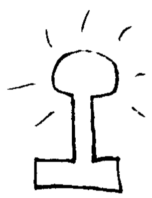
Problem 1

[3] a) $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}$ $\qquad V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}$

$|\vec{E}|_{max}$ occurs at surface of Van-de-Graaf generator

-1 if they assume E is constant.



$|\vec{E}| = \frac{V}{r} \Rightarrow V = |\vec{E}| \cdot r$

$= 3\times10^{6}\ N/C \times 0.5\ m$

$\boxed{V = 1.5 \times 10^{6}\ \text{Volts}}$

If they solve for Q instead of V => -2

[4] b) $\mathcal{U} = \frac{1}{2} C V^2$ $\qquad V = E \cdot d$ $\qquad C = \frac{\varepsilon_0 A}{d}$

(or $= \frac{1}{2}\varepsilon_0 \vec{E}^2 \cdot (\text{Volume})$)

$\Rightarrow \mathcal{U} = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(E d)^2 = \frac{1}{2}\varepsilon_0 A d E^2$

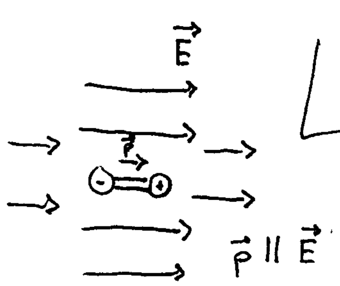
$= \frac{1}{2} \cdot 8.85\times10^{-12}(1\,m^2)\cdot(0.01\,m)(3\times10^{6}$

$\boxed{\mathcal{U} = 0.40\ J}$

-2 if you use voltage from part (a)

[3] c) $\mathcal{U} = -\vec{p}\cdot\vec{E} = -(6.2\times10^{-30}\ C\cdot m)\cdot 3\times10^{6}\ N/C$ (since $\theta = 0$ between $\vec{p}$ & $\vec{E}$)

$= -1.86\times10^{-23}\ J$



$\boxed{\mathcal{U} = -1.16\times10^{-4}\ eV}$

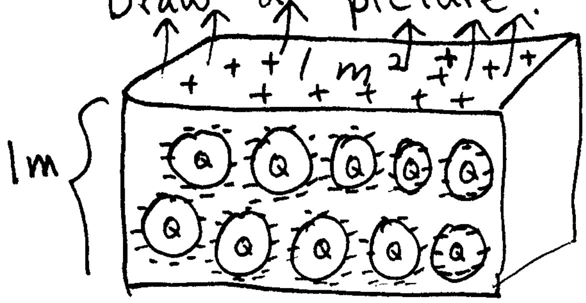
-1 if it's not in eV

$\vec{p} \parallel \vec{E}$

$\vec{p}\cdot\vec{E} = |\vec{p}||\vec{E}|\cos\theta_0$

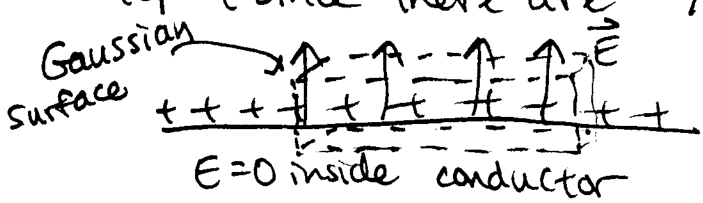
Midterm 2, McKee Spring 2011, Problem 2 Solution

Draw a picture!



What's going on: Charges inside cavities attract negative charges from the conductor. Since the conductor is originally neutral, then an equal amount of positive charge must accumulate on the outside surface of the slab.

Each cavity gets $-Q = -10^{-6}\,C$ charge surrounding it, which contributes $+Q = 10^{-6}\,C$ to the surface charge, which causes the $\vec{E}$ field above the slab. Assuming the charge is mostly on the top and bottom of the slab (the sides are negligible since it is large), then there is $\sigma = 5Q/m^2$ ~~on~~ surface charge density on top (since there are 10 cavities$/m^2$).



