

Problem 1. The impact between the block  $A$  and pan  $B$  must be treated separately; the solution is therefore divided into three parts.

Part 1. Let  $v_A$  be the velocity with which the block  $A$  hits the pan  $B$ . From the conservation of energy,

$$\begin{aligned} \Delta T + \Delta V_g &= 0 \\ \Rightarrow m_A gh &= \frac{1}{2} m_A v_A^2 \\ \Rightarrow v_A &= 6.26 \text{ m/s} \end{aligned} \quad \downarrow$$

Part 2. For the impact between block  $A$  and pan  $B$ , linear momentum is conserved. Let  $v$  be the common velocity of the block and pan after impact. Then

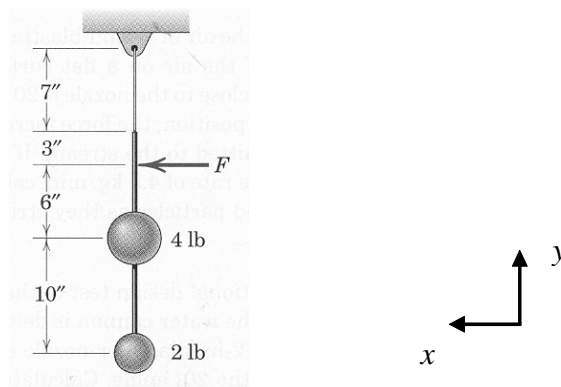
$$\begin{aligned} m_A v_A + m_B v_B &= (m_A + m_B) v \\ \Rightarrow 30(6.26) &= (30 + 10)v \\ \Rightarrow v &= 4.70 \text{ m/s} \end{aligned} \quad \downarrow$$

Part 3. Denote by  $x$  the maximum deflection of the pan, measured from its static equilibrium position. Initially the spring supports the weight of the pan, thus the spring has a static compression of  $\delta = m_B g / k = 4.91 \times 10^{-3} \text{ m}$ . When the pan reaches the position of maximum deflection, the spring has a total compression of  $x + \delta$ . From the conservation of energy,

$$\begin{aligned} \Delta T + \Delta V_g + \Delta V_e &= 0 \\ \Rightarrow \frac{1}{2} (m_A + m_B) v^2 + \frac{1}{2} k \delta^2 &= -(m_A + m_B) g x + \frac{1}{2} k (x + \delta)^2 \\ \Rightarrow kx^2 + 2(k\delta - 392)x - 884 &= 0 \\ \Rightarrow x &= 0.225 \text{ or } -0.196 \end{aligned}$$

Thus the maximum deflection of the pan is  $x = 0.225 \text{ m}$

Problem 2.



Let  $G$  be the center of mass of the system. Upon application of the horizontal force  $F$ ,  $G$  will move with an acceleration  $a_G$  in the direction of  $F$ . From the generalized equation of motion,

$$\begin{aligned} \sum F_x &= m a_G \\ \Rightarrow 12 &= \left( \frac{4 + 2}{32.2} \right) a_G \end{aligned}$$

$$\Rightarrow a_G = 64.4 \text{ ft/sec}^2 \quad \longleftarrow$$

In order to determine the angular momentum of the system about  $G$ , it is necessary to find its position. Let  $G$  be located at a distance  $d$  from the 2-lb ball along the rigid rod. By moment balance,

$$4(10 - d) = 2d$$

$$\Rightarrow d = \frac{20}{3} = 6.67 \text{ in}$$

Let the angular position of the rigid rod be specified by an angle  $\theta$  measured from any convenient fixed direction. Each ball moves in a circle relative to  $G$ . For the entire system,

$$H_G = \sum \rho_i (m_i \dot{\rho}_i) = \sum m_i \rho_i^2 \dot{\theta}$$

$$= \frac{4}{32.2} \left( \frac{3.33}{12} \right)^2 \dot{\theta} + \frac{2}{32.2} \left( \frac{6.67}{12} \right)^2 \dot{\theta} = 0.0288 \dot{\theta} \text{ lb-ft-sec}$$

With respect to the mass center  $G$ ,

$$\sum M_G = \dot{H}_G$$

$$\Rightarrow 12 \left( \frac{6 + 3.33}{12} \right) = 0.0288 \ddot{\theta}$$

$$\Rightarrow \ddot{\theta} = 325 \text{ rad/sec}^2 \quad \curvearrowright$$

Problem 3.

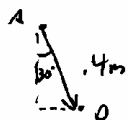
(a)  $\mathbf{V}_a = 0.5\mathbf{i}$  m/s . Because the engine rests on the driver axles, the engine must share the same velocity as the center of the axles: point A on the driver.

(b) 
$$\vec{V}_S = \vec{V}_A + \vec{\omega}_{dr} \times \vec{r}_{S/A} = 0.5\hat{i} \text{ m/s} + (-2.2\hat{k} \frac{\text{rad}}{\text{s}}) \times (-0.95\hat{j} \text{ m})$$

$$\boxed{\vec{V}_S = -1.59\hat{i} \text{ m/s}}$$

This velocity proves that the driver is slipping on the rail. Otherwise  $\mathbf{V}_s$  would be equal to zero.

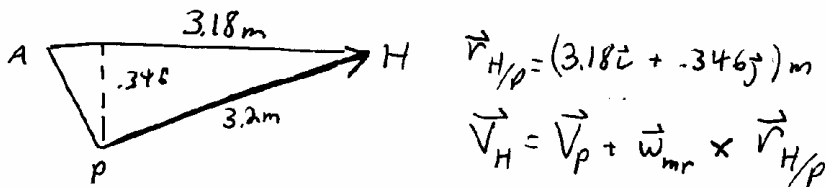
(c)



$\vec{r}_{P/A} = (.2\hat{i} - .346\hat{j}) \text{ m}$ ,  $\vec{V}_P = \vec{V}_A + \vec{\omega}_{dr} \times \vec{r}_{P/A}$

$$\vec{V}_P = 0.5\hat{i} \frac{\text{m}}{\text{s}} + (-2.2\hat{k} \frac{\text{rad}}{\text{s}}) \times (.2\hat{i} - .346\hat{j}) \text{ m}$$

$$\vec{V}_P = (-.261\hat{i} - .44\hat{j}) \frac{\text{m}}{\text{s}}$$



$$V_H\hat{i} = (-.261\hat{i} - .44\hat{j}) \frac{\text{m}}{\text{s}} + \dot{\theta}_{mr}\hat{k} \times (3.18\hat{i} + .346\hat{j}) \text{ m}$$

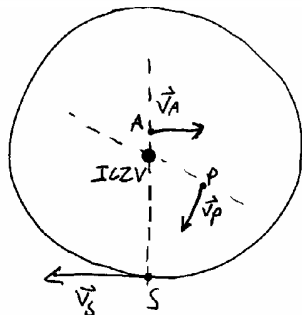
j component:  $0 = -.44 \frac{\text{m}}{\text{s}} + (3.18 \text{ m})(\dot{\theta}_{mr})$

$$\dot{\theta}_{mr} = .138 \frac{\text{rad}}{\text{s}}$$

i component:  $V_H = -.261 \frac{\text{m}}{\text{s}} - (.346 \text{ m})\dot{\theta}_{mr}$

$$\boxed{\vec{V}_H = -.309\hat{i} \text{ m/s}}$$

(d)



(e) The angular velocity of the Side Rod is always zero because the drivers all rotate at the same speed and the Side Rod only translates and does not rotate.