Midterm #2
Physics 7C Fall 2011

Each problem is worth 10 points, for a total of 40 points

Write neatly. Show all work. Explain your solutions clearly. Number all problems. Start each problem on a new page. If you are asked to substitute a result from a previous part, which you were not able to solve, please clearly define a variable to stand in for your would-be solution to the previous part. *Read all questions carefully before answering.*

1. Consider an array of 4 evenly spaced antennae, all lying on one line, with distance d between each one (see Figure 1). All antennae act as point sources, and emit radio waves (wavelength λ) in phase with one another. Very far away there is a long line of radio detectors (essentially a screen) parallel to the line of the emitting antennae. This setup is identical to 4-slit interference. Take the amplitude of radio waves from any one antenna in the array to be E_0 (all antennae emit waves of the same amplitude).

   (a) As usual, we will be labeling the position along the detector line by the angle θ with respect to the normal to the line of the antennae. Show that the path length difference between neighboring antennae to a point on the detector line labeled by θ is $d \sin\theta$.

   (b) Accurately draw a phasor diagram (properly labeling all angles and lengths) for the radio waves reaching a location along the detector line labeled by θ. Ignore the phasor rotation with respect to time. Draw the first phasor starting at the origin and pointing along the x-axis of your diagram. Start the next phasor at the tip of the first one and so on. Do not confuse the angle between phasors with the position on the screen (θ).

   (c) What is the amplitude of the radio wave at the center of the detector line ($\theta = 0$)? Draw the corresponding phasor diagram.

   (d) Find the angle θ corresponding to the first interference minimum. Draw the corresponding phasor diagram.

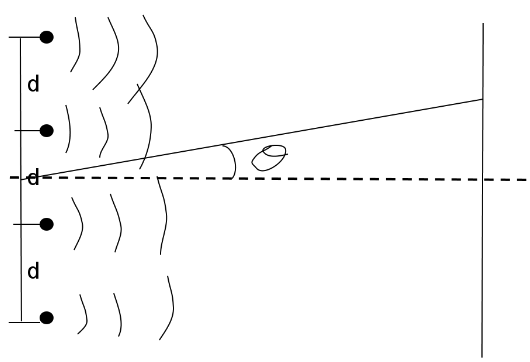

Figure 1: Figure for Problem 1

2. *Multiple Choice*

   (a) Which of the following is the same in all inertial frames?

      i. Momentum of a particle

      ii. Length of an object

      iii. Rest energy of a particle, $E_{rest} = mc^2$

      iv. The order in which two spacelike events occur

   (b) A particle has energy E_0 and momentum $\vec{p} = p_x\hat{x} + p_y\hat{y} + p_z\hat{z}$ in frame S. What is its energy in a frame S', moving with velocity $\vec{v} = v\hat{x}$ with respect to S?

      i. $E' = \frac{E_0 - vp_x}{\sqrt{1-v^2/c^2}}$

      ii. $E' = E_0\sqrt{1 - v^2/c^2}$

      iii. $E' = \frac{E_0 - v|\vec{p}|}{\sqrt{1-v^2/c^2}}$

      iv. $E' = \frac{p_x c - vE_0/c}{\sqrt{1-v^2/c^2}}$

3. A certain train station has a length of 5m. A train speeds by without stopping. Consider the following three spacetime events:

   - Event A: The front of the train enters the station.
   - Event B: The back of the train enters the station.
   - Event C: The front of the train leaves the station.

   For an observer on the station, the time between events A and B is 1.5×10^{-8}s while the time between events A and C is 3.1×10^{-8}s. The speed of light is $c = 3 \times 10^8$m/s.

   (a) Find the spacetime intervals between A and B, A and C, and B and C.

   (b) From the point of view of a passenger on the train, how long did it take for the front of the train to pass through the station?

   (c) Show that there does not exist a frame for which events A and B are simultaneous.

4. Spaceship A leaves Earth and heads for a star 18 lightyears (1.7×10^{17}m) away at speed $v = 0.6c$ (relative to Earth), turns around and heads back to Earth at the same speed. Spaceship B leaves Earth at the instant when spaceship A arrives at the star (simultaneous in Earth's frame). Spaceship B also heads for the star at speed $v = 0.6c$ relative to Earth, but it will not turn around. All passengers of ship A were 20 years old when they boarded the ship, and all of them had twins that stayed behind on Earth. Those twins then became the passengers of ship B when it finally departed Earth. The speed of light is $c = 3 \times 10^8$m/s.

   (a) Draw an accurate spacetime diagram showing the paths of the two ships as seen in Earth's reference frame.

   (b) When the two spaceships meet, how much time has passed on Earth since the departure of spaceship A?

   (c) When the two spaceships meet, a passenger on ship A transfers to ship B. How old are the passengers of ship B when this happens, and how old is the new passenger?