Solutions to prob. 1

a) $p = 1\text{atm} = 1.01325 \times 10^5$ Pa

$V = 1\text{cm}^3 = 10^{-6}\ \text{m}^3$

$N = 10^{20}$

$k_B = 1.38 \times 10^{-23}$ J/k

$$\boxed{T = \frac{pV}{N k_B} = 72.3\,\text{k}}$$

Marks

(2) ≡ ideal gas law

(1) ≡ correct N

(1) ≡ V in correct units

(1) ≡ P in correct units

(1 point deducted if answer was miscalculated w/ the correct numbers).

b) $N(v \to v+\Delta v) \doteq f(v) \cdot \Delta v = 4\pi N \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot v^2 \cdot e^{-mv^2/(2k_B T)} \cdot \Delta v$

$T = 100\,\text{k}$

$m = m_{O_2} = 2 \cdot A \cdot u = 2 \cdot 16 \cdot 1.66 \times 10^{-27}\ \text{kg} = 5.31 \times 10^{-26}\ \text{kg}$

$$N(1000 \to 1001) \doteq 4\pi N \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot (1000\,\text{m/s})^2 \cdot e^{-m\cdot(1000\,\text{m/s})^2/(2k_B T)} \cdot 1\,\text{m/s}$$

$$N(300 \to 301) \doteq 4\pi N \cdot \left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot (300\,\text{m/s})^2 \cdot e^{-m(300\,\text{m/s})^2/(2k_B T)} \cdot 1\,\text{m/s}$$

$$\boxed{\therefore \frac{N(1000 \to 1001)}{N(300 \to 301)} = \left(\frac{1000}{300}\right)^2 \cdot e^{-m/2k_B T\left((1000\,\text{m/s})^2 - (300\,\text{m/s})^2\right)} = 2.77 \times 10^{-7}}$$

Marks:

(2) for $\int_0^\infty f(v)\,dv = N$ (either directly or indirectly stated)

(2) for $f(v) = 4\pi N \cdot \left(m/(2\pi k_B T)\right)^{3/2}\ v^2 \cdot e^{-mv^2/2k_B T}$

(1) for having an expression of $N(1000 \to 1001)/N(300 \to 301)$ unsolved

(2) for having the correct mass

(2) for $N(v \to v+\Delta v) \doteq f(v) \cdot \Delta v$ (if a calculator was used, marks were awarded only if final answer was correct)

(1) correct final answer.

McKee problem # 2

February 28, 2011

The key thing to notice is that the wind from the chinook is *fast* so it does not exchange heat with its surrounding. This means that the process is *adiabatic* [**5 pts.**]. Also, we are told that the air molecules are *diatomic*, which means $d = 5$ [**1 pt.**] and $\gamma = \frac{d+2}{d} = \frac{7}{5}$ [**1 pt.**]. Finally, since we are told the wind is strong it is safe to assume that the chinook entirely displaces the air originally in Denver.

Using the ideal gas law $PV = NkT$ [**1 pt.**], we can rewrite the adiabatic equation in terms of P and T:

$$\begin{aligned} P_1 V_1^\gamma &= P_2 V_2^\gamma \\ \Rightarrow P_1 \left(\frac{NkT_1}{P_1}\right) &= P_2 \left(\frac{NkT_2}{P_2}\right) \\ \Rightarrow & \boxed{P_1^{1-\gamma} T_1^\gamma = P_2^{1-\gamma} T_2^\gamma.} \end{aligned} \tag{0.1}$$

Fully solving for this equation, or using any other method to find an expression for T from the adiabatic equation, gave you [**5 pts.**]. Partial credit was given to students who had the right idea but did not quite get to the final equation.

Plugging in the given values, we find $T_2 = 287K = 14°C$ so $\boxed{\Delta T = 12°C}$ [**2 pts.**].

Common mistakes: (i) Assuming that this is a constant volume process, and solving for the temperature from the ideal gas law: $P_1/T_1 = P_2/T_2$. In fact, volume is changing as can easily be seen from the adiabatic equation $PV^\gamma = const$.

(ii) Using the calorimetry equation $\Delta Q = C_V \Delta T$. This is incorrect (a) because there is no heat flow, $Q = 0$, and (b) because this equation can only be used in constant volume processes, while the volume of the wind is changing since the process is adiabatic.

