ME 105 Midterm #2

4/20/04

Problem 1

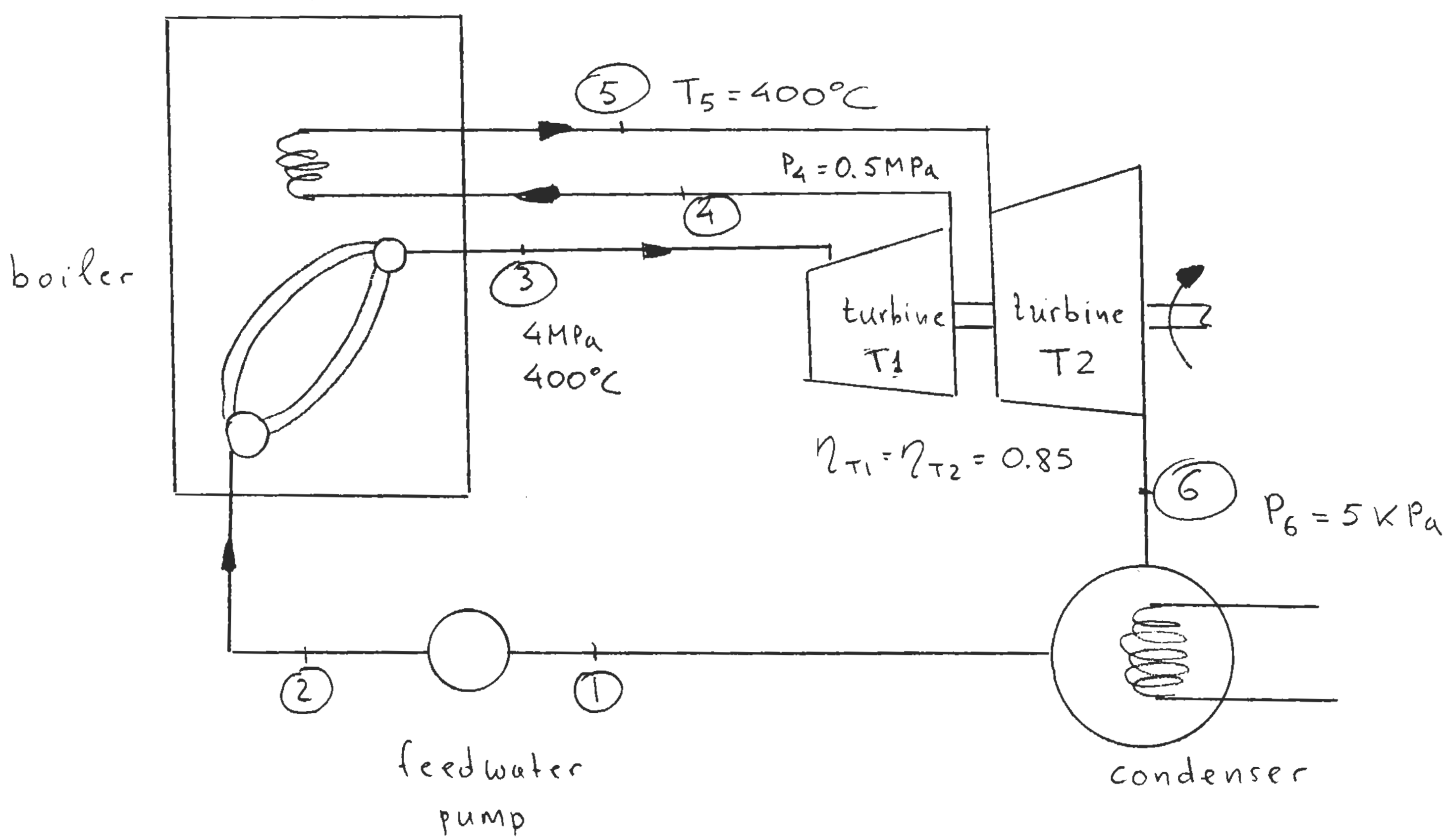
A well insulated piston-cylinder device contains 0.02 m^3 of saturated vapor refrigerant R-134a at a pressure of 1 MPa. The refrigerant is allowed to expand in a reversible manner till the pressure drops to 0.4 MPa. Please determine:

(a) The final temperature in the cylinder
(b) The work done on the refrigerant.

