ME C176/BIOE C119 Mid-Term Exam, Fall 2011 Name: GSIs

Thursday, October 6, 2011, 12 :30 PM–2:00 PM.

Please write your name at the top of each page as indicated and write all answers in the space provided. If you need additional space, write on the back sides. Do not remove or add any pages. *Good luck!*

---

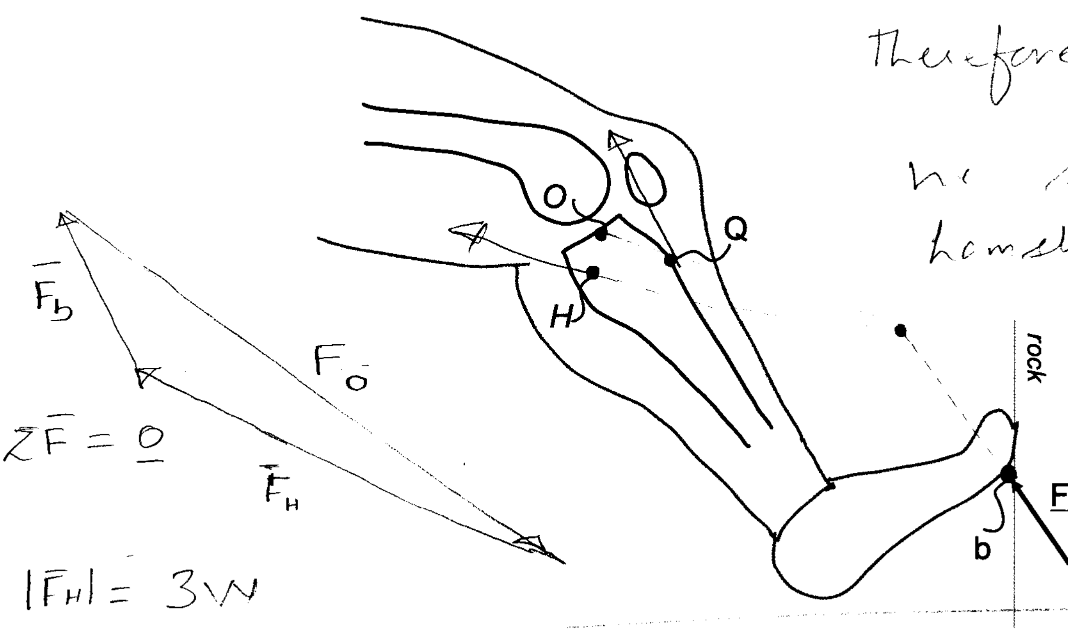
1. (35 Points) Statics

(i) [5 points] A rock climber is scaling a crack using a "layback" technique, during which simultaneous hand-pulling and foot-pushing against vertical rock surfaces allows the climber to support his or her weight using only frictional forces. Let's first focus on the climber's lower leg and foot. Assume as shown below that the joint contact force at the knee acts at point ***O***, and that the quadriceps muscle acts at point ***Q*** and the hamstrings at point ***H***. The rock-reaction force $\underline{\mathbf{F}}_b$ acts on the foot, as drawn, at point **b**.

Assuming only one muscle group is active, which one must it be? Provide a *mathematical basis* for your answer.

$\underline{r}_{H/O} \times \underline{F}_H < 0$; $\underline{r}_{Q/O} \times \underline{F}_Q > 0$; $\underline{r}_{b/O} \times \underline{F}_b > \underline{0}$

Therefore, for $\Sigma \underline{M}^O = 0$ we should consider hamstrings active, and assume quadriceps free in the model.



