NE101 Mid Term #1

(1) For parts a, b, c need to conserve electric charge, numbers of neutrons, protons

(a) $${}^{7}_{3}Li_{4} + {}^{1}_{1}H_{0} \rightarrow \underline{\underline{{}^{4}_{2}He_{2}}} + {}^{4}_{2}He_{2}$$

(b) $${}^{22}_{11}Na_{11} \longrightarrow \underline{\underline{{}^{22}_{10}Ne_{12}}} + e^{+} + \nu_e$$

(c) $${}^{233}_{92}U_{141} + {}^{1}_{0}n_{1} \rightarrow {}^{99}_{40}Zr_{59} + {}^{133}_{52}Te_{81} + \underline{\underline{2n}}$$

(d) $${}^{235}_{92}U_{143} \longrightarrow \longrightarrow \longrightarrow \text{eventually } {}^{207}_{82}Pb_{125}$$

$$\Delta A = 235 - 207 = 28$$

$\therefore$ To change A by 28 requires

$\underline{\underline{7\ \alpha}}$ decays ($\Delta A = 4/\alpha$ decay)

7 α decays would also produce $\Delta Z = 14$

($\Delta Z = 2/\alpha$ decay)

But $\Delta Z (U \rightarrow Pb) = 10$

$\therefore$ Need $\underline{\underline{4\ \beta^{-} \text{ decays}}}$ to bring total

ΔZ back to 10

② ⓐ $M(^{55}Cr) = 24 \times M(^1H) + 31 \times M_n - B(^{55}Cr)/c^2$

$M(^{55}Mn) = 25 \times M(^1H) + 30\, M_n - B(^{55}Mn)/c^2$

$\therefore\ M(^{55}Cr) - M(^{55}Mn) = M_n - M(^1H) + \dfrac{B(^{55}Mn) - B(^{55}Cr)}{c^2}$

$A = 55$ for both ^{55}Cr and ^{55}Mn

$\therefore$ only need to consider Coulomb and Symmetry terms in binding energy formula

$B'(^{55}Mn) = -\dfrac{a_c (25)(24)}{55^{1/3}} - \dfrac{a_{sym} (5)^2}{55} = -124.05$ MeV

$B'(^{55}Cr) = -a_c \dfrac{(24)(23)}{55^{1/3}} - \dfrac{a_{sym} (7)^2}{55} = -125.00$ MeV

$\therefore\ M(^{55}Cr) - M(^{55}Mn) = (1.00866501 - 1.007825)u + 0.95 \dfrac{MeV}{c^2}$

$\Delta E \quad = \Delta M_0 c^2$

$\therefore\ \boxed{\Delta E = +1.732 \text{ MeV}}$

ⓑ $^{55}_{24}Cr_{31}$ is more massive than $^{55}_{25}Mn_{30}$

$\therefore$ $\boxed{\text{Expect } ^{55}Cr \xrightarrow{\beta^-} {}^{55}Mn}$

also has more neutrons than ^{55}Mn

$\therefore$ moves down mass parabola via β^- decay

(3) Production rate $= R = \phi \sigma n$

(a) $\therefore R_{56Mn} = (1\times10^{13}/cm^2/sec)\left(\frac{5\text{ grams}}{55\text{ grams/mole}}\times 6.02\times10^{23}\frac{\text{atoms}}{\text{mole}}\right)1.3\times10^{-26}\,cm^2$

$= 7.1\times10^{9}/sec$

$R_{54Mn} = 5.5\times10^{7}/sec$

$\lambda_{54Mn} = \frac{\ln 2}{t_{1/2\,54Mn}} = 2.57\times10^{-8}/s$

$N(t) = \frac{R}{\lambda}(1-e^{-\lambda t})$

$\lambda_{56Mn} = 7.7\times10^{-5}/sec$

$\therefore A(t) = R(1-e^{-\lambda t})$

In our case $t = 1$ hour

Since $t_{1/2}(^{54}Mn) = 312$ days, $e^{-\lambda t} \approx 1-\lambda t$

$\therefore A(^{54}Mn) = \lambda_{54Mn} R_{54Mn}(3600\text{ sec}) = \underline{\underline{5100/sec}}$

$A(^{56}Mn) = R_{56Mn}(1-e^{-\lambda_{56Mn}t})$

$= \underline{\underline{1.7\times10^{9}/sec}}$

(b) After irradiation, $A = A_0 e^{-\lambda t} = A_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$

$t = 5$ days

For ^{54}Mn: $A = 5040/sec$ (i.e. about 1% less than A at end of irradiation)

For ^{56}Mn: $A = A_0\left(\frac{1}{2}\right)^{\frac{120\text{ hrs}}{2.5\text{ hrs}}} = A_0\left(\frac{1}{2}\right)^{48} = 3.5\times10^{-15}A_0$

$\therefore \underline{\underline{A \approx 0}}$!

(4) (a) $^{27}Al = {}^{26}Al + n$

$I^{\pi}(^{26}Al) = 5^{+}$

Assume $l_n = 0 \rightarrow \therefore J(^{27}Al) = \vec{5} + \vec{\tfrac{1}{2}} = \tfrac{11}{2}$ or $\tfrac{9}{2}$

$\pi(^{27}Al) = (-1)^0 = +$

Assume $l_n = 1 \rightarrow J(^{27}Al) = \vec{5} + \vec{\tfrac{3}{2}}$ or $\vec{5} + \vec{\tfrac{1}{2}}$

$= \tfrac{13}{2}, \tfrac{11}{2}, \tfrac{9}{2}, \tfrac{7}{2}$

$\pi(^{27}Al) = (-1)^1 = -$

Assume $l_n = 2 \rightarrow J^{\pi}(^{27}Al) = \vec{5} + \vec{\tfrac{5}{2}}$ or $\vec{5} + \vec{\tfrac{3}{2}}$

$= \tfrac{15}{2}, \tfrac{13}{2}, \tfrac{11}{2}, \tfrac{9}{2}, \tfrac{7}{2}, \tfrac{5}{2}$

$\pi(^{27}Al) = (-1)^2 = +$

(b) $Q = [m(^4He) + m(^9Be) - m(^{12}C) - m(n)]c^2$

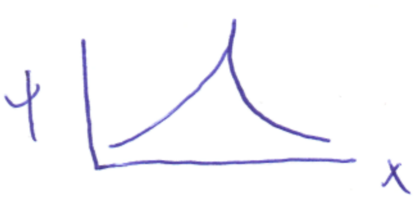
$= [4.002603u + 9.012182u - 12.0000u - 1.0086650u] \times 931.502\ MeV/u$

$= +\underline{\underline{5.701\ MeV}}$

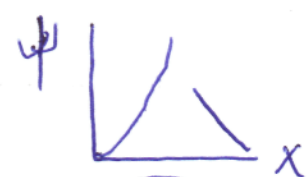
$m(^{12}C) \equiv 12.000$ AMU

1 Amu $\equiv \tfrac{1}{12} m(^{12}C)$

(c)



$\frac{d\psi}{dx}$ not continuous



ψ not continuous

ψ double valued

ψ can't be normalized