EE100 Test 1 Solution

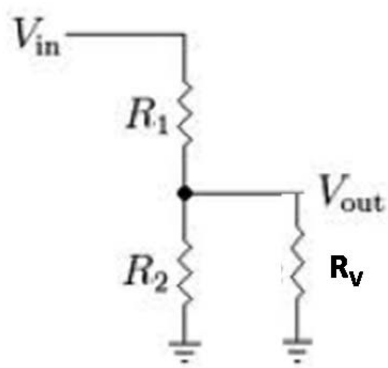
1)

a) Vout = $(\frac{R2}{R1+R2})$ Vin

b)

i. High resistance because you don't want the resistance of the voltmeter to affect the voltage at node Vout. Equivalently, by having a very high resistance, voltmeter will not draw any current to it, leaving the original circuit unperturbed.

ii.

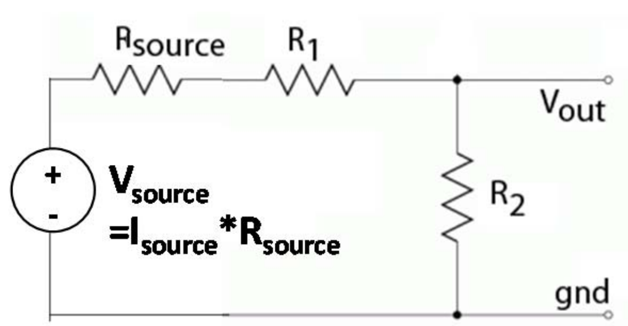


Vout = $(\frac{R2 \,//\, Rv}{R2 \,//\, Rv + R1})$ Vin

= $\frac{(\frac{R2*Rv}{R1+Rv})}{(\frac{R2*Rv}{R1+Rv})+R1}$ Vin

= $\frac{R2*Rv}{R2*Rv+R1(R2+Rv)}$ Vin

c)

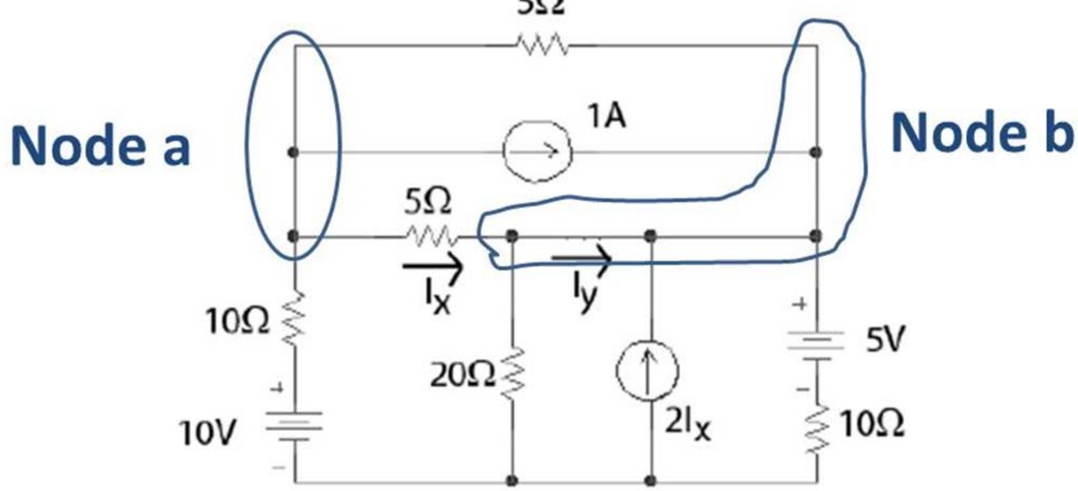


Vout = $(\frac{R2}{R1+R2+Rsource})$ Vsource = $(\frac{R2}{R1+R2+Rsource})$ (Isource * Rsource)

= $\frac{R2*Rsource}{R1+R2+Rsource}$ Isource

2. a)

Consider the following circuit:



Node a :

$$\frac{Va - 10}{10} + \frac{Va - Vb}{5} + 1 + \frac{Va - Vb}{5} = 0$$

$$Va = \frac{4}{5}Vb$$

Node b :

$$\frac{Vb - Va}{5} + \frac{Vb}{20} - 2Ix + \frac{Vb - 5}{10} - 1 + \frac{Vb - Va}{5} = 0$$

$$Ix = \frac{Va - Vb}{5} = -\frac{Vb}{25}$$

Therefore,

$$\frac{Vb - Va}{5} + \frac{Vb}{20} - 2(\frac{Va - Vb}{5}) + \frac{Vb - 5}{10} - 1 + \frac{Vb - Va}{5} = 0$$

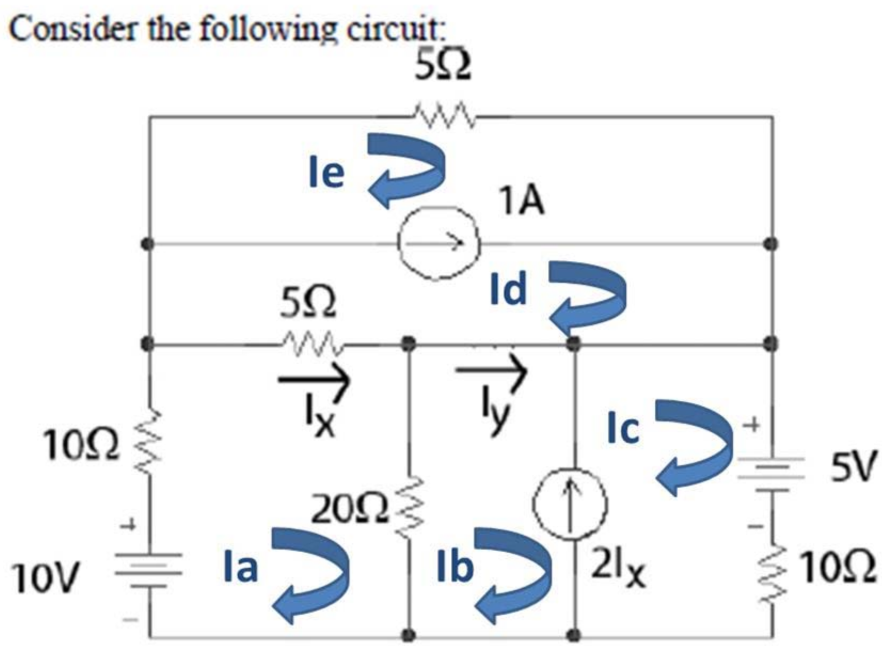
Solve for Va and Vb.

Perform KCL at the node containing Ix and Iy currents:

$$-Ix+\frac{Vb}{20}+Iy=0$$

$$Iy=Ix-\frac{Vb}{20}=\frac{Va-Vb}{5}-\frac{Vb}{20}=\frac{Va}{5}-\frac{Vb}{4}=-\frac{9}{100}Vb$$

b)



Mesh a:

$$-10+10Ia+5(Ia-Id)+20(Ia-Ib)=0 \qquad \text{Eq.1}$$

Supermesh b & c:

$$20(Ib-Ia)+5+10Ic=0 \qquad \text{Eq.2}$$

Supermesh d & e:

$$5(Id-Ia)+5Ie=0 \qquad \text{Eq.3}$$

Constraint# 1:

$$2Ix = Ic - Ib \text{ and } Ix = Ia - Id$$

⇨ $2(Ia - Id) = Ic - Ib$
⇨ $2Ia - 2Id - Ic + Ib = 0$ Eq. 4

Constraint# 2 :

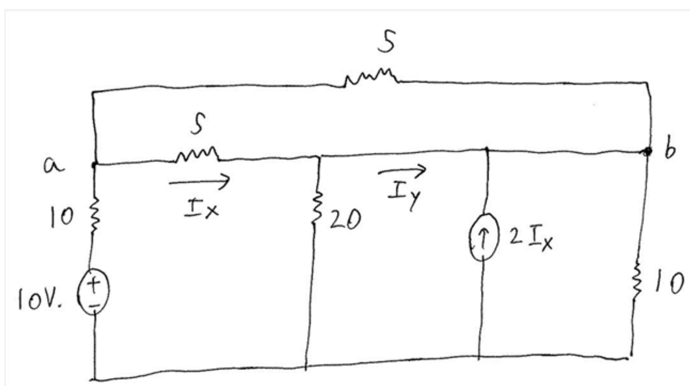
$$Id - Ie = 1$$ Eq. 5

Five equations, 5 unknowns => solve for Ia,Ib,Ic,Id, and Ie

c) Nodal analysis is easier in this case since there are only two node equations to solve as compared to 5 equations in the mesh analysis technique.

