Midterm #1 Solutions
Physics 7C Fall 2011

1. (a) (*3 points*) Plane waves near the Earth's surface, satisfying the conditions given are

$$\mathbf{E} = E_0\hat{x}\cos\left(\frac{2\pi}{\lambda}z - \frac{2\pi c}{\lambda}t\right)$$

$$\mathbf{B} = \frac{E_0}{c}\hat{y}\cos\left(\frac{2\pi}{\lambda}z - \frac{2\pi c}{\lambda}t\right)$$

(b) (*2 points*) $I = \frac{1}{2}c\epsilon_0 E_0^2$ You may also express it as in part (c).

(c) (*2 points*)

$$I = \frac{P_S}{4\pi d_E^2} = \frac{1}{2}c\epsilon_0 E_0^2$$

$$E_0 = \sqrt{\frac{P_S}{2\pi c\epsilon_0 d_E^2}}$$

$$\mathbf{E} = \sqrt{\frac{P_S}{2\pi c\epsilon_0 d_E^2}}\hat{x}\cos\left(\frac{2\pi}{\lambda}z - \frac{2\pi c}{\lambda}t\right)$$

$$\mathbf{B} = \frac{1}{c}\sqrt{\frac{P_S}{2\pi c\epsilon_0 d_E^2}}\hat{y}\cos\left(\frac{2\pi}{\lambda}z - \frac{2\pi c}{\lambda}t\right)$$

(d) i. (*1 point*)

$$\frac{1}{\infty} + \frac{1}{d_i} = \frac{1}{f}$$

$$d_i = f$$

ii. (*2 points*)

$$P = I \cdot A = \frac{P_S}{4\pi d_E^2}\frac{\pi D^2}{4}$$

2. (a) (*3 points*) See Figure 1.

$$\tan\theta_0 = \frac{x}{d_0}$$

$$\tan\theta' = \frac{x}{d'}$$

$$\frac{d'\tan\theta'}{d_0\tan\theta_0} \approx \frac{d'\theta'}{d_0\theta_0} = 1$$

$$\frac{\theta_0}{\theta'} \approx \frac{n_1}{n_2} \text{ (Snell's law)}$$

$$d' = \frac{n_1}{n_2}d_0$$

(b) (*3 points*) See Figure 2.

Figure 1: Problem 2(a)

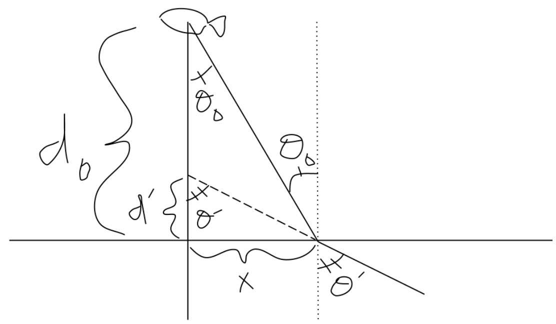
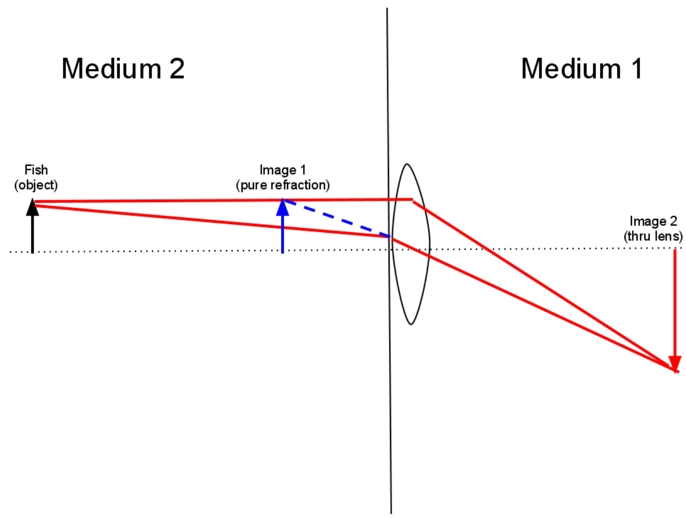


Figure 2: Problem 2(b)



(c) (*2 points*)

$$\frac{1}{d'} + \frac{1}{d_i} = \frac{1}{f_1}$$

$$d' = \frac{n_1}{n_2} d_0$$

$$\frac{1}{\frac{n_1}{n_2} d_0} + \frac{1}{d_i} = \frac{1}{f_1}$$

$$\frac{n_2}{d_0} + \frac{n_1}{d_i} = \frac{n_1}{f_1}$$

(d) (*2 points*) No matter where the outside light ray comes from, it is refracted to an angle $\theta \leq \theta_{crit}$. Thus the field of view is given by $\alpha = 2\theta_{crit} = 2\sin^{-1}\frac{n_2}{n_1}$.

