Physics 137B: Midterm 1 September 27, 2011

Problem 1: Variational Method (5 points)

Use a Gaussian trial function to obtain the lowest upper bound you can on the ground state energy of the linear potential $V(x) = \alpha|x|$.

Hint: The following integrals may be useful:

$$\begin{aligned} \int_0^\infty dx e^{-yx^2} x^2 &= \frac{\sqrt{\pi}}{4} y^{-\frac{3}{2}} \\ \int_0^\infty dx e^{-yx^2} &= \frac{\sqrt{\pi}}{2\sqrt{y}} \end{aligned} \tag{1}$$

Problem 2: WKB Approximation (5 points)

For spherically symmetrical potentials we can apply the WKB approximation to the radial part. In the case $l = 0$ the answer is

$$\int_0^{r_0} p(r) dr = (n - 1/4)\pi\hbar, \tag{2}$$

where r_0 is the turning point (in effect, we treat $r = 0$ as an infinite wall). Exploit this formula to estimate the allowed energies of a particle in the logarithmic potential

$$V(r) = V_0 \ln(r/a) \tag{3}$$

(for constants V_0 and a). Treat only the case $l = 0$. Show that the spacing between the levels is independent of mass.

Hint: The following integral may be useful:

$$\int_0^{r_0} \sqrt{\log \frac{r_0}{r}} dr = r_0 \frac{\sqrt{\pi}}{2} \tag{4}$$

Problem 3: Rotating Rigid Body (7 points)

Consider an electrically neutral body that can rotate about a fixed axis, with moment of inertia I. The Hamiltonian for the system is

$$H_0 = \frac{L^2}{2I}, \tag{5}$$

where L is the angular momentum.

(a) What are the possible energy values and what is the degeneracy for each energy eigenvalue?

(b) Now suppose that an electron is embedded in the rotating body at some fixed position off the axis, and an equal positive charge is placed on the axis. Correspondingly, we add a new term to our Hamiltonian, of the form
$H' = \lambda S \cdot L$
where λ is small and S is the electron spin operator. Use first order perturbation theory to determine the energies and degeneracies of the $l = 0$ and $l = 1$ eigenstates.

Problem 4: Interaction with a magnetic field (8 points)

Consider a spin-1/2 particle with gyromagnetic ratio γ in a magnetic field $\vec{B} = B'\hat{x} + B_0\hat{z}$, where $\hat{x}$ and $\hat{z}$ are mutually orthogonal vectors of unit length and $B_0 \gg B'$. Treating B' as a perturbation, $H' = -\gamma S_x B'$ and $H_0 = -\gamma S_z B_{0,z}$, calculate the first- and second-order shifts in energy and first-order shift in wave function for the ground state.