

**UNIVERSITY OF CALIFORNIA
COLLEGE OF ENGINEERING**

**E7: INTRODUCTION TO COMPUTER PROGRAMMING
FOR SCIENTISTS AND ENGINEERS**

Professor Raja Sengupta
Spring 2010
Final Exam—May 11, 2010

[85 points ~ 3 hours]

Question	Points	Question	Points
Part I	5	Part VII	5
Part II	5	Part IX	5
Part III	5	Part X	5
Part IV	5	Part XI	17
Part V	5	Part XII	13
Part VI	5		
TOTAL			85

Notes:

1. Your exam should have 38 pages. Check this before you begin.
2. You may use a calculator, your notes, and the textbook on this examination as necessary provided that you do not impede those sitting next to you. No electronic devices are permitted.
3. Use a #2 pencil and green scantron sheet to record your answers. Bubble in your solution to each question on the corresponding space on your scantron. There is one correct answer for each question. Multiple bubbles, incomplete bubbles, or stray marks will cause your solution to be marked incorrect.
4. Please write your **name**, **student ID number**, and **discussion section** on your scantron for identification purposes.

Part I:

Questions 1-5: Basic Programming

1. Given the following:

```
>> a = [1,2,3,4]
```

What answer does MATLAB give if you type the command `size(a)`?

- a) `ans = 1`
- b) `ans = 4`
- c) `ans = 1 4`
- d) `ans = 4 1`
- e) none of the above

2. Given the following

```
>> a = [1,2,3,4]; b = [1;2;3;4];
```

Which of the following MATLAB commands will return an error?

- a) `b * a`
- b) `a * b`
- c) `b .* a`
- d) `a .* b'`
- e) none of the above

3. Select the correct MATLAB command to fill in the blank:

```
>> x = pi  
>> _____ ( x , pi )  
ans =  
1
```

- a) `isa`
- b) `isequal`
- c) `strcmp`
- d) `type`
- e) `lt`

4. Select the correct MATLAB command to fill in the blank:

```
>> _____ ('%1.4f is the value of %s', pi, 'Pi')
ans =
3.1416 is the value of Pi
```

- a) isa
- b) sprintf
- c) disp
- d) num2str
- e) none of the above

5. Select the correct MATLAB command to fill in the blank:

```
>> x = 1;
>> y = 0;
>> _____ (x, y)
ans =
     1
```

- a) and
- b) not
- c) xor
- d) lt
- e) isequal

Part II:

Questions 6-10: Linear Algebra

6. Consider a 3x3 input matrix A and the following Matlab command and output:

```
>> det(A)
ans =
     0
```

Given the preceding information, what will the output be of the following command?

```
>> inv(A)
```

a)
ans =
 0 0 0
 0 0 0
 0 0 0

b)
ans =
 1 0 0
 0 1 0

```
0 0 1
```

c)

Warning: Matrix is singular to working precision.

ans =

```
Inf    Inf    Inf
Inf    Inf    Inf
Inf    Inf    Inf
```

d) Not enough information to determine

7. Which MATLAB command can be used to find a solution for $A*x = b$

a) `pinv(A,b)`

b) `inv(A,b)`

c) `mldivide(A,b)`

d) `inv(b)*A`

e) None of the above

Given the following system of linear equations and the matrices $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, answer questions #8-10.

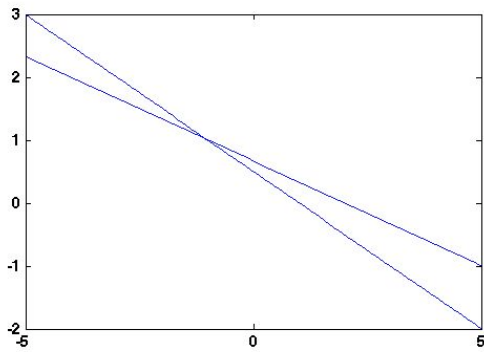
$$2x + 4y = 2$$

$$x + 3y = 2$$

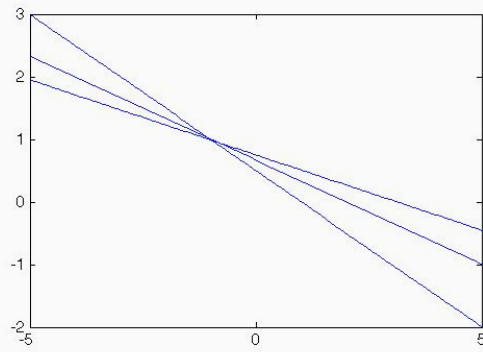
$$x + 2y = 1$$

8. If you were to graph on the same plot each of the linear equations, which of the following plots is a graph of all three lines? (Note: each of the plots below shows three lines).

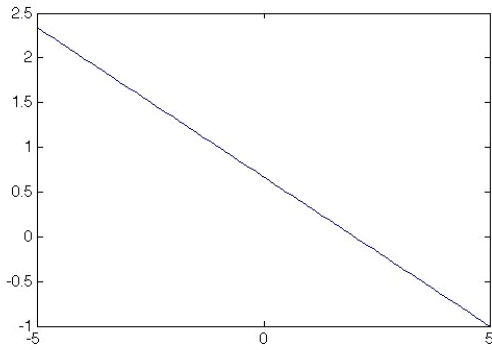
a)



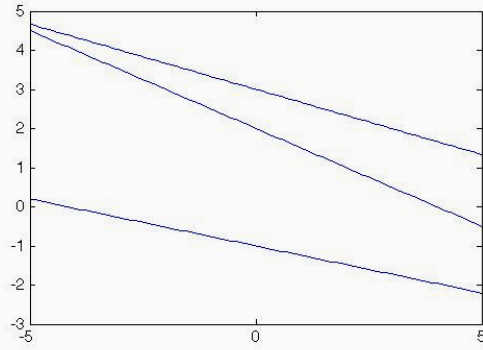
c)



b)



d)



9. Given the A and B **defined above**, which of the following statements will return `ans = 1` ?

- a) `isequal(rank(A) ,rank(b))`
- b) `isequal(rank(A) ,rank([A,b]))`
- c) `le(rank(A) ,rank(b))`
- d) `lt(rank(A) ,rank([A,b]))`
- e) all of the above

10. Given the A and B **defined above**, which of the following MATLAB commands returns an error message?

- a) `mldivide(A,b)`
- b) `inv(A)*b`
- c) `A\b`
- d) all of the above
- e) b and c only

Part III:

Questions 11-15: Computer Representation of Numbers

11. What is the single precision exponent field of the IEEE 754 representation of the decimal number 128?

- a) 00000000
- b) 10000110
- c) 01111111
- d) 00001000
- e) 01000000

12. What is the single precision fraction field of the IEEE 754 representation of the decimal number 128?

- a) 0000 0000 0000 0000 0000 000
- b) 1000 0000 0000 0000 0000 000
- c) 1111 1110 0000 0000 0000 000
- d) 0100 0000 0000 0000 0000 000

For questions 13-15, consider the following numbers A and B, expressed in IEEE 754 single precision format

A = 0 0111 1111 0000 0000 0000 0000 0000 000

B = 0 1000 0000 0000 0000 0000 0000 0000 000

13. What is the result of the addition $A + B$, expressed as a decimal number?

- a) 1
- b) 1.5
- c) 3
- d) 5

14. What is the result of the addition $A + B$, expressed in single precision IEEE 754 format?

- a) 0 0111 1111 1000 0000 0000 0000 0000 000
- b) 0 0111 1111 1111 1111 1111 1111 1111 111
- c) 0 1000 0000 1000 0000 0000 0000 0000 000
- d) 0 1100 0000 1000 0000 0000 0000 0000 000

15. Consider the numbers x and y , which are stored in IEEE 754 single precision form, and which satisfy the inequality $A < x < y < B$. What is the smallest possible value of $y - x$? Express your answer in decimal notation

- a) 0
- b) 2^{-24}
- c) 2^{-23}
- d) 1

Part IV:

Questions 16-20: Root Finding

16. Which of the following commands will return true?

- a) `gt(fzero(@cos,0.5*pi), 0.2*pi) && lt(fzero(@cos,0.5*pi), 0.8*pi)`
- b) `gt(fzero(@sin,0.5*pi), 0.2*pi) && lt(fzero(@sin,0.5*pi), 0.8*pi)`
- c) `gt(fzero(@exp,0.5*pi), 0.2*pi) && lt(fzero(@exp,0.5*pi), 0.8*pi)`
- d) `gt(fzero(@sqrt,0.5*pi), 0.2*pi) && lt(fzero(@sqrt,0.5*pi), 0.8*pi)`

17. Given a function $f(x) = x^2 - 10x$, which of the following roots will Newton's method converge to given the initial estimate $x_0 = 7$?

- a) 10
- b) 0
- c) 7
- d) Newton's method will not converge with these inputs

18. Given a function $f(x) = x^2 - 10x$, which of the following will Newton's method converge to given the initial estimate $x_0 = 5$?

- a) 10
- b) 0
- c) 5
- d) Newton's method will not converge with these inputs

19. What will be the output of the following Matlab command?

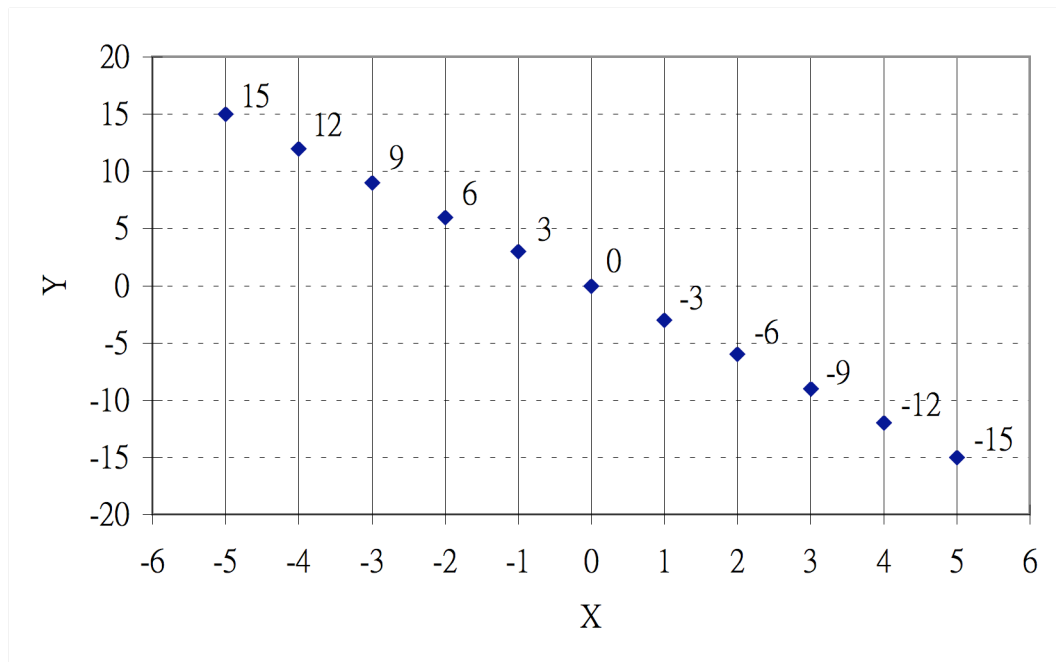
```
>> fzero(@(x) 15, 0)
```

- a) 1
- b) 0
- c) 2
- d) none of the above

20. Which of the following statements is true?

- a) Bisection method always finds a root for any function
- b) Newton's method always finds a root for any function
- c) The root found using Newton's method may be different for two different initial guesses
- d) Newton's method will never diverge

Part V:
Questions 21-25: Interpolation



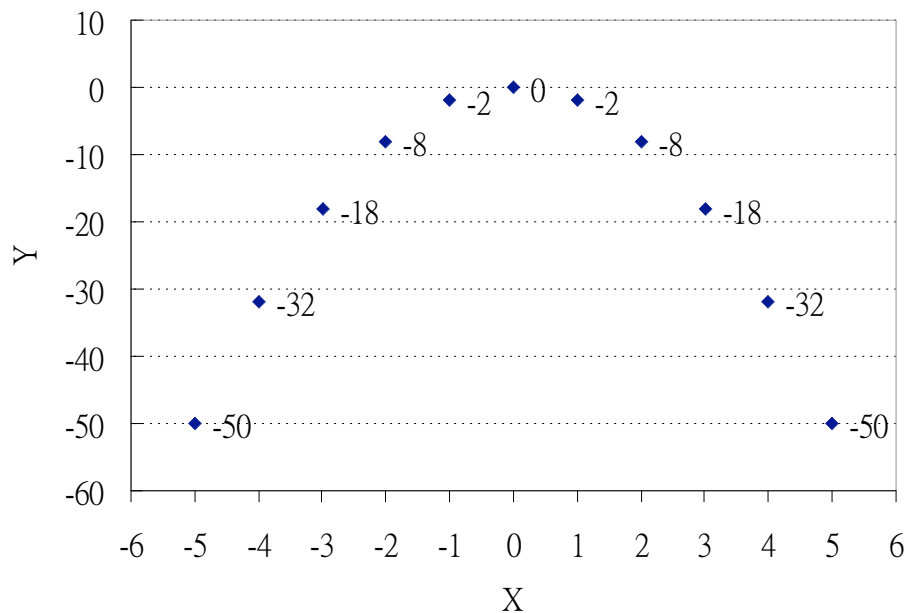
Answer Problems 21 and 22 based on the data in this chart.