Problem 2

Consider a steam power plant that operates on a reheat Rankine cycle and has a net power output of 2 MW. Steam enters the turbine at 4 MPa and 400°C. The extraction to the reheat occurs at 0.5 MPa and the steam enters the second stage of the turbine at 400°C. The adiabatic efficiency of both stages of the turbine, $\eta_{T1} = \eta_{T2} = 0.85$. The condenser pressure is 5 kPa. Please:

(a) Show the cycle on T-s diagram
(b) Determine the mass flow rate of steam through the boiler
(c) Calculate the thermal efficiency of the cycle.



Problem 3

An air-standard cycle is executed in a closed system with 1 kg of air and consists of the following three processes:

1-2:	Isentropic compression from 100 kPa and 20°C to 1 MPa
2-3:	Isobaric (i.e. constant pressure) heat addition of 1000 kJ
3-1:	Heat rejection to the initial state following the relation $P = c \cdot v$, where P is the pressure, v is the specific volume and c a constant.

Assuming that the specific heats C_p and C_v are constant, please:

(a) Show the cycle on P-v and T-s diagrams
(b) Calculate the cycle net work
(c) Determine the cycle thermal efficiency.

ME 105 Midterm #2 Solutions 4/19/04

Problem 1

reversible + adiabatic ⟹ isentropic $s_2 = s_1$

$P_1 = 1.0$ MPa, saturated vapor $\Big\} \Rightarrow$ $v_1 = v_{g@1MPa} = 0.0202$ m^3/Kg

$u_1 = u_{g@1MPa} = 247.77$ KJ/Kg

$s_1 = s_{g@1MPa} = 0.9043$ KJ/Kg K

$$m = \frac{V}{v_1} = \frac{0.02}{0.0202} = 0.99 \text{ Kg}$$

$P_2 = 0.4$ MPa, $s_2 = s_1$ $\Big\} \Rightarrow$ $x_2 = \dfrac{s_2 - s_{f@0.4MPa}}{s_{g@0.4MPa} - s_{f@0.4MPa}} \Rightarrow$