$\Sigma \bar{F} = \underline{0}$

$|F_H| = 3W$

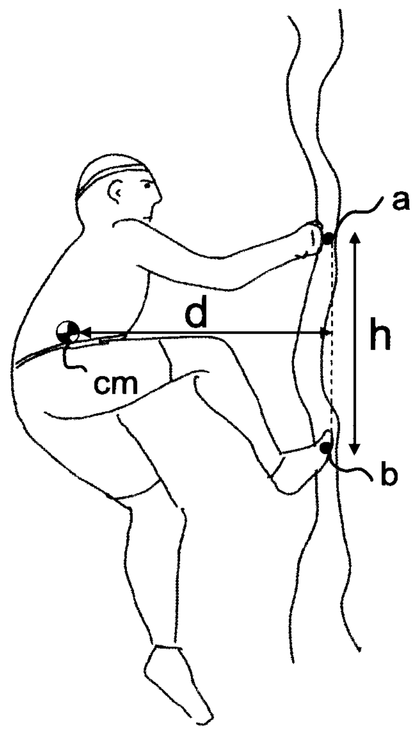
$|\bar{F}_O| = 3.5W$

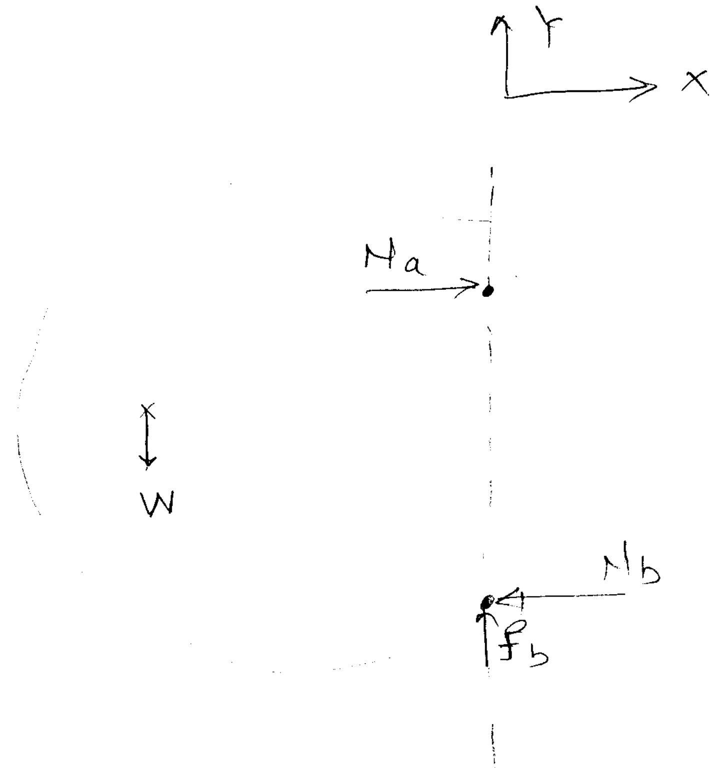
$\boxed{\text{Hamstrings must be active}}$

(ii) [10 points] Use the 3-vector force triangle technique to estimate the force that this muscle exerts on the tibia. Express your estimate as a multiple of $\mathbf{F}_b$. (Draw lines of force acting through the appropriate points in the figure above. Use the space beside the figure for your triangle. Label all forces and indicate their direction).

(iii) Now, let's analyze the whole person (and no longer assume the foot reaction force is at any pre-specified angle). As depicted below, the climber is making contact with the rock at points **a** and **b**. Assume that frictional forces act at the foot along the vertical dotted line, and that the coefficient of static friction is $\boldsymbol{\mu}$ for the foot-rock interface. Assume that the vertical component of the force at the hand is zero.

Draw a free body diagram <u>for the climber</u> for this situation. Indicate clearly and label all the forces acting on the climber. Write out the force- and moment-balance equations for static equilibrium.





$$\sum F_x = 0 \Rightarrow \boxed{N_a - N_b = 0} \quad (\mathrm{I})$$

$$\sum F_y = 0 \Rightarrow \boxed{f_b - W = 0} \quad (\mathrm{II})$$

$$\sum M_b = 0 \Rightarrow \boxed{N_a h = W d} \quad (\mathrm{III})$$

(iv) [10 points] In order to exert minimal hand force, skilled climbers learn to put their body in an optimal position, which can vary depending on the climbing conditions. To gain insight into the underlying biomechanics of such practices, assume that the climber's foot is on the verge of slipping. The coefficient of friction for the hand-rock interface is zero. For this situation, derive an expression relating the friction coefficient **μ** to the ratio **d/h** — for which **d** is the distance between the rock and the climber's center of mass and **h** is the distance between the climber's hand and foot. State and justify any key assumptions for your analysis.

Based on your analysis, what advice would you give to a beginning rock climber.

$f_b = \mu N_b$ → On the verge of slipping

∴ $\mu N_b = W$ — from II

$N_b = N_a$ — from I

$d/h = N_a/W$ → from III

$d/h = N_a/W = N_b/W = W/\mu/W$.