21. Using linear interpolation, what is the value of Y corresponding to $X = 4.5$?

- a) -13.25
- b) -13.5
- c) -13.75
- d) -14

22. Using cubic spline interpolation, what is the value of Y corresponding to $X = -2.5$?

- a) 7.25
- b) 7.5
- c) 7.75
- d) 8



Answer Problem 23, 24, and 25 based on data in this chart. The data points can be fit exactly by a quadratic polynomial.

23. Using linear interpolation, what is the value of Y corresponding to $X = 3.5$?

- a) -24
- b) -24.5
- c) -25
- d) -25.5

24. Using cubic spline interpolation, what is the value of Y corresponding to $X = 2.5$?

- a) -12
- b) -12.5
- c) -13
- d) -13.5

25. Given the set of data in the above chart, the cubic spline interpolation method will:

- a) always produce y estimates less than or equal to linear interpolation
- b) always produce y estimates greater than or equal to linear interpolation
- c) always produce y estimates equal to linear interpolation
- d) none of above

Part V:

Questions 26-30: Regression

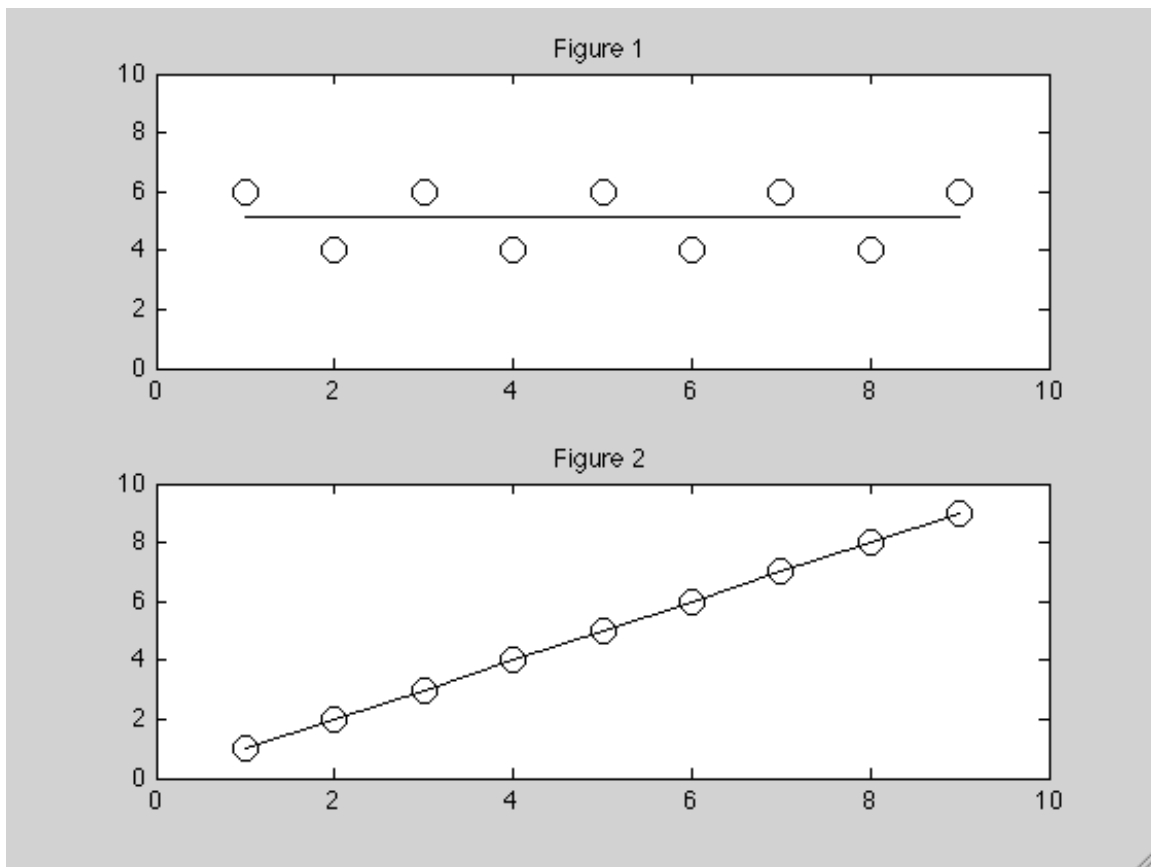
26. Which of the following commands would return the coefficients (a and b) corresponding to the least-squares fit of the model $y = a \sqrt{x} + b$? Use the following as a predefined data set (as column vectors in Matlab):

$$x = [x_1 \ x_2 \ \dots \ x_n]'$$

$$y = [y_1 \ y_2 \ \dots \ y_n]'$$

- a. $Y = y$; $X = [\text{ones}(n,1), x.^{0.5}]$; $\text{sol} = \text{inv}(X' * X) * (X' * Y)$; $a = \text{sol}(1)$; $b = \text{sol}(2)$
- b. $Y = y$; $X = [x.^{0.5}, \text{ones}(n,1)]$; $\text{sol} = \text{inv}(X' * X) * (X' * Y)$; $a = \text{sol}(1)$; $b = \text{sol}(2)$
- c. $\text{sol} = \text{polyfit}(x,y,1)$; $a = \text{sol}(1)$; $b = \text{sol}(2)$
- d. $\text{sol} = \text{polyfit}(x.^{0.5},y,2)$; $a = \text{sol}(1)$; $b = \text{sol}(2)$
- e. none of the above

27. Consider the two plots below. Each represents a set of data and best fit line. Which of the following statements regarding the coefficient of determination (R^2) is true?



- a. Figure 1 has $R^2 = 0$ and Figure 2 has $R^2 = 0$
- b. Figure 1 has $R^2 = 0$ and Figure 2 has $R^2 = 1$
- c. Figure 1 has $R^2 = 1$ and Figure 2 has $R^2 = 0$
- d. Figure 1 has $R^2 = 1$ and Figure 2 has $R^2 = 1$
- e. None of the above

28. Which of the following commands will determine the proportionality constant k corresponding to the least-squares fit of the model $y = kx$? Use the following as a predefined data set (as column vectors in Matlab). Assume $n > 1$:

$$x = [x_1 \ x_2 \ \dots \ x_n]'$$

$$y = [y_1 \ y_2 \ \dots \ y_n]'$$

- a. $A = x$; $b = y$; $k = \text{pinv}(A)*b$
- b. $A = x$; $b = y$; $k = \text{inv}(A)*b$
- c. $k = \text{polyfit}(x,y,1)$
- d. $k = \text{polyfit}(x,y,0)$
- e. None of the above

29. Which of the following models CANNOT be estimated by a least squares model? t is the independent variable and y is the dependent variable.