3. (a) (*1 point1*)

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_o}$$

$$d_i = \frac{f_o d_o}{d_o - f_o}$$

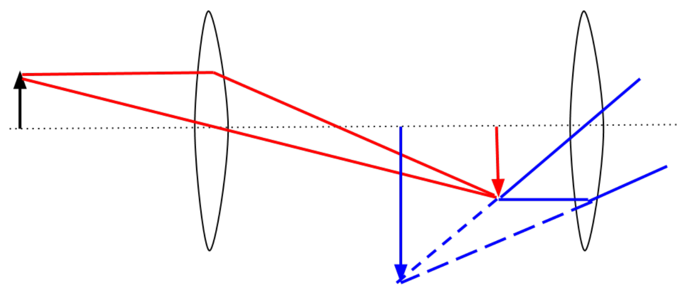
(b) (*2 points*) Let d be the distance from the first image where we will place the eyepiece. The image distance is $-d_1$, so

$$\frac{1}{d} + \frac{1}{-d_1} = \frac{1}{f_e}$$

$$d = \frac{d_1 f_e}{d_1 + f_e}$$

(c) (*2 points*) See Figure 3.

Figure 3: Problem 3(c)



(d) (*1 point*)

$$m_o = -\frac{d_i}{d_o} = \frac{f_o}{f_o - d_o}$$

(e) (*1 point*)

$$m_e = -\frac{-d_1}{d} = \frac{d_1 + f_e}{f_e}$$

(f) (*1 point*)

$$M = m_o m_e = \frac{f_o}{f_e} \frac{d_1 + f_e}{f_o - d_o}$$

(g) (*1 point*) $h = d_1 \alpha_{min}$

(h) (*1 point*) The eye sees the magnified image $h' = |M| h_o$. Thus, at the limit of resolution

$$h_o = \frac{\alpha_{min} d_1}{|M|}$$

4. (a) (*1 point*) The straight path is d, simple trigonometry shows the reflected path is $2\sqrt{H^2 + (d/2)^2}$, so

$$\Delta\ell = \sqrt{4H^2 + d^2} - d$$

(b) (*2 points*) Remembering the extra phase shift from reflection,

$$\Delta\phi = \frac{2\pi}{\lambda_0}\Delta\ell + \pi$$

$$\Delta\phi = \frac{2\pi}{\lambda_0}\left(\sqrt{4H^2 + d^2} - d + \frac{\lambda_0}{2}\right)$$

(c) (*3 points*)

$$\Delta\phi = 2\pi m$$

$$\left(m + \frac{1}{2}\right)\lambda_0 = \sqrt{4H^2 + d^2} - d$$

$$\left[\left(m + \frac{1}{2}\right)\lambda_0 + d\right]^2 = 4H^2 + d^2$$

$$\left[\left(m+\frac{1}{2}\right)\lambda_0\right]^2 + 2d\left(m+\frac{1}{2}\right)\lambda_0 = 4H^2$$

$$d = \frac{2H^2}{(m+\frac{1}{2})\lambda_0} - \frac{(m+\frac{1}{2})\lambda_0}{2}$$

(d) (*3 points*) A calculation similar to part (c) (but with $m + 1/2 \to m$) yields

$$d = \frac{2H^2}{m\lambda_0} - \frac{m\lambda_0}{2}$$

(e) (*1 point*) Using the answer to part (d) and plugging in $m \leq 0$ does not yield a finite, positive solution for d, and so are not allowed. Increasing m decreases the first term, while increasing the second (negative) term, thus causes d to decrease. So the largest possible distance occurs for $m = 1$,

$$d = \frac{2H^2}{\lambda_0} - \frac{\lambda_0}{2}$$