(iii) Integrating to find the work in an adiabatic process and using the first law to try to solve for T. It is true that for an adiabatic process ($Q = 0$), the first law gives

$$\Delta E_{int} = Q - W = -W. \tag{0.2}$$

However, trying to use this to solve for T results in a tautology:

$$\begin{aligned}
\Delta E_{int} &= -W \\
\Rightarrow \frac{d}{2}Nk(T_2 - T_1) &= -\int_{V_1}^{V_2} PdV \\
&= -P_1V_1^{\gamma}\int_{V_1}^{V_2}\frac{dV}{V^{\gamma}} \\
&= \frac{P_1V_1^{\gamma}}{\gamma-1}(V_2^{1-\gamma} - V_1^{1-\gamma}) \\
&= \frac{1}{\gamma-1}(P_2V_2 - P_1V_1) \\
&= \frac{d}{2}(P_2V_2 - P_1V_1) \\
&= \frac{d}{2}Nk(T_2 - T_1). \qquad (0.3)
\end{aligned}$$

(In the third line we substituted the adiabatic equation, and in the final line we used the ideal gas law.) Trying to solve this for T is like trying to solve for T from the equation $1 = 1$.

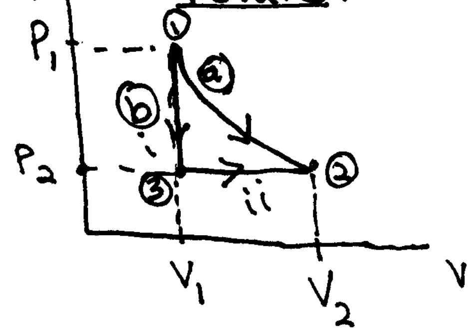
Note: Instead of starting from the adiabatic equation, some students started going through the steps needed to derive it. The correct argument goes like this: starting from the first law in infinitesimal form with $Q = 0$,

$$\begin{aligned}
&\frac{d}{2}NkdT = -PdV \\
\Rightarrow &\frac{d}{2}d(PV) = -PdV \\
\Rightarrow &\frac{d}{2}VdP = -\left(\frac{d}{2}+1\right)PdV \\
\Rightarrow &\frac{dP}{P} = -\left(\frac{d+2}{d}\right)\frac{dV}{V} \\
\Rightarrow &\int_{P_1}^{P_2}\frac{dP}{P} = -\gamma\int_{V_1}^{V_2}\frac{dV}{V} \\
\Rightarrow &\ln P_2 - \ln P_1 = -\gamma\ln V_2 + \gamma\ln V_1 \\
\Rightarrow &P_1P_2^{-1} = V_1^{-\gamma}V_2^{\gamma} \\
\Rightarrow &\boxed{P_1V_1^{\gamma} = P_2V_2^{\gamma}.} \qquad (0.4)
\end{aligned}$$

Of course you didn't need to derive the equation to use it.

Problem 3, McKee Midterm 1, Spring 2011

Picture!



a) monatomic ideal gas

$\Rightarrow$ 3 degrees of freedom

$P_1V_1 = P_2V_2$. $\quad T_1 = \frac{P_1V_1}{Nk} = T_2$

isothermal $\Rightarrow \Delta T = 0 \Rightarrow \Delta E = 0$.

1st law $\quad \Delta E = Q - W = 0 \Rightarrow Q = W$. (

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{NkT_1\,dV}{V} = P_1V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$\boxed{Q_a = P_1V_1 \ln\left(\frac{V_2}{V_1}\right)} \qquad \left(\text{or } Q_a = P_2V_2 \ln\left(\frac{V_2}{V_1}\right)\right)$$

b) isovolumetric $\Rightarrow W = 0$, $\Rightarrow Q = \Delta E$

$$T_3 = \frac{P_2V_1}{Nk_B} \qquad T_1 = \frac{P_2V_2}{Nk_B} \qquad \Delta E = \frac{d}{2} Nk\,\Delta T$$

$$= \frac{3}{2}\cancel{Nk}\left(\frac{P_2V_1}{\cancel{Nk}} - \frac{P_2V_2}{\cancel{Nk}}\right)$$

$$Q_i = \frac{3}{2} P_2(V_1 - V_2)$$

isobaric : since $T_1 = T_2$, $\Delta E_i = -\Delta E_{ii}$

$\Delta E_{ii} = +\frac{3}{2} P_2(V_2 - V_1)$; $\quad W_{ii} = P_2(V_2 - V_1)$

$$Q_{ii} = \Delta E_{ii} + W_{ii} = \frac{5}{2} P_2(V_2 - V_1)$$

$$Q_b = Q_{ii} + Q_i = P_2(V_2 - V_1) \qquad \boxed{Q_b = P_2(V_2 - V_1)}$$

Grading Rubric #3

5 pt. isothermal.

4 pts if no work.

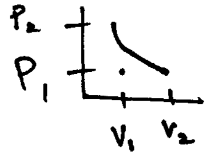
~~3~~ 4 pts if not written in terms of given $P_1 V_1$ etc.

2 pts W

~~4~~ 1 pts. $\Delta E = 0 \Rightarrow Q = W$.

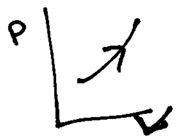
2 pts. $T_1 = \frac{P_1 V_1}{Nk}$.