$$x_2 = \frac{0.9043 - 0.2399}{0.9145 - 0.2399} = 0.985$$

$T_2 = T_{sat@0.4MPa} = \underline{8.93°C}$

$u_2 = u_{f@0.4MPa} + x_2 (u_{g@0.4MPa} - u_{f@0.4MPa}) \Rightarrow$

$u_2 = 61.69 + 0.985 (231.97 - 61.69) \Rightarrow$

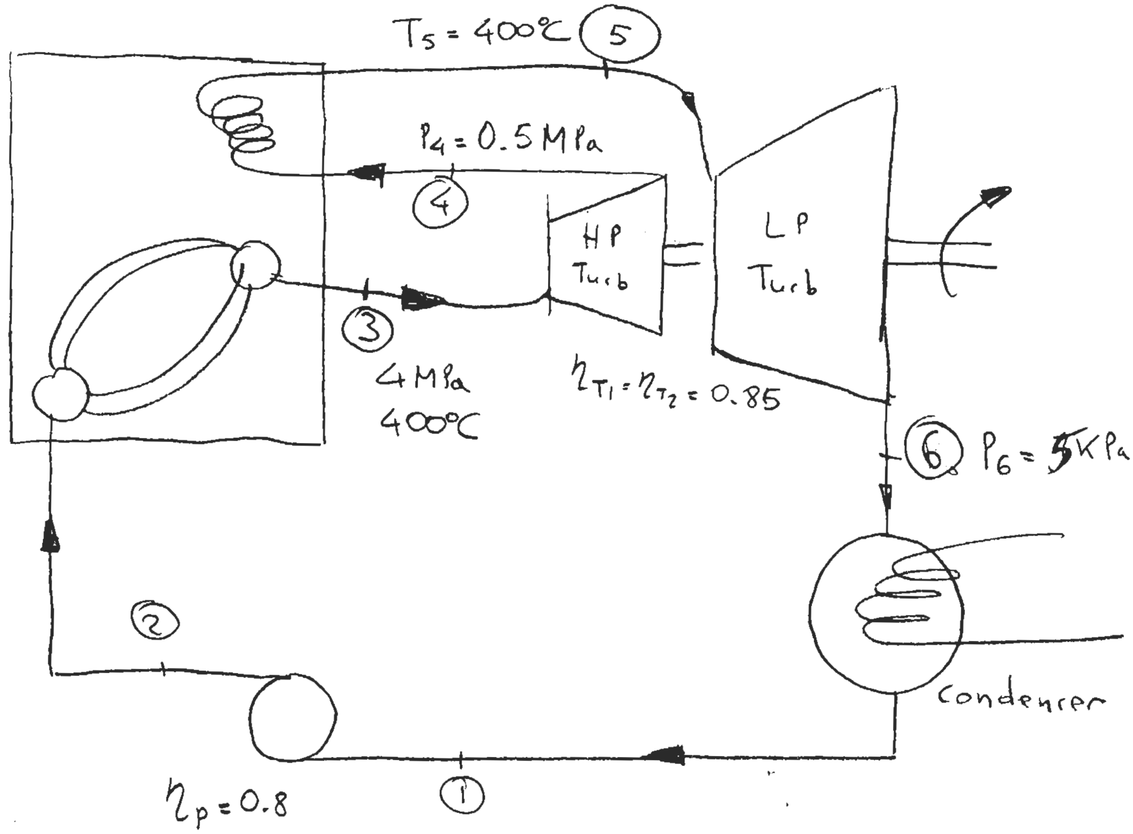
$u_2 = 229.4$ KJ/Kg

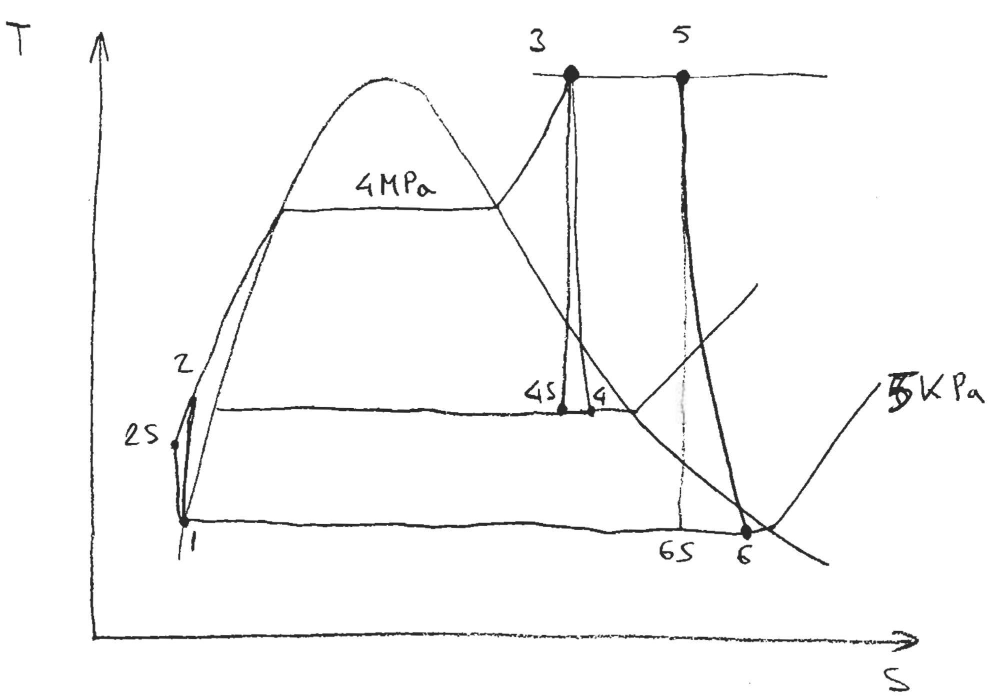
$\cancel{Q}^{\,0} - W = m(u_2 - u_1) \Rightarrow W = -m(u_2 - u_1) \Rightarrow$

