conversely, low stacking fault energy means high stacking fault width. Stacking fault inhibits dislocation movement. Because when a dislocation has to cross-slip onto another plane, it must re-associate (i.e. remove the stacking fault area) and then glide followed by disassociation again. Therefore a wide stacking fault area, or low stacking fault energy will necessarily cause the material to have higher yielding strength. Conversely, a high stacking fault energy, which has low stacking fault width, will be easier for the dislocation to be mobile and therefore tends to be softer.