−2



−2

−2



−1 for $PV \ln\left(\frac{V_2}{V_1}\right)$

−2 for $Q = P_4 V\left(1 - \frac{V_1}{V_2}\right)$

~~8~~ pts isovolumetric

pts. $W = 0$; $Q = \Delta E$

2 pts T_2, T_1

−1 for writing $(T_2 - T_1)$ (= 0) not in givens

$Q_i = \frac{3}{2} V_1 (P_2 - P_1)$

−1 for not knowing $d = 3$

pts. isobaric.

4 — ΔE → 2 pts T_2, T_m.

→ 1 pts. $3/2$.

→ 1 pt. $\Delta E = \frac{d}{2} Nk\Delta T$.

1 pt $P\Delta V$ ← 3 — W

pts

1 pt $P = P_2$

1 pt $V_2 - V_1$

$Q = \frac{5}{2} P_2 (V_2 - V_1) = \frac{5}{2} V_1 (P_1 - P_2)$

$\frac{3}{2} P_2 V_1 - \frac{3}{2} P_1 V_1$

$+ \frac{5}{2} P_2 V_2 - \frac{5}{2} P_2 V_1$

$=$

$\frac{5}{2} P_2 V_2 - \frac{3}{2} P_1 V_1 - P_2 V_1$

$= V_1 (P_1 - P_2)$

for combining $Q_i + Q_{ii}$

−2 for $-Q_i + Q_{ii} = Q_{tot}$.

−2 for $(T_i - T_f)$, $(\Delta T_i = \Delta T_{ii})$

Problem 4

Givens: Refrigerator with T_L= -17ºC, T_H = 25ºC,

m = 0.4 kg, c_{ice} = 0.5 cal g^{-1} K^{-1}, L_f = 333 kJ kg^{-1}

c_{water} = 4186 J kg^{-1} K^{-1} (1 pt)

$$c_{ice}=(0.5\frac{cal}{gK})(\frac{4.185\,J}{cal})(\frac{1000\text{g}}{kg})=2092\frac{J}{kg\,K}$$

$$Q_L=m\,c_{water}\Delta T_1+m\,L_f+m\,c_{ice}\Delta T_2$$

(6 pts)

$$Q_L=(0.4)(4186)(25)+(0.4)(333000)+(0.4)(2092)(17)=1.89*10^5\,J$$

Carnot Refrigerator: $COP=\frac{T_L}{T_H-T_L}=\frac{(273-17)}{(273+25)-(273-17)}=6.095$ (5 pts)

$$COP=\frac{Q_L}{W}\quad\rightarrow\quad W=\frac{Q_L}{COP}=1.89*10^5/6.095\;=\;\mathbf{3.11*10^4\,J}$$ (3 pts)

Common mistakes:

- saying that Q is W and ignoring COP.
- using the COP for a heat pump instead of a refrigerator
- using ΔE = Q – W, which applies to the working substance of the refrigerator, not the water
- unit conversion errors

McKee Midterm 1

Problem 5

a. $\Delta S = \int \frac{dQ}{T}$

But since T is constant

$\Delta S = \frac{1}{T}\int dQ = \frac{Q}{T}$ [2 points]

$= \frac{-m_w L_v - m_w c_{steam} \Delta T_{balloon}}{T}$ [2 points]

$= \frac{1}{423\text{ K}}\left(\left(1\text{ g}\right)\left(2260\text{ J/g}\right) + \left(4.186 \frac{J}{cal}\right)\left(.48 \frac{cal}{gK}\right)\left(1\text{ g}\right)\left(50\text{ K}\right)\right)$

$= -5.58$ J/K [3 points]

b. $\Delta S = \Delta S_{vaporization} + \Delta S_{heating}$

$\Delta S_{vaporization} = \int \frac{dQ}{T} = \frac{1}{T_{boiling}}\int dQ = \frac{Q_{vaporization}}{T_{boiling}} = \frac{m_w L_v}{T_{boiling}}$

$= \frac{2260\text{ J}}{373\text{ K}}$ [4 points]

$\Delta S_{heating} = \int \frac{dQ}{T} = \int \frac{mc\,dT}{T} = mc\ln\left(\frac{T_f}{T_i}\right)$

$= (1\text{ g})\left(.48 \frac{cal}{gK}\right)\left(4.186\text{ J/K}\right)\ln\left(\frac{423}{373}\right)$ [6 point

$\Delta S = 6.31$

c. Several different right answers:

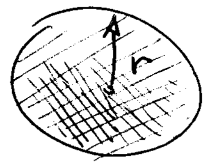
- The steam exchanges heat at constant T while the balloon exchanges heat at varying T, so although $|Q_{steam}| = |Q_{balloon}|$, $|\Delta S_{steam}| \neq |\Delta S_{balloon}|$
- The H_2O underwent a change in phase, which implies a change from an ordered to disordered state.
- The process is irreversible and no heat is exchanged from the universe, so $\Delta S \geq 0$.

etc.