$W = -0.99 (229.4 - 247.77) \Rightarrow \boxed{W = 18.16 \text{ KJ}}$

Rankine Cycle with Reheat

$\dot{W}_{net} = 2MW \rightarrow$ (a) $\dot{m} = ?$
(b) $\eta_{th} = ?$





$$\left.\begin{array}{l} P_3 = 4\,MPa \\ T_3 = 400°C \end{array}\right\} \Rightarrow \begin{array}{l} h_3 = 3213.6 \text{ KJ/Kg} \\ s_3 = 6.7690 \text{ KJ/(Kg K)} \end{array}$$

$$P_{4s} = 0.5\,MPa \Rightarrow \begin{array}{l} s_f = 1.8607 \text{ KJ/Kg K} \\ s_{fg} = 4.9606 \text{ KJ/Kg K} \end{array}$$

$$x_{4s} = \frac{s_3 - s_f}{s_{fg}} \Rightarrow x_{4s} = \frac{6.7690 - 1.8607}{4.9606} = 0.99$$

$$h_{4s} = h_f + x_{4s} h_{fg} = 640.23 + 0.99 \cdot 2108.5 = 2727.6 \text{ KJ/Kg}$$

$$h_3 - h_4 = \eta_{T1} \cdot (h_3 - h_{4s}) \Rightarrow h_4 = h_3 - \eta_{T1} (h_3 - h_{4s}) \Rightarrow$$

$$h_4 = 3213.6 - 0.85 (3213.6 - 2727.6) \Rightarrow$$

$$h_4 = 2800.5 \text{ KJ/Kg}.$$

$$\left.\begin{array}{l} T_5 = 400°C \\ P_5 = 0.5\,MPa \end{array}\right\} \Rightarrow \begin{array}{l} h_5 = 3271.9 \text{ KJ/Kg} \\ s_5 = 7.7938 \text{ KJ/Kg K} \end{array}$$

$$P_{6s} = 5\,KPa \Rightarrow \begin{array}{l} s_f = 0.4764 \text{ KJ/(Kg K)} \\ s_{fg} = 7.9187 \text{ KJ/(Kg K)} \end{array}$$

$$x_{6s} = \frac{s_5 - s_f}{s_{fg}} = \frac{7.7938 - 0.4764}{7.9187} = 0.924$$

$$h_{6s} = h_f + x_{4s} h_{fg} = 137.82 + 0.924 \cdot 2423.7 \Rightarrow$$

$$h_{6s} = 2377.3 \text{ KJ/Kg}$$

$$h_5 - h_6 = \eta_{T2} \cdot (h_5 - h_{6s}) \Rightarrow$$

$$h_6 = h_5 - \eta_{T2} (h_5 - h_{6s}) \Rightarrow$$

$$h_6 = 3271.9 - 0.85 (3271.9 - 2377.3) \Rightarrow h_6 = 2511.5 \text{ KJ/Kg}$$

$$W_T = W_{T_1} + W_{T2} = (h_3 - h_4) + (h_5 - h_6) \Rightarrow$$

$$W_T = (3213.6 - 2800.5) + (3271.9 - 2511.5) \Rightarrow$$

$$W_T = 1173.5 \text{ KJ/Kg} \qquad \dot{m} = \frac{\dot{W}_T}{W_T} = \frac{2000}{1173.5} = 1.7 \text{ Kg/s}$$

$$W_{sp} = v (P_2 - P_1) = 0.001005 \cdot (4000 - 5) = 4 \text{ KJ/Kg}$$

$$W_p = \frac{W_{sp}}{\eta_{sp}} = \frac{4}{0.8} = 5 \text{ KJ/Kg} \qquad h_2 = h_1 + W_p = 137.82 + 5 = 142.82 \text{ KJ/Kg}$$

