Midterm #2 Solutions
Physics 7C Fall 2011

1. (a) (*3 points*) Looking at Figure 1, we see that $\sin\theta = \frac{\Delta\ell}{d} \implies \Delta\ell = d\sin\theta$

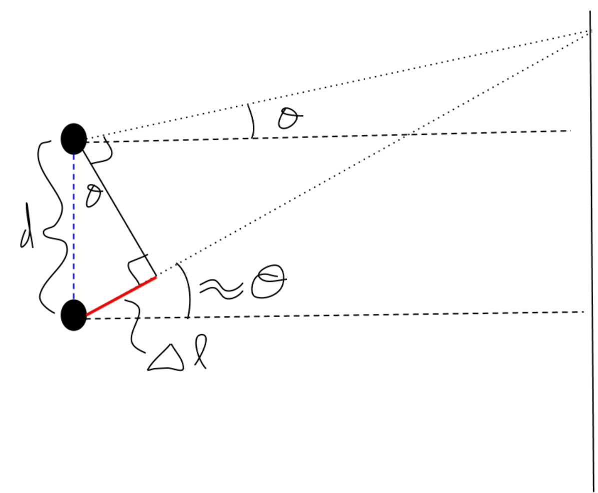

Figure 1: Figure for problem 1(a)

   (b) (*3 points*) The path length difference $\Delta\ell = d\sin\theta$ leads to a phase difference (between neighboring antennae) of $\delta = k\Delta\ell = \frac{2\pi}{\lambda}d\sin\theta$. Thus we are interfering 4 waves, and the total electric field is given by

$$E_{tot} = E_0\cos(\omega t) + E_0\cos(\omega t + \delta) + E_0\cos(\omega t + 2\delta) + E_0\cos(\omega t + 3\delta)$$

   The corresponding phasor diagram is shown in Figure 2 (E_0 and E_{tot} in the figure label the lengths of the phasors).

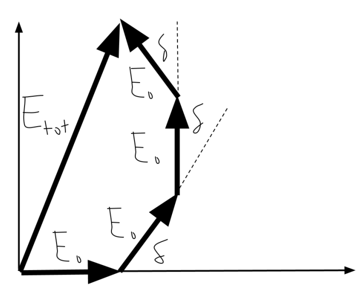

Figure 2: Figure for problem 1(b)

   (c) (*2 points*) $\theta = 0 \implies \delta = 0$. Looking at Figure 3, we see that $E(\theta = 0) = 4E_0$

   (d) (*2 points*) In order to achieve the phasor diagram of Figure 4 (which is the first time the phasors sum to 0, hence the first minimum), we need $\delta = \pi/2$. Thus $\pi/2 = \frac{2\pi}{\lambda}d\sin\theta \implies d\sin\theta = \lambda/4 \implies \theta = \sin^{-1}(\frac{\lambda}{4d})$.

2. (a) (*5 points*) iii

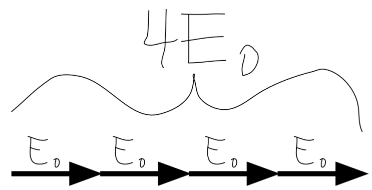

Figure 3: Figure for problem 1(c)

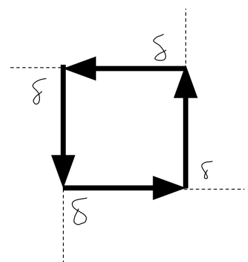

Figure 4: Figure for problem 1(d)

(b) (*5 points*) i

3. (a) (*3 points*)

$$(\Delta s)^2_{AB} = (3 \times 10^8 m/s \times 1.5 \times 10^{-8} s)^2 = 20.25 m^2$$
$$(\Delta s)^2_{AC} = (3 \times 10^8 m/s \times 3.1 \times 10^{-8} s)^2 - (5m)^2 = 61.5 m^2$$
$$(\Delta s)^2_{BC} = (3 \times 10^8 m/s \times 1.6 \times 10^{-8} s)^2 - (5m)^2 = -1.96 m^2$$

(b) (*3 points*) The time taken for the train to pass the platform to an observer on the train is the proper time between events A and C. So $\tau_{AC} = \sqrt{(\Delta s)^2_{AC}}/c = \sqrt{61.5 m^2}/c = 26.1 ns$.

(c) (*4 points*) If an observer sees A and B occur simultaneously, then the spacetime interval for that observer would satisfy $(\Delta s)^2_{AB} = -(\Delta x)^2_{AB} \leq 0$. But this contradicts the fact that $(\Delta s)^2_{AB} = 20.25 m^2 > 0$. So there is no frame where A and B are simultaneous.

4. (a) (*3 points*) See Figure 5.

(b) (*3 points*) Since the speeds of Spaceship A and B are the same, the meeting occurs halfway betweeen Earth and the star. A one-way trip to or from the star by Spaceship A takes $18ly/0.6c = 30$ years in Earth's frame. So Spaceship B leaves Earth 30 years after Spaceship A and meets ship A another 15 years (half the time to the star) after that. Thus the total time passed is 45 years.

(c) (*4 points*) A person's age is determined by how long they live. So we want to calculate the proper time along each worldline, up to the point of meeting. For the passenger of Spaceship A, we have

$$\tau_A = (\sqrt{30^2 - 18^2} + 0.5\sqrt{30^2 - 18^2}) \text{ years} = 36 \text{ years}$$

For a passenger of spaceship B we have

$$\tau_B = 30 \text{ years} + \sqrt{15^2 - 9^2} \text{ years} = 42 \text{ years}$$

Thus the passenger of ship A is 56 years old, and the passengers of ship B are 62 years old just after the transfer.

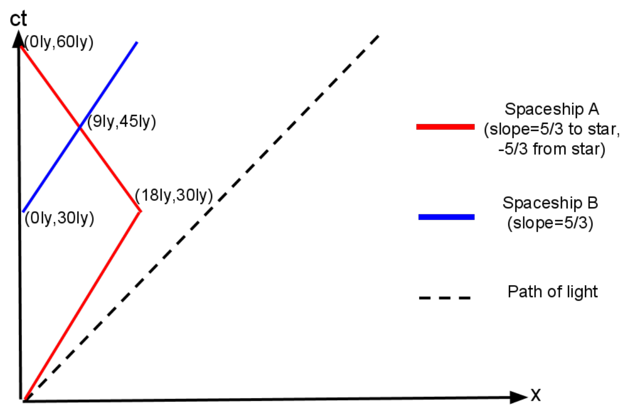

Figure 5: Figure for problem 4(a)