d) Superposition

Voltage source : 10V



Node a:

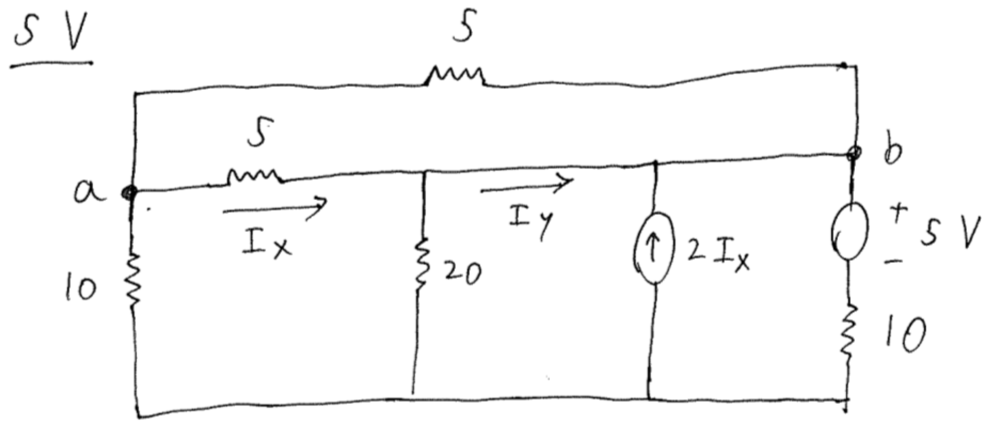
$$\frac{Va - 10}{10} + \frac{Va - Vb}{5} + \frac{Va - Vb}{5} = 0$$

Node b:

$$\frac{Vb - Va}{5} + \frac{Vb}{20} - 2\left(\frac{Va - Vb}{5}\right) + \frac{Vb}{10} + \frac{Vb - Va}{5} = 0\,, \quad NOTE: Ix = \left(\frac{Va - Vb}{5}\right)$$

$$Iy, 10\,V = Ix - \frac{Vb}{20} = \frac{Va}{5} - \frac{Vb}{4}$$

Voltage source : 5V



Node a:

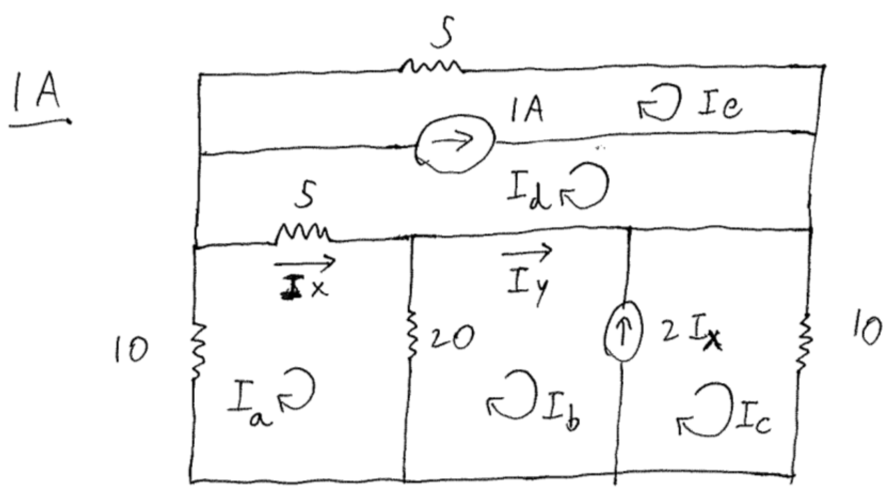
$$\frac{Va}{10}+\frac{Va-Vb}{5}+\frac{Va-Vb}{5}=0$$

