UNIVERSITY OF CALIFORNIA, BERKELEY   College of Engineering

Statics and Mechanics of Materials (ME85/C30)

Mid-term Examination (Spring 2011)

Problem 1.

A pole is fixed (clamped) at point B and tethered by a rope, as shown in Figure 1. The tension in the rope is $100N$. Ignore the weight of the pole. If the pole is in equilibrium, determine the reactions (both the force and the moment) acting at point B. Hints:

$$\mathbf{M}_B = \mathbf{r}_{BA} \times \mathbf{F}_A \qquad (1)$$

$$\mathbf{F}_A = \|\mathbf{F}_A\| \frac{\mathbf{r}_{AC}}{\|\mathbf{r}_{AC}\|} \qquad (2)$$

where $\|\mathbf{F}_A\|$ is the magnitude of $\mathbf{F}_A$, and $\mathbf{r}_{BA} = \mathbf{r}_A - \mathbf{r}_B$, $\mathbf{r}_{AC} = \mathbf{r}_C - \mathbf{r}_A$. For vector cross product,

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad (3)$$

and the equilibrium conditions in three-dimensional space are,

$$\sum \mathbf{F}_i = 0, \quad \text{and} \quad \sum \mathbf{M}_B = 0 .$$
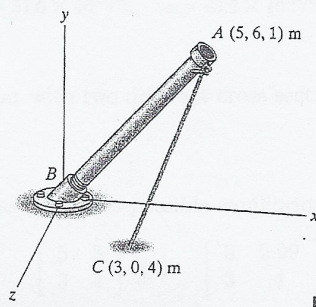
(30 points)

Figure 1: A pole and a rope

Problem 2. A planar circular three-hinge arch consists of two segments as shown in Fig. 2. Determine the reaction forces at A and B caused by the application of a vertical force P at C.
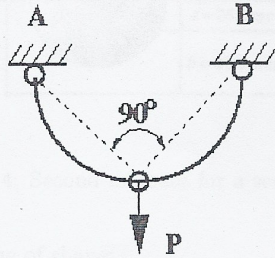

Figure 2: A three-hinge arch truss system

(25 points)

Problem 3.

For a two-bar system shown in the figure, determine (a) the compressive force in the bars after a temperature rise of $96^o C$, and (b) stresses in each bar.

Hints:

First write down the displacement compatibility condition. Thermal strain: $\epsilon_T = \alpha \Delta T$; $\Delta = \dfrac{L}{EA} P$.

(25 points)

Bronze
$A = 1500 \text{ mm}^2$
$E = 105 \text{ GPa}$
$\alpha = 21.6 \times 10^{-6}/^{\circ}C$

Aluminum
$A = 1800 \text{ mm}^2$
$E = 73 \text{ GPa}$
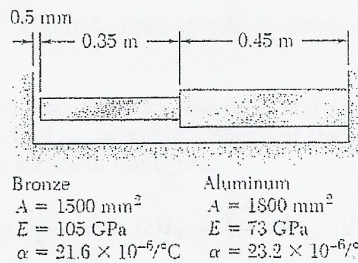$\alpha = 23.2 \times 10^{-6}/^{\circ}C$

Figure 3: An axially deformed bar with two different cross sections and two different materials.

Problem 4.

A. Which of the following stress states are possible equilibrium stress state ?

$(a): \begin{bmatrix} 5 & 3 \\ 3.01 & 4 \end{bmatrix}$, $(b): \begin{bmatrix} 5 & 3 \\ 3 & 4 \end{bmatrix}$, $(c): \begin{bmatrix} 0 & 2.99 \\ 3 & 4 \end{bmatrix}$, $(d): \begin{bmatrix} -5 & 3 \\ 3 & -3 \end{bmatrix}$,

B. Which of the following statements are incorrect: The shear strain
(a) is a relative elongation;
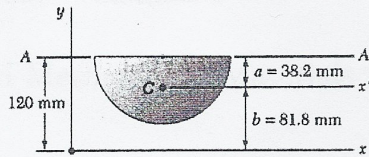(b) is the change of angle;
(c) has nothing to do with temperature;

Figure 4: Second moment for a semi-circle.

(d) has something to do with change of shape;
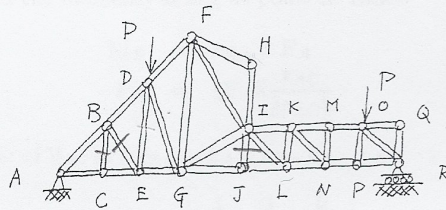(e) has something to do with change of volume.

Figure 5: Find zero force members.

C. The semi-circle shown in Fig. 4 has the radius r, and area $A = \pi r^2/2$, and the moment of inertia with respect to axis AA', $I_{AA'} = \pi r^4/8$. Which of the following is the moment of inertia w.r.t. x-axis for the semi-circle shown in the figure:

$(a) I_x = \dfrac{\pi r^4}{8} + (a+b)^2 A; \quad (b) I_x = \dfrac{\pi r^4}{8} + a^2 A; \quad (c) I_x = \dfrac{\pi r^4}{8} + b^2 A; \quad (d) I_x = \dfrac{\pi r^4}{8} - a^2 A + b^2 A; \quad (e) I_x = \dfrac{\pi r^4}{8} + (a^2 + b^2) A.$

Hint: Parallel axis theorem:

$$I_x = I_{Cx} + d^2 A$$

(D) For the structure shown in Fig. 5, which of the following members are not the zero-force member ?

$(a) CE; \quad (b) BD; \quad (c) OQ; \quad (d) DE; \quad (e) OP; \quad (f) IJ.$

(20 points)