- a. $y = a*t^2 + b*t$
- b. $y = a/t + b$
- c. $y = a*\sin(t) + b*\cos(t)$
- d. $y = t^a + b$

30. What is the output of the following code?

```
>> x = [1:10]';  
>> y = 0;  
>> for j = 2:10  
>>     y(j,1) = y(j-1,1) + 0.5*j;  
>> end  
>> myfit = polyfit(x,y,1);  
>> a = myfit(1);  
>> b = myfit(2);  
>> sum(y - (a*x + b))
```

- a) ans =
1.3000
- b) ans =
2.1000
- c) ans =
3.4000
- d) ans =
-3.4000
- e) None of the above

Part VII:

Questions 31-35: Numerical Differentiation

31. A first order forward difference method derivative approximation can be defined as follows. Assume that f is differentiable everywhere.

$$F_h(x_i) = \frac{f(x_i + h) - f(x_i)}{h}$$

A second-order central difference method derivative approximation can be defined as follows:

$$G_h(x_i) = \frac{f(x_i + h) - f(x_i - h)}{2h}$$

Define $F(x_i) = \lim_{h \rightarrow 0} F_h(x_i)$ and $G(x_i) = \lim_{h \rightarrow 0} G_h(x_i)$. Which of the following statements is true?

- a) $F(x_i)$ will always be greater than $G(x_i)$
- b) $F(x_i)$ will always be less than $G(x_i)$
- c) $F(x_i)$ will be equal to $G(x_i)$

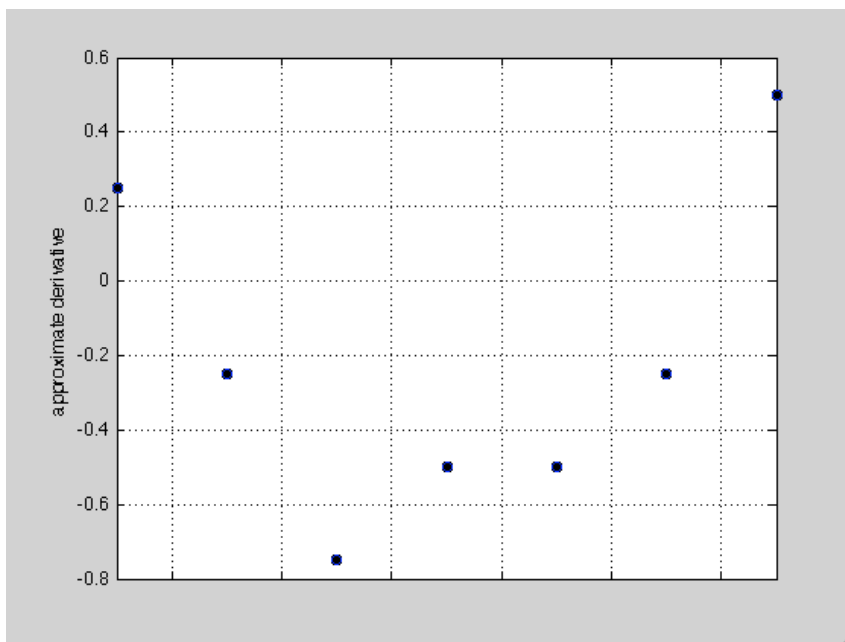
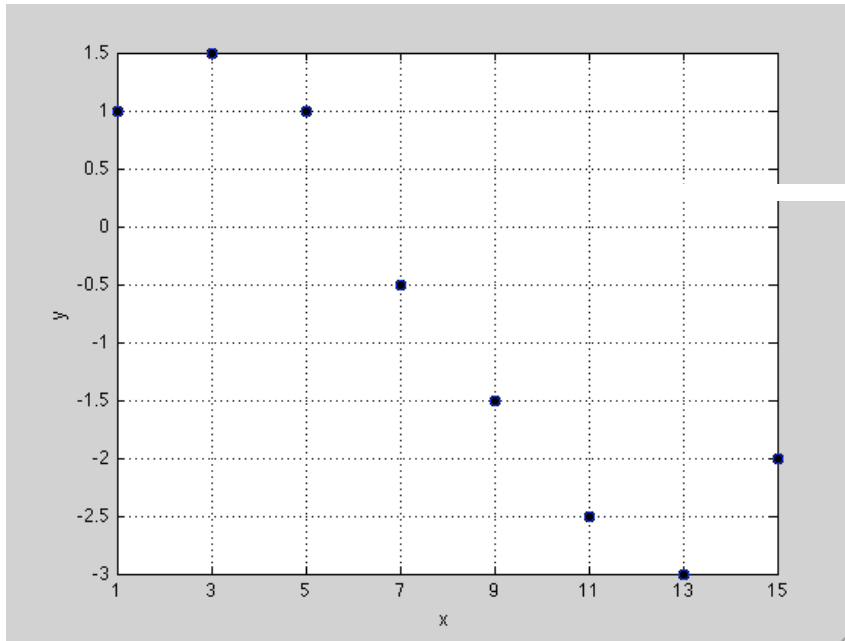
32. Given vectors of position values and their corresponding time coordinates, (*position* and *time*, respectively), an *acceleration* vector is calculated using 2nd order central differences applied twice in succession. For how many values of *time* can *acceleration* be calculated (i.e., what is $\text{length}(\text{acceleration})$)?

- a) $\text{length}(\text{time}) - 4$
- b) $\text{length}(\text{time}) - 3$
- c) $\text{length}(\text{time}) - 2$
- d) $\text{length}(\text{time}) - 1$
- e) $\text{length}(\text{time})$

Problems 33-34 refer to the following figures.

The top figure is a plot of a discrete data set, y versus x .

You then input this data set into Raja's code, which returns a discrete approximation to the derivative (bottom figure). Unfortunately, Raja refuses to let you see the code, and also forgot to output the x values corresponding to the derivatives, hence the x -axis is unlabeled.