∴ $\boxed{d/h = 1/\mu}$

Advice

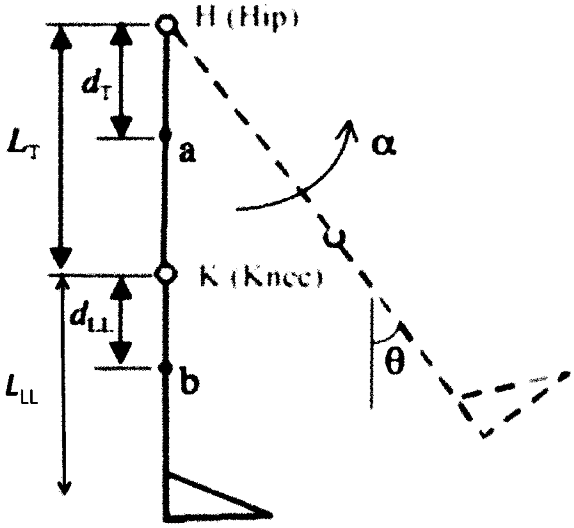
To minimize hand force, d↓ & h↑

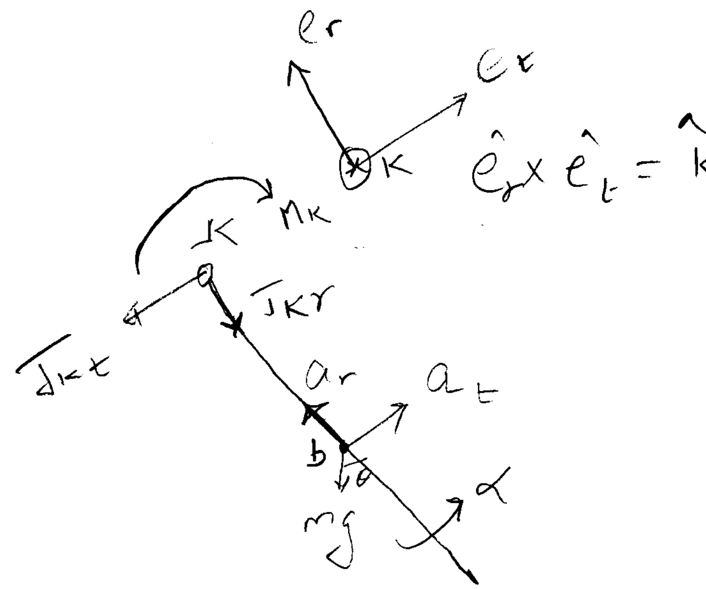
To maintain Equilibrium, when the friction force becomes dynamic, d↑ & h↓

2. (35 Points) Dynamics

An individual stands erect and executes a straight-leg kick, as shown schematically below. The observed angular acceleration for their leg is α when the leg is at the angle θ. At this instant, assume that the mass center of the thigh is at point a; the mass center of the "lower leg" (all the leg from the knee down, including the foot) is at point b; the center of the hip joint, point H, is a fixed point in space; and the "lower leg" acts as a single rigid body.

(i) [10 points] Draw a labeled free body diagram for (only) the "<u>lower leg</u>". Make sure to include and label any accelerations.





J_{kr}, J_{kt} → knee Joint resultant forces

a_r, a_t → radial and tangential accelerations.

α → angular acceleration.

mg → weight of the lower leg.

(ii) [12 points] Write out the equations of motion for the lower leg in terms of the quantities in your free body diagrams. *You can write the equations either in the component form (two for linear motion and one for angular motion) or in the vector form (one for linear motion and one for the angular motion).*

Recall:

About an accelerating point P, $\Sigma \underline{M}^P = I^{cm}\underline{\alpha} + (\underline{r}_{cm/P} \times m\underline{a}_{cm})$

About a fixed point (o), $\Sigma \underline{M}^o = I^o\underline{\alpha}$, where $I^o = I^{cm} + m\|\underline{r}_{o/cm}\|^2$

$$J_{Kr} + mg\cos\theta = -ma_r$$

$$a_r = \omega^2 r = \omega^2 (L_T + d_{LL})$$

$$\therefore \boxed{J_{Kr} + mg\cos\theta = -m\omega^2 (L_T + d_{LL})}$$

$$J_{Kt} + mg\sin\theta = -ma_t$$

$$a_t = \alpha r = \alpha (L_T + d_{LL})$$

$$\therefore \boxed{J_{Kt} + mg\sin\theta = -m\alpha (L_T + d_{LL})} \quad (II)$$

$$a_{cm} = a_t \hat{e}_t + a_r \hat{e}_r = \alpha (L_T + d_{LL}) \hat{e}_t + \omega^2 (L_T + d_{LL}$$

$$(M_K + mg\sin\theta \, d_{LL})\hat{k} = -I_{cm} \alpha \hat{k} + -d_{LL} \hat{e}_r \times m a$$

$$\boxed{M_K + mg\sin\theta \, d_{LL} = -I_{cm}\alpha + m\alpha d_{LL}(L_T + d} \quad (III$$

(iii) [13 points] Derive an expression for the magnitude of the resultant moment acting at the knee joint in terms of <u>only</u> the mass and geometry parameters shown in the boxes below and the angular acceleration α.