$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\oint \vec{E}\cdot da = \underset{(top)}{EA} + \underset{(bottom + sides)}{0} = \frac{\sigma A}{\epsilon_0} \Rightarrow \vec{E} = \frac{\sigma}{\epsilon_0}\overset{(up)}{\hat{z}} = \frac{5Q}{\epsilon_0} \text{ up.}$$

$$\boxed{\vec{E} = 5.65\times 10^{5}\,\frac{N}{C}\ \text{\underline{up}}^{*}}$$

(or $\frac{V}{m}$ units, ok too). since $\frac{N}{C} = \frac{V}{m}$

* must put in direction of field

Rubric for Problem 2, McKee Midterm 2
Spring 2011, Physics 7B

10 points

+ 1 pt Gaussian surface drawing

+ 2 pts Gauss's law correctly used for Gaussian surface

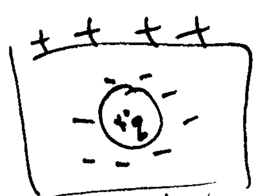
+ 1 pt correct units $\frac{N}{C}$ or $\frac{V}{m}$

+ 1 pt direction (either a drawing or a clear $\hat{z}$ or $\hat{y}$ whichever was away from plane, or words ok)

+ 1 pt calculations, or correct algebra

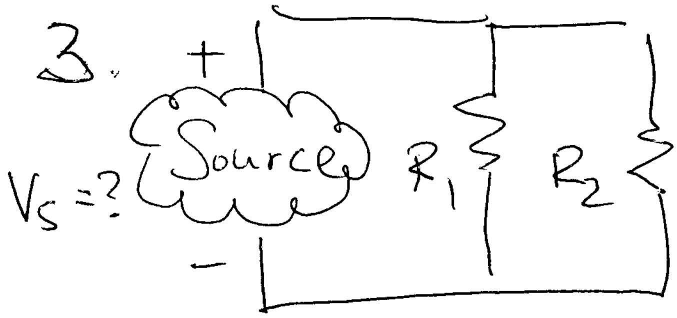
+ 2 pts. correct surface charge for Gaussian surface

+ 2 pts. explanation of what's going on (drawing ok or words)



→ needed to mention/draw negative charge on inner surface of cavities balanced by positive charge on outside of conductor.

3.

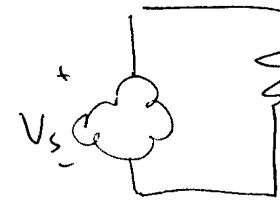


$P_s = 100\,W$
$R_1 = 10\,k\Omega$
$R_2 = 5\,k\Omega$

Solution + Rubric

a. $V_s = ?$ [15 points]

$$P_s = 100\,W = \frac{V_s^2}{R_{eq}}$$



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10}{3}\,k\Omega$$

$$R_{eq} \cong 3.3\,k\Omega$$

$$V_s = \sqrt{P_s \cdot R_{eq}}$$
$$= \sqrt{100\,W \times 3.3 \times 10^3\,\Omega}$$
$$V_s \simeq 577\,V$$

Rubric

$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$: + 5 pts

$P_s = \frac{V_s^2}{R_{eq}}$: + 5 pts

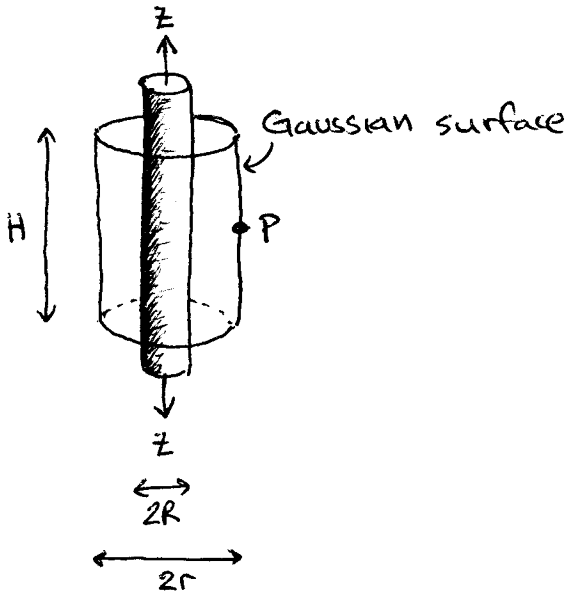
$V_s = \sqrt{P_s R_{eq}}$: + 5 pts (incorrect numerical result: − 2 pts)