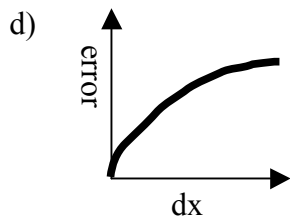
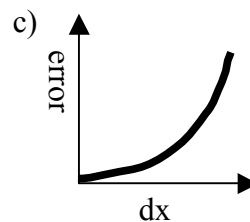
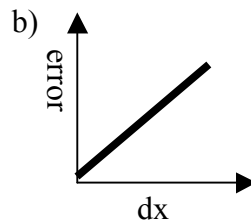
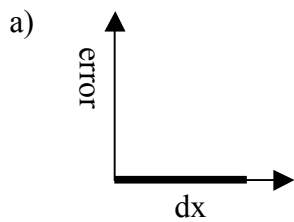
33. For the discrete data in the top figure, what is the approximate derivative at $x = 5$, using a 2nd order central difference? (Ignore bottom figure for this question)

- a) 2
- b) 0.5
- c) -0.5
- d) -1
- e) -2

34. Raja finally remembers that the x-values corresponding to the derivatives are some subset of the original x-values (i.e., a subset of the numbers [1 3 5 7 9 11 13 15]). Which finite difference approximation could Raja have used to generate the data in the bottom figure?

- a) He could have used 1st order forward difference but not 1st order backward difference
- b) He could have used 1st order backward difference but not 1st order forward difference
- c) 2nd order central difference
- d) either 1st order forward difference or 1st order backward difference could have been used
- e) none of the above could have been used

35. Suppose a 2nd order central difference approximation is used to estimate the derivative of a cubic function $y(x)$. The step size is dx , and error is defined as the absolute value of the difference between the derivative and its numerical estimate. Which of the following is an accurate sketch of the error versus dx ?



Part VIII:

Questions 36-40: Numerical Integration

36. Consider integrating the function $f(x) = 10x-3$

- a) Trapezoidal rule is more accurate than Simpson's rule.
- b) Trapezoidal rule is exact but Simpson's rule is not
- c) Simpson's rule is exact but trapezoidal rule is not
- d) Trapezoidal and Simpson's rules are both exact

37. Consider the integral $\int_0^2 x^2 dx$. Using Simpson's Rule to evaluate the integral, what's the best number of subintervals to use?

- a) 1
- b) 2
- c) 3
- d) 4
- e) The more the better

38. Consider the function $f(x) = 1 + 2x - x^2 + 8x^3$. What is the Simpson's rule integration error using only one subinterval over the interval $[0, 1]$?

- a) 0.667
- b) 1.333
- c) 0.333
- d) 0
- e) 0.125

39. Consider the function $f(x) = x^3$. What is the trapezoidal rule integration error with one subinterval over the interval $[0, 1]$?

- a) 0
- b) 0.125
- c) 0.250
- d) 0.333
- e) 0.667

40. Consider the function $f(x) = \sin(t)$. What is the trapezoidal rule integration error with five subintervals over the interval $[0, 2\pi]$?

- a) 0
- b) 0.588
- c) 0.951
- d) 1.176
- e) 1.571

Part IX:

Questions 41-50: Cell and Structure Arrays

41. What is the output of the following code?

```
>> A(1,1)={ [1 2; 3 4] }; %(1,1) enclosed in parentheses
>> A(1,2)={ [1 2; 3 4] }; %(1,2) enclosed in parentheses
>> A{2,1}='abc'; %{2,1} enclosed in curly brackets
>> A{2,2}={'abc'}; %{2,2} enclosed in curly brackets
>> A
```

a)

```
A =
    {1x1 cell}    [2x2 double]
    {1x1 cell}    'abc'
```

b)

```
A =
    [2x2 double]    [2x2 double]
    'abc'          {1x1 cell}
```

c)

```
A =
    [2x2 double]    {1x1 cell}
    {1x1 cell}     {1x1 cell}
```

d)

```
A =
    {1x1 cell}    {1x1 cell}
    {1x1 cell}    {1x1 cell}
```

42. Consider the following Matlab session:

```
>> B = {1 2};  
>> class(B(1)) %(1) enclosed in parentheses  
ans =  
cell  
>> class(B{1}) %{1} enclosed in curly brackets  
ans =  
double
```

What will be the output of the following command?

```
>> B(1) + 1
```

a)

```
ans =  
    2
```

b)

```
ans =  
    3
```

c)

```
ans =  
    1
```

d) This code will return an error

e) None of the above

43. What is the output of the following code?

```
>> B={'a'};  
>> B={B,'b'};  
>> B={B,'c'};  
>> size(B)
```

a)

```
ans =  
    1    1
```

b)

```
ans =  
    1    2
```

c)

```
ans =  
    1    3
```

d) None of the above

Problems 44 and 45 will be based on the follow code:

```
>> oldBinaryTree.name = 'Beethoven';  
>> oldBinaryTree.data = {'Piano Trio'};  
>> oldBinaryTree.left = [];  
>> oldBinaryTree.right = [];  
>> updatedBinaryTree.data = {oldBinaryTree.data{1} 'String  
Quartet'}
```

44. What's the output of the above code?

a)

```
updatedBinaryTree =  
  data: {'Piano Trio' 'String Quartet'}
```

b)

```
updatedBinaryTree =  
  name: 'Beethoven'  
  data: {'Piano Trio' 'String Quartet'}  
  left: []  
  right: []
```

c)

```
updatedBinaryTree =  
  name: 'Beethoven'  
  data: {'String Quartet'}  
  left: []  
  right: []
```

d) None of the above

45. Based on the above code, what is the output of the following command?

```
>> size(updatedBinaryTree)
>> size(updatedBinaryTree.data)
```

a)

```
ans =
     1     1
ans =
     1     2
```

b)

```
ans =
     1     4
ans =
     1     2
```

c)

```
ans =
     1     3
```

??? Reference to non-existent field 'data'.

d)

```
ans =
     1     1
ans =
     1     4
```

Problems 46 and 47 will be based on the follow code:

```
>> s(1).type = 'big';
>> s(2).type = 'little';
>> s(1).color = 'red';
>> s(2).color = 'red';
>> s(1).x = 3;
>> s(2).x = 4;
```

46. Given s defined above, what is the output of the following command?

```
>> size(s), size(s(1).type), size(s.type)
```

The following code may be useful to you as a hint:

```
>> class(s.type)
??? Error using ==> class
The CLASS function must be called from a class constructor.
```

a)

```
ans =
     1     2
ans =
     1     3
ans =
     1     2
```

b)

```
ans =
     1     2
ans =
     1     1
ans =
     1     2
```

c)

```
ans =
     1     2
ans =
     1     3
```

```
??? Error using ==> size
Dimension argument must be a positive integer scalar in the
range 1 to 2^31.
```

d)

```
ans =
     1     2
ans =
     1     1
ans =
     1     3
```

47. Given `s` defined above, what is the output of the following command?

```
>> s(2).color
```

a)

```
ans =  
      []
```

b)

```
ans =  
      {}
```

c)

```
ans =  
red
```

d)

```
??? Reference to non-existent field 'color'.
```

Problems 48 and 49 will be based on the follow code:

```
A{1}=@(t,y)t;
```

```
A{2}=[0:2];
```

```
A{3}=0
```