Problem 6

$r = 2.76$ cm
$m = 1.00$ kg
$\varepsilon = 0.065$
$T_i = 300$K
$T_f = 2.73$K

Initial temperature T_i



Lead sphere, emissivity ε

mass m radius r

Final temperature T_f

Atomic weight $A = 208$

$d = 6$ degrees of freedom

- How long till sphere cools from T_i to T_f?

Sphere is losing energy due to blackbody radiation at rate $P = \varepsilon\sigma A_s(T^4(t) - \underbrace{T^4_{background}}_{\text{ignore}})$, but we can ignore $T^4_{background}$. (where A_s is the surface area of the sphere)

We also have $-P = \frac{dQ}{dt}$, where the minus sign accounts for the fact that $\frac{dQ}{dt}$ has to be negative.

If we know the heat capacity C, we could write $dQ = C\,dT$ to give:

$$\frac{dQ}{dt} = C\frac{dT}{dt} = -\varepsilon\sigma A_s T^4(t).$$

We can solve this by multiplying by dt and dividing by $T^4(t)$:

$$C\frac{dT}{T^4} = -\varepsilon\sigma A_s\,dt$$

$$C\int_{T_i}^{T_f}\frac{dT}{T^4} = -\varepsilon\sigma A_s\int_0^t dt$$

$$-\frac{C}{3}\left(\left(\frac{1}{T_f}\right)^3 - \left(\frac{1}{T_i}\right)^3\right) = -\varepsilon\sigma A_s t$$

$$\boxed{t = \frac{C}{3\varepsilon\sigma A_s}\left(\frac{1}{T_f^3} - \frac{1}{T_i^3}\right)}$$

To find C, we use the definition $C = \frac{dQ}{dT}$. The first law of thermo holds, so we have $dQ = dE_{int} + dW$. Since the sphere does no appreciable work on its surroundings, we have $dW = 0$, so $dQ = dE_{int}$. For a solid with 6 degrees of freedom, we can use the equipartition theorem to write $dE_{int} = \frac{6}{2}Nk_B dT = 3Nk_B dT$

This gives us: $C = \frac{dQ}{dT} = \frac{dE_{int}}{dT} = \frac{3Nk_B dT}{dT} = \boxed{3Nk_B = C}$

We need N, which we can get as $N = \frac{m}{m_{atom}}$, where $m_{atom} = A \cdot 1u$

and $1u = 1.66 \times 10^{-27}$ kg. We finally get:

$$t = \frac{3\left(\frac{m}{A \cdot 1.66 \times 10^{-27}}\right) k_B}{3 \varepsilon \sigma (4\pi r^2)} \left[\frac{1}{T_f^3} - \frac{1}{T_i^3}\right]$$

$$t = \frac{3 \times \left(\frac{1.00\,\text{kg}}{208 \cdot 1.66 \times 10^{-27}\,\text{kg}}\right) \times 1.38 \times 10^{-23}\,\text{J/K}}{3 \times 0.065 \times 5.67 \times 10^{-8}\,\text{W/m}^2\text{K}^4 \times 4\pi \times [2.76 \times 10^{-2}\,\text{m}]^2} \left[\frac{1}{(2.73\text{K})^3} - \frac{1}{(300\text{K})^3}\right]$$

$$\boxed{t = 5.57 \times 10^{10}\ \text{s}}$$

Problem 6 Rubrik: (20 points total)

$P = \varepsilon\sigma(\text{Area})T^4$ 3 ~~4~~ pts.

$P = \frac{dQ}{dt}$ 2 pts.

$\frac{dQ}{dt} = C\frac{dT}{dt}$ 2 pts.

$C\frac{dT}{dt} = -\varepsilon\sigma(\text{Area})T^4$ 4 pts.

$t = \frac{C}{3\varepsilon\sigma(\text{Area})}\left(\frac{1}{T_f^3} - \frac{1}{T_i^3}\right)$ 5 pts.

$C = \frac{dQ}{dT} = \frac{6}{2}Nk_B = 3Nk_B$ 4 pts.

$N = \frac{m}{m_{atom}}$

$m_{atom} = A \times 1.66\times10^{-27}\,\text{kg}$

~~Bl~~ 2 pts.

Correct numerical answer: 2 pts.

$7 + 5 + 4 + 2 + 2 = 20$ pts.

Common problems:

$$\frac{\Delta Q}{\Delta t} = \varepsilon \sigma A T_{init.}^4 \text{, not a fcn. of time}$$

$$\frac{\Delta Q}{\Delta t} = \varepsilon \sigma A \left(T_{init}^4 - T_{final}^4 \right) \text{ just plain wrong.}$$

used C_p instead of C_v