Midterm #1
Physics 7C Fall 2011

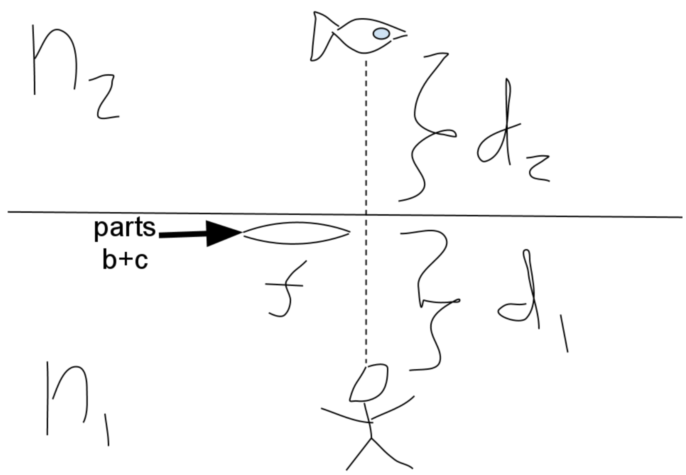
Each problem is worth 10 points, for a total of 40 points

Write neatly. Show all work. Explain your solutions clearly. Number all problems. Start each problem on a new page. If you are asked to substitute a result from a previous part, which you were not able to solve, please clearly define a variable to stand in for your would-be solution to the previous part.

1. The sun emits EM waves with vacuum wavelength λ. The sun radiates a time-averaged power P_S. By the time these waves reach the surface of Earth (a distance d_E away from the sun), they are essentially plane waves. Denote the radius of the earth by R_E.
   (a) Taking $\hat{z}$ to be towards the center of the Earth, $z = 0$ at the surface of the Earth, and $\mathbf{E}(t = 0) = E_0\hat{x}$ at the surface of the Earth, write down the expressions for $\mathbf{E}$ and $\mathbf{B}$ (at the surface of the Earth) in terms of λ, z, t, and E_0, the electric field amplitude.
   (b) What is the time-averaged intensity (time-averaged magnitude of the Poynting vector) of EM radiation hitting the Earth's surface? (Hint: $\bar{S} = \frac{1}{2}\epsilon_0 c E_0^2$)
   (c) Write E_0 in terms of P_S, d_E, and R_E. Rewrite your solutions to part (a) by eliminating E_0.
   (d) Suppose we want to use a converging lens (focal length f) to focus the Sun's energy (perhaps to burn a dry leaf).
      i. What height above the ground should we hold the magnifying glass so as to form a sharp image on the ground (you may assume that the distance to the sun is essentially infinite)?
      ii. Assume all of the energy hitting the lens is focused onto the image. If the lens has diameter D, how much power is delivered to the spot on the ground?

2. Recall from class that looking directly down at a fish in water, you see its image at a lesser depth. Now consider that you are located in a fluid with index of refraction n_1, the fish is located in a fluid with index of refraction n_2. The line of sight connecting you to the fish is perpendicular to the interface of the two fluids (thus it is appropriate to use the small angle approximation for light rays). You are a distance d_1 away from the interface, while the fish is a distance d_2 from the interface.
   (a) How far from the interface does the fish appear to be?
   (b) Suppose we place a lens very close to the interface, but entirely in the fluid with index n_1. The focal length of the lens in that fluid is f_1. Draw a ray diagram to see that the image is formed as a result of two optical "devices" (the fluid interface and the lens).
   (c) Relate the final image distance d_i to the initial object distance $d_0 = d_2$. Rewrite the relation to make it look as much as possible like the thin lens equation..
   (d) Remove the lens, and take $n_1 > n_2$. As you look towards the interface, you see everything that is within the 2nd fluid in a limited angle (it does not take up your entire field of view). Why is this? Find the angular size in which you see everything within the 2nd fluid.

Figure 1: Figure for Problem 2



3. Let's design a microscope. We will use two converging lenses. The objective lens has focal length f_0, while the eyepiece has focal length f_e. We want the image to be viewed at the standard reading distance (from the eyepiece), which we will call d_1 ($d_1 > 0$). The object to be viewed is placed a distance d_0 from the objective lens.

   (a) Take $d_0 > f_0$ and find the distance from the objective lens where the first image is created.
   (b) Using the first image as the object for the eyepiece, find how far away from this first image to place the eyepiece so that the second image is virtual, and a distance d_1 from the eyepiece. Use this lens positioning for all the parts below.
   (c) Draw a ray diagram of the microscope.
   (d) Find the linear magnification of the objective lens.
   (e) Find the linear magnification of the eyepiece.
   (f) The total magnification of the microscope is the product of the linear magnification of the objective of the eyepiece. What is the total magnification of the microscope?
   (g) The eye can resolve an object whose angular size is larger than $\alpha_{min} \ll 1$. To what linear size does this correspond at standard reading distance, d_1?
   (h) If the microscope has total magnification M, how small can the object be so that its image through the microscope is at the limit of eye resolution found in part (g)?

4. A cell phone tower transmits electromagnetic waves with vacuum wavelength λ_0. A person with a cell phone stands a horizontal distance d away. The source of EM waves is a point at the top of the tower, a height H above the ground. The person holds the phone the same height H above the ground. The EM waves can reach the phone either via a straight, direct path or via a path that reflects once off the ground (assume the law of reflection holds).

   (a) Find the path length difference between the two possible paths.
   (b) Find the phase difference between the two possible paths.
   (c) Find the horizontal distances away from the tower where the person holding the phone would experience completely *constructive* interference.
   (d) Find the horizontal distances away from the tower where the person holding the phone would experience completely *destructive* interference.

(e) Find the maximum distance, beyond which completely destructive interference is impossible. (Hint: Your solutions in part (d) should correspond to integers. Find the smallest integer, below which the distances in part (d) are no longer positive. Show that for integers larger than this minimum integer, the distances in part (d) decrease with increasing integers. Hence conclude that the distance corresponding to this minimum integer is the maximum possible distance for completely destructive interference.)

Figure 2: Figure for Problem 4