48. What's the output of the above code?

a)

```
A =  
      0
```

b)

```
??? Conversion to cell from function_handle is not possible.
```

c)

```
A =  
      [@(t,y)t]      [1x3 double]      [0]
```

49. Given A defined above, what is the output of the following command?

```
>> [T, Y]=ode45(A{1}, A{2}, A{3}) ;  
>> [T Y]
```

a)

ans =

```
      0      1.0000  
 1.0000      1.5000  
 2.0000      3.0000
```

b)

ans =

```
      0      0  
 1.0000      1.0000  
 2.0000      2.0000
```

c)

ans =

```
      0      0  
 1.0000      0.5000  
 2.0000      2.0000
```

d) Matlab will return an error

50. Given the following function:

```
function updatedBinaryTree = insertInTree(oldBinaryTree, name, data)  
if isempty(oldBinaryTree) % if the oldBinaryTree is empty, then the  
new record is the updatedBinaryTree  
    updatedBinaryTree.name = name;  
    updatedBinaryTree.data = {data};  
    updatedBinaryTree.left = [];  
    updatedBinaryTree.right = [];  
  
elseif strcmp(oldBinaryTree.name, name)  
    % if the record's name is the same as the oldBinaryTree's name  
field, then concatenate the record's data to the the oldBinaryTree's  
data field  
    updatedBinaryTree = oldBinaryTree;  
    updatedBinaryTree.data =  
{oldBinaryTree.data{1:length(oldBinaryTree.data)} data};  
  
elseif strcmp(sort({char(name) char(oldBinaryTree.name)}),  
{char(name) char(oldBinaryTree.name)}),  
    % if the name to insert is before the name of current node the  
the  
    % oldBinary Tree, then go to the left child  
    updatedBinaryTree = oldBinaryTree;
```

```
        updatedBinaryTree.left =
insertInTree(oldBinaryTree.left,name,data);

else
    updatedBinaryTree = oldBinaryTree;
    updatedBinaryTree.right =
insertInTree(oldBinaryTree.right,name,data);
end

end
```

What is the output of the following commands?

```
>> Score(1).name='luke';
>> Score(1).score=61;
>> Score(2).name='beyonce';
>> Score(2).score=80;
>> myTree=[];
>> for i=1:length(Score)
>>     myTree=insertInTree(myTree,Score(i).name,Score(i).score);
>> end
>> size(myTree)
```

a)

```
ans =
     1     1
```

b)

```
ans =
     1     2
```

c)

```
ans =
     1     4
```

d)

```
ans =
     4     4
```


Part X:

Questions 51-55: Ordinary Differential Equations

51. What is the largest value k such that the classic Runge-Kutta algorithm (NOT Matlab's numerical implementation of the algorithm) can solve $dy/dt=t^k$ with zero error?

- a) $k=2$
- b) $k=3$
- c) $k=4$
- d) $k=5$

52. The command `ode45(@myFunc, [0 10], 0)` can be used to solve:

- a) A 2nd order differential equation modeled by myFunc.m
- b) A 1st order differential equation modeled by myFunc.m
- c) Two 1st order differential equations modeled by myFunc.m
- d) None of the above

53. Consider the ODE $y''(t)+y'(t)+y(t)=0$ with initial conditions $y(0) = 0$ and $y'(0)=5$. Which of the following plots the solution $y(t)$?

- a) `[y,t]=ode45(@ (t) [y(2); -y(2)-y(1)], [0 10], [0 5]); plot(t,y(:,1))`
- b) `[t,y]=ode45(@ (t,y) [y(1); -y(2)-y(1)], [0 10], [0 5]); plot(t,y(:,2))`
- c) `[t,y]=ode45(@ (t,y) [-y(2)-y(1); y(1)], [0 10], [5 0]); plot(t,y(:,1))`
- d) `[t,y]=ode45(@ (t,y) [y(2); -y(2)-y(1)], [0 10], [0 5]); plot(t,y(:,1))`

54. Consider the ODE $y'(t)+y(t)=0$ with initial condition $y(0)=1$. Which of the following statements is correct?

- a) `ode45(@ (t,y) -y, [0:0.1:3], 1)`; will solve the ODE with step size 0.1
- b) `ode45(@ (t,y) -y, [0:0.1:3], 1)`; will solve the ODE with step size 1
- c) `ode45(@ (t,y) -y, [0:0.1:3], 1)`; will not solve the ODE
- d) `ode45(@ (t,y) -y, [0:0.1:3], 1)`; will solve the ODE but the step size is not fixed by the command

55. Consider a function $f(x)$ with complexity $O(n^2)$ and another function $g(x)$ with complexity $O(\log(n))$. What is the complexity of the function $h(x) = f(x) + g(x)$?

- a) $O(n^2)$
- b) $O(\log(n))$
- c) $O((n^2) * \log(n))$
- d) $O((n^2) + \log(n))$

Part XI:

Questions 56-72: Computational Complexity

For questions 56 through 58, consider the following two Matlab functions.

```
function array = bubbleSort(array)
n = length(array);
swapped = true;
while swapped
    swapped = false;
    for i=1:n-1
        if array(i) > array(i+1)
            swapped = true;
            tmp = array(i);
            array(i) = array(i+1);
            array(i+1) = tmp;
        end
    end
end
end
```

```
function array = anotherSort(array)
n = length(array);
swapped = true;
count = 1;
while swapped
    swapped = false;
    for i=1:n-count
        if array(i) > array(i+1)
            swapped = true;
            tmp = array(i);
            array(i) = array(i+1);
            array(i+1) = tmp;
        end
    end
    count=count+1;
end
end
```

56. What is the complexity of bubbleSort when it is given a sorted array as an input?

- a) $O(n)$
- b) $O(\log(n))$
- c) $O(n^2)$
- d) exponential complexity

57. What is the worst-case complexity of bubbleSort?

- a) $O(n)$
- b) $O(\log(n))$
- c) $O(n^2)$
- d) exponential complexity

58. Which of bubbleSort or anotherSort is the better algorithm to use?

- a) bubbleSort, because it has lower complexity
- b) anotherSort, because it runs in less time in most cases
- c) anotherSort, because it has lower complexity
- d) neither, because they have the same complexity

59. What is the complexity of the following function in terms of n ?

```
function [out] = tribonacci(n)
if n < 4
    out = 1;
else
    out = tribonacci(n-1)+tribonacci(n-2)+tribonacci(n-3);
end
end
```

- a) $O(n)$
- b) exponential complexity
- c) $O(n^3)$
- d) $O(\log(n))$
- e) none of the above