Node b:

$$\frac{Vb-Va}{5}+\frac{Vb}{20}-2(\frac{Va-Vb}{5})+\frac{Vb-5}{10}+\frac{Vb-Va}{5}=0$$

$$Iy,5\,V=Ix-\frac{Vb}{20}=\frac{Va}{5}-\frac{Vb}{4}$$

Current source : 1A



Mesh a:

$$10Ia+5(Ia-Id)+20(Ia-Ib)=0$$

Supermesh b & c:

$$20(Ib - Ia) + 10Ic = 0$$

Supermesh d & e:

$$5(Id - Ia) + 5Ie = 0$$

Constraint# 1:

$$2Ix = Ic - Ib \text{ and } Ix = Ia - Id$$

$2(Ia - Id) = Ic - Ib$

$2Ia - 2Id - Ic + Ib = 0$

Constraint# 2 :

$Id - Ie = 1$

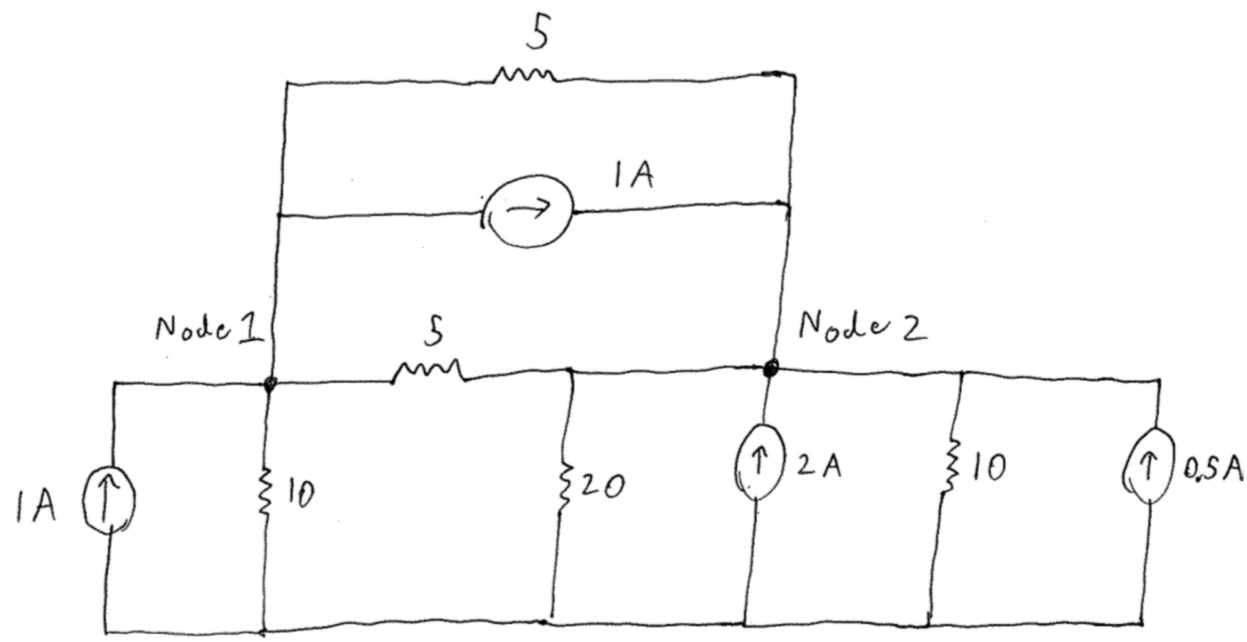
Solve for Ia, Ib, Ic, Id, and Ie.

$Iy, 1A = Ib - Id$

Therefore,

$Iy = Iy, 10V + Iy, 5V + Iy, 1A$

e) Need to make sure that all the sources are either independent current sources or independent voltage sources. Let's convert all the sources into independent currents sources using source transformation and perform nodal analysis by inspection.



Conductance Matrix:

$$G11 = \frac{1}{10} + \frac{1}{5} + \frac{1}{5} = \frac{1}{2}$$

$$G22 = \frac{1}{5} + \frac{1}{20} + \frac{1}{10} + \frac{1}{5} = \frac{11}{20}$$

$$G12 = -\frac{1}{5} - \frac{1}{5} = -\frac{2}{5}$$

$$G21 = G12$$

Source Vector:

$It1 = 1 - 1 = 0A$

$It2 = 2 + 1 + 0.5 = 3.5\ A$

$$\begin{bmatrix} \frac{1}{2} & \frac{-2}{5} \\ \frac{-2}{5} & \frac{11}{20} \end{bmatrix} \begin{bmatrix} v1 \\ v2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3.5 \end{bmatrix}$$