Mass Properties	Geometry Properties
m — Mass of the lower leg I^{cm} — Mass moment of inertia of the lower leg about its center of mass	L_T — Distance from hip to knee joint centers d_{LL} — Distance from knee joint center to the center of mass of the lower leg θ — Angle between the leg and the vertical axis at the instant α is measured

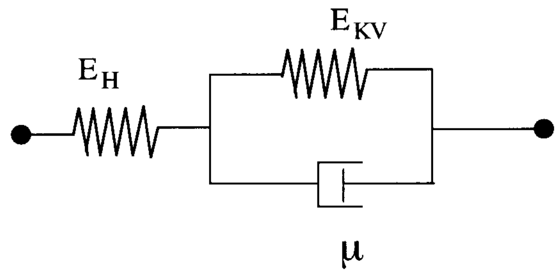
Magnitude of resultant Moment, M_k
using equation (III) from part (ii)

$$\boxed{\overline{M_K} = \left(-mg\sin\theta\, d_{LL} - I_{cm}\alpha + m\alpha\, d_{LL}(L_1 + d\right.}$$

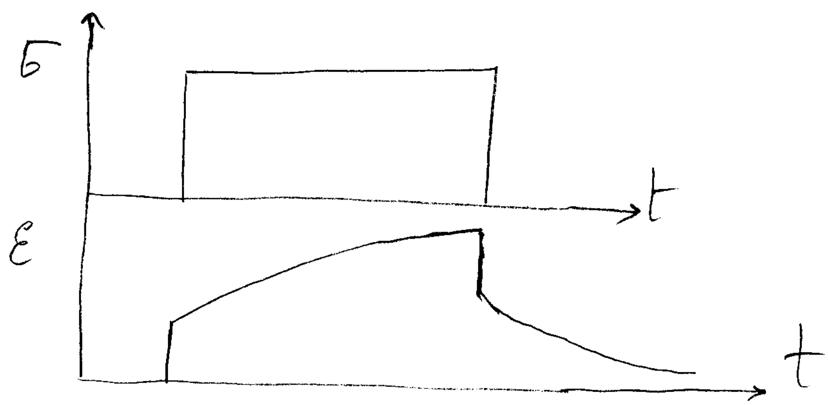
$\hat{k}$ is the direction inside the plane of the paper

3. (30 Points) Viscoelasticity

(i) [10 points] The general time-dependent behavior of many soft tissues, and even bone, can be modeled using spring-dashpot models, such as that shown below for the "standard linear solid".



Draw (qualitatively) the creep function for this model:



(ii) [10 points] Assume the following values for the spring and dashpot elements of the standard linear solid model: $\mathbf{E_H}$ = 10 N/m^2, $\mathbf{E_{KV}}$ = 20 N/m^2, μ = 5 N*Sec/m^2. For an applied constant stress of σ_0 = 10 N/m^2, calculate the following for this model:

A) Initial value of the total strain:

$$\varepsilon_{initial} = \frac{\sigma_0}{E_{eff\text{-}ini}}$$

$$E_{eff\text{-}ini} = E_H$$

$$\therefore \varepsilon_{initial} = \frac{10}{10} = 1$$

B) Steady-state value of the total strain:

$$\varepsilon_{ss} = \frac{\sigma_0}{E_{eff-ss}}$$

$$\frac{1}{E_{eff-ss}} = \frac{1}{E_M} + \frac{1}{E_{KV}}$$

$$\varepsilon_{ss} = 10\left(\frac{1}{10} + \frac{1}{20}\right) = 1$$

(iii) [10 points]

• Briefly explain the concepts of the "loss modulus" and "storage modulus" for a linearly viscoelastic material.

Storage Modulus: Measures the stored energy corresponding to the elastic portion

Loss-Modulus Measures the energy dissipated as heat, corresponding to the viscous portion

• Describe briefly how the storage modulus is typically measured in experiments?

Apply Sinusoidal loading $\sigma = \sigma_0 \sin \omega t$

Measure strain response $\varepsilon = \varepsilon_0 \sin(\omega t - \delta)$

δ: phase difference.

$E_S = \frac{\sigma_0}{\varepsilon_0} \sin \delta$ $\quad E_L = \sigma_0/\varepsilon_0 \cos \delta$.