60. What is the complexity of the following function in terms of n ?

```
function [out] = tribonacci(n)
% assume n is at least 4
out = zeros(1,n);
out(1:3) = 1;
for i = 4:n
    out(i) = out(i-1) + out(i-2) + out(i-3);
end
out = out(n);
end
```

- a) $O(n)$
- b) exponential complexity
- c) $O(n^3)$
- d) $O(\log(n))$
- e) none of the above

61. What is the complexity of the following function in terms of n ?

```
function [out] = mat_mult(A,B,n)
% out = A*B, assume A and B are n x n square matrices

out = zeros(n,n);
for i = 1:n
    for j = 1:n
        out(i,j) = A(i,:) * B(:,j);
    end
end
end
```

- a) $O(n)$
- b) $O(n^2)$
- c) $O(n^3)$
- d) $O(n^4)$
- e) greater complexity than $O(n^4)$

62. What is the complexity of the following function in terms of n ?

```
function [out] = all_perms(V, n)
% out is an array that contains every possible permutation
% of the row vector V made of distinct numbers where each
% row of out is a permutation of V. n is the amount of
% columns of V to be used, and is the same as the number of
% columns in 'out'. n<=length(V(1,:))
out = [];
if n > 0
    N = factorial(n-1);
    for i = 1:n
        out = [out ;
[V(i)*ones(N,1),all_perms(V(V~=V(i)),n-1)]];
    end
end
end
```

Example:

```
>> all_perms([1 2 4 3],3)
```

```
ans =
```

```
     1     2     4
     1     4     2
     2     1     4
     2     4     1
     4     1     2
     4     2     1
```

- a) $O(n)$
- b) $O(n^2)$
- c) $O(n^3)$
- d) $O(n^4)$
- e) greater complexity than $O(n^4)$

63. What is the complexity of the following function in terms of n ?

```
function [out] = reduction(n)

while n > 1
    n = n/3;
end
end
```

- a) $O(n)$
- b) exponential complexity
- c) $O(n^3)$
- d) $O(\log(n))$
- e) none of the above

64. What is the complexity of a function that returns:
The mean of a length- n vector of numbers.

- a) $O(1)$
- b) $O(n)$
- c) $O(n^2)$
- d) $O(\log(n))$
- e) none of the above

65. What is the complexity of a function that returns:
All possible combinations of the first 10 letters of the alphabet.

- a) $O(1)$
- b) $O(n)$
- c) $O(n^2)$
- d) $O(\log(n))$
- e) none of the above

66. Suppose you have an alphabet with n letters. What is the complexity of a function that prints all words of length n from the alphabet?

- a) $O(1)$
- b) $O(n)$
- c) Exponential complexity
- d) $O(\log(n))$
- e) none of the above

67. Let m be an $n \times n$ matrix in Matlab. What is the complexity of a function that returns:

$m(3, 3)$

- a) $O(1)$
- b) $O(n)$
- c) $O(n^2)$
- d) $O(\log(n))$
- e) none of the above

68. What is the complexity of a function that returns:

A display of all i, j indices such that $M(i, j)$ equals a certain number

N . M is an $n \times n$ matrix.

- a) $O(1)$
- b) $O(n)$
- c) $O(n^2)$
- d) $O(\log(n))$
- e) none of the above

69. What is the complexity of multiplying an n -bit binary positive integer by 8 using the most efficient algorithm?

- a) $O(1)$
- b) $O(\log(n))$
- c) $O(n)$
- d) $O(n^3)$
- e) none of the above

Use the following descriptions for Questions 70-72. In a classroom setting, it is common for the instructor to give a handout to the class. Given the following two strategies for distributing a handout to a class, answer the following questions.

Strategy 1: The instructor hands the first student the stack of handouts. The student takes a handout, then passes the stack to the another student who has no handouts. This process is repeated until every student has a handout.

Strategy 2: The instructor hands the first student the stack of handouts. The student takes a handout, divides the stack in two and then passes the each stack to another student who has no handouts. This process is repeated until every student has a handout.

70.

Which strategy is most likely to be the faster strategy for a class of size n ?

- a) Strategy 1
- b) Strategy 2
- c) The two strategies will likely take the same amount of time
- d) none of the above

71.

For a class of size n , what is the complexity of Strategy 1?

- a) $O(1)$
- b) $O(\log(n))$
- c) $O(\sqrt{n})$
- d) $O(n)$
- e) none of the above

72.

For a class of size n , what is the complexity of Strategy 2?

- a) $O(1)$
- b) $O(\log(n))$
- c) $O(\sqrt{n})$
- d) $O(n)$
- e) none of the above

Part XII:

Questions 73-85: Sort/Search, Data Structures, Comprehensive

73. A company stores its employee salary records in a structure array. Which of the following statements is true?

- a) If the array is sorted by name the complexity of retrieving salary given a name cannot be less than $O(n)$
- b) If the array is sorted by salary the complexity of retrieving salary given a name is $O(\log(n))$
- c) If the array is unsorted, the complexity will be the same as if the array is sorted by name
- d) If there are 100 records in the array and it is sorted by name, salary information on a given employee can always be found by comparing the name to fewer than 10 records

74. Given the differential equation $dy/dt = t^2 + 1$ and the initial condition $t_0 = 0$ and $y(0) = 1$, give the 4th order Runge-Kutta estimate for $y(2)$. You may assume one step.

- a) $y(2) = 0$
- b) $y(2) = 1$
- c) $y(2) = 5/3$
- d) $y(2) = 8/3$
- e) None of the above

75. What is the complexity of the 4th Order Runge-Kutta method in terms of n where n is the number of discrete points, t_i , for which a $y(t_i)$ is desired.

- a) $O(1)$
- b) $O(n)$
- c) $O(n^2)$
- d) $O(n^4)$
- e) None of the above

76. Given an $m \times n$ matrix, A , and an $m \times 1$ column vector, b , where $m = 10n$, what is the computational complexity of finding the least squares solution to $Ax = b$? You should make this determination from the formula $x = (A^T A)^{-1} A^T b$, and you may assume that taking an inverse is $O(n^3)$.

- a) $O(1)$
- b) $O(n)$
- c) $O(n^3)$
- d) $O(n^{10})$
- e) The complexity is greater than $O(n^{10})$

77. What is the complexity of a search on a minimum height binary tree?

- a. $O(n)$
- b. $O(n^2)$
- c. $O(\log n)$
- d. $O(n^{0.5})$

78. Create a binary tree out of the following records, adding them in the order given using Lastname as the key. What is the height of the resulting tree?