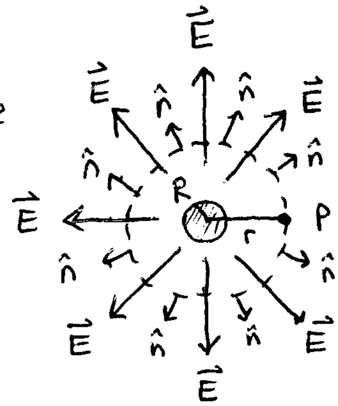
b. $P_{5k\Omega} = \frac{V_s^2}{5k\Omega} \cong 66.7\,W$ + 5 pts

or calculate I_{5k} and use $P_{5k\Omega} = I_{5k} \cdot V_s$

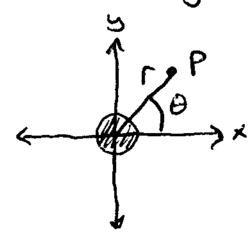
(No numerical answer: −2 pts)

Solutions to #4





$r^2 = x^2 + y^2$



(a) Use Gauss's Law to find $\vec{E}$ at P:

$$\oint \vec{E}\cdot\hat{n}\,dA = E\oint dA = E(2\pi r H) \quad , \quad Q_{encl} = \lambda H$$

$$\rightarrow \oint \vec{E}\cdot\hat{n}\,dA = \frac{Q_{encl}}{\epsilon_0} \rightarrow E(2\pi r H) = \frac{\lambda H}{\epsilon_0} \rightarrow \underbrace{\boxed{E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}}}_{\text{magnitude}}$$

The electric field is $\boxed{\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\hat{r} \quad , \quad r = \sqrt{x^2+y^2}}$. units: $\frac{\text{charge/length}}{\epsilon_0 \cdot \text{length}}$ ✓

(b) Choose reference on surface of cylinder. Then V=0 when r=R:

$$V(r) - \cancelto{0}{V(R)} = -\int_R^r \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r'}\,dr' = -\frac{1}{2\pi\epsilon_0}\lambda \ln(r/R)$$

$$\boxed{V(r) = -\frac{1}{2\pi\epsilon_0}\lambda \cdot \ln\left(\sqrt{x^2+y^2}/R\right)}$$

(c) Use superposition:

$$V_{TOTAL} = V_1 + V_2 = -\frac{1}{2\pi\epsilon_0}\lambda \ln\left(\sqrt{x^2+y^2}/R\right) - \frac{1}{2\pi\epsilon_0}\lambda \ln\left(\sqrt{x^2+z^2}/R\right)$$

$$\boxed{V_{TOTAL} = -\frac{1}{2\pi\epsilon_0}\lambda \ln\left(\sqrt{(x^2+y^2)(x^2+z^2)}/R^2\right)}$$

And the reference point must be at $\boxed{\vec{r}_{ref} = (R, 0, 0)}$.

Common mistakes:

- Choosing reference $V \to 0$ as $r \to \infty$ in part (b)
- Not drawing a Gaussian surface and instead using Gauss's Law to find $\vec{E}$ on surface of cylinder. $(r = R)$
- Try to find $\vec{E}_{TOT} = \vec{E}_1 + \vec{E}_2$ instead of $V_{TOT} = V_1 + V_2$. Note: Not incorrect, but much more difficult.

−2pts • Thinking that V is a vector and finding components. This mistake cost points!

Rubric:

Part (a)

+1 − Draw field lines
+1 − $Q_{enc} = \lambda H$
+1 − $\oint \vec{E} \cdot \hat{n}\, dA = E(2\pi r H)$
+1 − $\vec{E} \propto \hat{r}$

−1 − wrong units
−2 − weird Gauss surface mistakes

Part (b)

+1 − Set up integral
+1 − Evaluate integral
+1 − Choose good reference
+1 − Convert to Cartesian

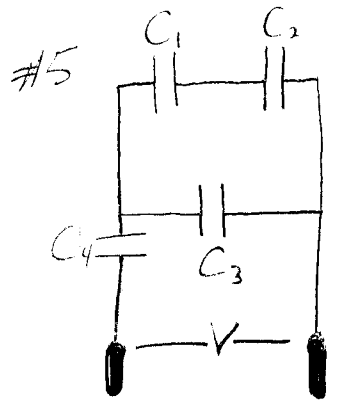
−2 − Write $V = \langle V_x, V_y, V_z \rangle$