$$\dot{W}_p = \dot{m} W_p = 1.7 \cdot 5 = 8.5 \text{ KW}$$

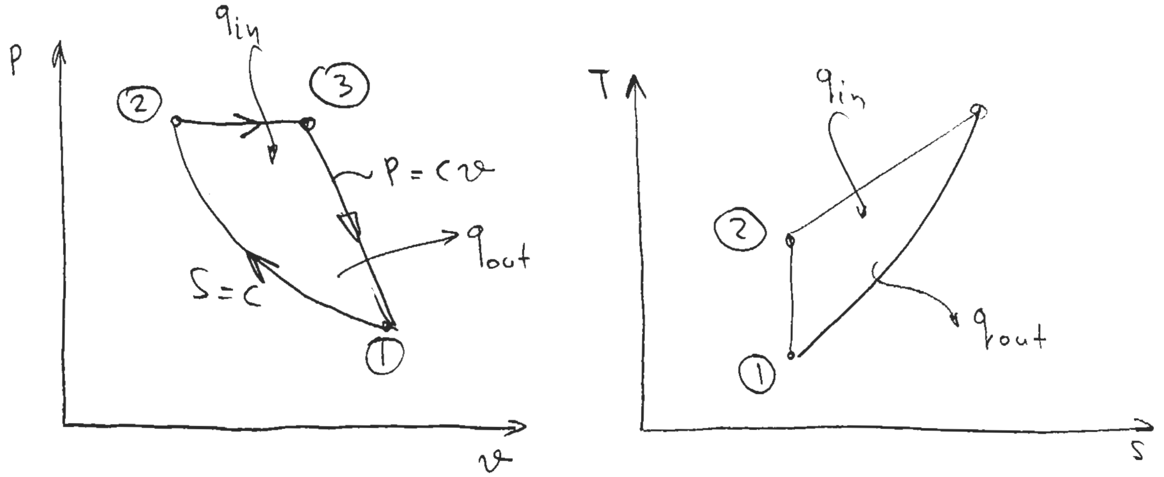
$$q_H = (h_3 - h_2) + (h_5 - h_4) = (3213.6 - 142.82) + (3271.9 - 2800.5) \Rightarrow$$

$$q_H = 3542.18 \text{ KJ/Kg}$$

$$W_{net} = W_T - W_p = 1173.5 - 5 = 1168.5 \text{ KJ/Kg}$$

$$\eta_{TH} = \frac{W_{net}}{q_H} = \frac{1168.5}{3542.18} = 0.323$$

Problem 3



$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}} = 293\left(\frac{1000}{100}\right)^{\frac{0.4}{1.4}} = 565.7\,K$$

$$Q_{in} = m(h_3 - h_2) = m\,C_p(T_3 - T_2) \Rightarrow$$

$$T_3 = \frac{Q_{in}}{m\,C_p} + T_2 \Rightarrow T_3 = \frac{1000}{1 \cdot 1.005} + 565.7 \Rightarrow T_3 = 1560\,K$$

$$w_{31} = \frac{P_3 + P_1}{2}(v_1 - v_3) = \frac{P_3 + P_1}{2}\left(\frac{R T_1}{P_1} - \frac{R T_3}{P_3}\right) \Rightarrow$$

$$w_{31} = \frac{1000 + 100}{2}\left(\frac{293}{100} - \frac{1560}{1000}\right) \cdot 0.287 \rightarrow w_{31} = 216\ KJ/Kg$$

$$Q_{31} - W_{31} = m(u_1 - u_3) \Rightarrow Q_{31} = m\,W_{31} + m\,C_v(T_1 - T_3)$$

$$Q_{31} = 1 \cdot [216 + 0.718(293 - 1560)] \Rightarrow Q_{31} = -693.7\,KJ$$

$$\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{|Q_{31}|}{Q_{in}} = 1 - \frac{693.7}{1000} \Rightarrow$$

$$\eta_{th} = 0.3063$$

$$W_{net} = Q_{in} - Q_{out} \Rightarrow W_{net} = 1000 - 693.7 = 306.3\,KJ$$