Firstname	Lastname	Favorite Ice Cream Flavor
David	Leung	Rocky Road
Anu	Sridharan	Cookie Dough
Timmy	Siauw	Cookies and Cream
Scott	Payne	Mint Chocolate Chip
Raja	Sengupta	Vanilla
Melissa	Humber	Strawberry
Travis	Walter	Chocolate

- a. 2
- b. 3
- c. 5
- d. 7

79. Luggage is loaded into an airplane as a stack. On this particular flight, there are seven passengers with one checked bag apiece, and each bag is identified by the person who owns it. After loading the bags on this flight, it is discovered that two of them possibly contain explosives and must be removed immediately for testing. The bags are arranged as follows:

Mr. Bond
Ms. Winfrey
Mr. Rogers
Mrs. Doubtfire
Mr. Pitt
Mr. Clooney
Mr. Birgineau

Mr. Clooney and Mr. Rogers are the owners of the suspicious bags. Bags can only be accessed from the top. What is the proper series of pop and push operations required to remove these bags while ensuring that the rest of the bags still get on the flight?

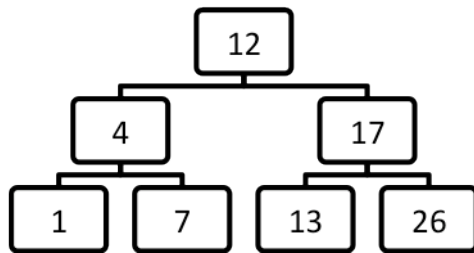
a)
Pop(Rogers)
Pop(Clooney)

b)
Pop(Birgineau)
Pop(Clooney)
Pop(Pitt)
Pop(Doubtfire)
Pop(Rogers)
Pop(Winfrey)
Pop(Bond)
Push(Birgineau)
Push(Pitt)
Push(Doubtfire)
Push(Winfrey)
Push(Bond)

c)
Pop(Bond)
Pop(Winfrey)
Pop(Rogers)
Pop(Doubtfire)
Pop(Pitt)
Pop(Clooney)
Push(Bond)
Push(Winfrey)
Push(Doubtfire)
Push(Pitt)

d)
Push(Rogers)
Push(Clooney)
Pop(Bond)
Pop(Winfrey)
Pop(Doubtfire)
Pop(Pitt)
Pop(Birgineau)

80. Consider this three layer binary tree, organized by layer as the numbers were entered starting at the root (12):



Which of the following could be a possible order in which these elements were entered into the tree?

- a) 12, 4, 26, 13, 1, 7, 17
- b) 12, 17, 26, 4, 7, 13, 1
- c) 12, 7, 3, 26, 17, 1, 4
- d) 12, 26, 17, 13, 4, 1, 7
- e) none of the above

81. Considering the binary tree in Problem 80, what would be the order of the elements if a post-order tree traversal was performed?

- a) 1,4,7,12,13,17,26
- b) 1,7,4,13,26,17,12
- c) 12,4,1,7,17,13,26
- d) None of the above

82. Consider this set of numbers in the following order:

7, 12, 17, 26, 1, 4, 3

If you were to organize these numbers in a binary tree in the given order and then performed a binary search within the tree for the number 26, how many steps would this search require?

- a) 1
- b) 2
- c) 4
- d) 6
- e) Not enough information is provided to determine this

83. Which of the following algorithms is **not** $O(1)$?

- a) a push operation on a stack
- b) a pop operation on a stack
- c) calculating the nth Fibonacci number iteratively
- d) inserting an element into a queue

84. Which of the following commands will produce a MATLAB error (assuming no variables were previously defined)?

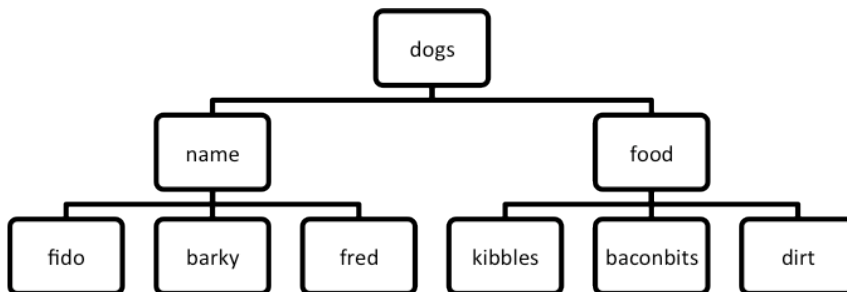
- a) `s.signal = 'stop'; s.color = 'red';`
- b) `signals = {'stop', 'go', 'slow'};`
- c) `colors = ['blue'; 'red'; 'green'; 'orange'];`
- d) `s(1).signal = 'go'; s(2).color= 'green';`

85.

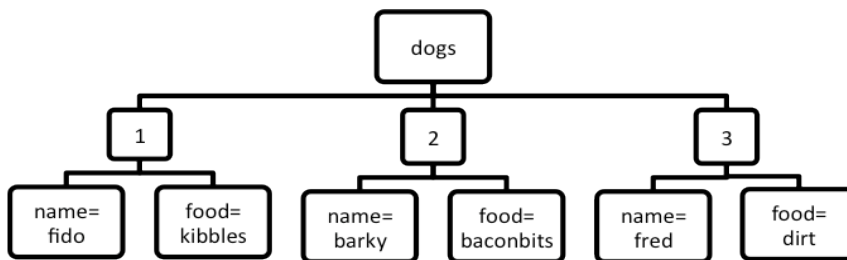
```
>> dogs(1).name = 'fido';  
>> dogs(2).name = 'barky';  
>> dogs(3).name = 'fred';  
>> dogs(1).food= 'kibbles';  
>> dogs(2).food= 'baconbits';  
>> dogs(3).food = 'dirt';
```

Which of the following diagrams best represents this structure's organization in Matlab?
Please note that answer choices c and d appear on the next page.

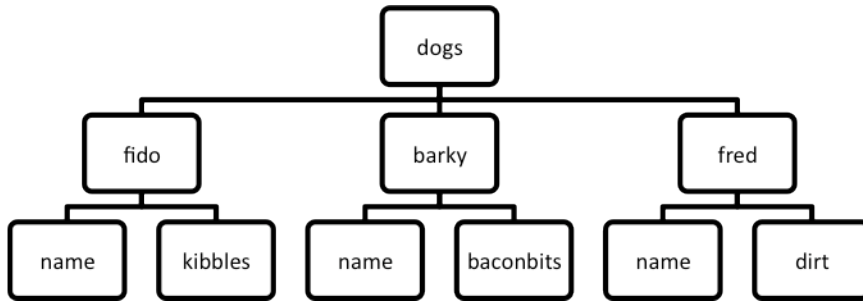
(a)



(b)



(c)



(d)