Part (c)

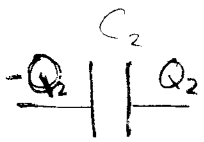
+2 − Use superp.
+2 − Add potential instead of $\vec{E}$
+2 − Choose good ref. point.
+1 − Evaluate sum correct

#5 Answer / Rubric

#5

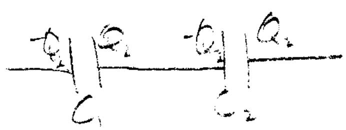


(1) 1



(2) $C_2 = \frac{Q_2}{V_2} \Rightarrow V_2 = Q_2/C_2$ 2

(3) by charge conservation $Q_1 = Q_2$ 2



(4) So $V_1 = Q_1/C_1 = Q_2/C_2$ 1

(5) Since $V_3 = V_1 + V_2$ since they are parallel 3

(6) $\Rightarrow Q_3 = C_3 V_3 = C_3(V_1+V_2) = C_3(Q_2/C_1 + Q_2/C_2) = Q_2\left(\frac{C_3}{C_1} + \frac{C_3}{C_2}\right)$ 2

(7) $Q_4 = Q_1 + Q_3 = Q_2 + Q_3 = Q_2 + Q_2\left(\frac{C_3}{C_1} + \frac{C_3}{C_2}\right)$ 2

(8) $V = V_1 + V_2 + V_4 = V_3 + V_4$ 2

(9) $V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_4}{C_4} = \frac{Q_2}{C_1} + \frac{Q_2}{C_2} + \left(Q_2 + Q_2\left(\frac{C_3}{C_1} + \frac{C_3}{C_2}\right)\right)\frac{1}{C_4}$

$= Q_2\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_4} + \frac{C_3}{C_4}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right)$ 5

#5 Common Errors

Algebra Mistakes

make sure to check the units of your final answer to check for some of these.

$$V = \frac{Q}{C} \quad \text{not} \quad QC \quad \text{or} \quad \frac{C}{Q}$$

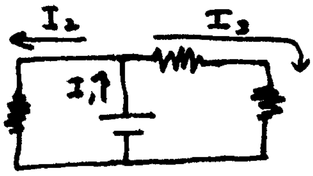
$$Q_4 = Q_{tot} = Q_2 + Q_3 \neq Q_1 + Q_2 + Q_3 + Q_4$$

$$V_{bt} = V_1 + V_2 + V_4 \neq V_1 + V_2 + V_3 + V_4$$

Lecture 1 Midterm 2

Problem 6

a) with the switch closed, the circuit looks like



Writing Kirchoff's laws:

(1) $I_1 = I_2 + I_3$

(2) $\mathcal{E} - (20\,\Omega) I_1 - (75\,\Omega) I_2 = 0$

(3) $\mathcal{E} - (20\,\Omega) I_1 - (30\,\Omega + 50\,\Omega) I_3 = 0$

(12 points)

Since the voltmeter reads 15 V

(4) $I_3 = \dfrac{15\text{ V}}{50\,\Omega} = .3\text{ A}$ (2 points)

Using (4) and (3) to solve for I_1

(5) $I_1 = \dfrac{\mathcal{E} - 24}{20}$

Using (5) and (1) in equation (2)

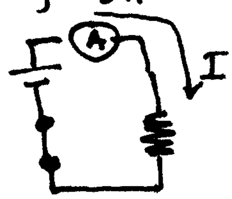
$$\mathcal{E} - (\mathcal{E} - 24) - 75\left(\frac{\mathcal{E}-24}{20} + .3\right) = 0$$

$$\Rightarrow \quad \mathcal{E} = \left(\frac{24}{75} + .3\right)20 + 24$$

$$= 36.4\text{ V}$$

(4 points)

b. Focusing on the rightmost loop



Kirchoff's law says

$25\text{V} - I\,(50\,\Omega) = 0$

$\Rightarrow \quad I = .5\text{ A}$ (7 points)