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Mechanical Behavior of Materials

Midterm 1 Solution

Problem 1

a) Relationship between torque and shear stress is given as:

$$\sigma_{\theta z} = \frac{Tr}{J}$$

Where $T = 3000\ \text{ft}\cdot\text{lbf}$ and the outer radius of the rod, $R = \frac{D}{2} = 1.25\ \text{in}$. J is the polar moment of inertia and is:

$$\begin{aligned} J &= \frac{\pi}{2}[R^4 - (R-t)^4] \\ &= \frac{\pi}{2}[1.25^4 - (1.25 - 0.125)^4] \\ &= 1.32\ \text{in}^4 \end{aligned}$$

The maximum stress occurs when $r = R$, thus:

$$\begin{aligned} \sigma_{\theta z} &= \frac{TR}{J} \\ &= \frac{3000(12)(1.25)}{1.32} \\ &= 34100\ \text{psi} \\ &= 34.1\ \text{ksi} \end{aligned}$$

Stress state:

$$\sigma = \begin{bmatrix} \sigma_{zz} & \sigma_{z\theta} & \sigma_{zr} \\ \sigma_{\theta z} & \sigma_{\theta\theta} & \sigma_{\theta r} \\ \sigma_{rz} & \sigma_{r\theta} & \sigma_{rr} \end{bmatrix} = \begin{bmatrix} 0 & 34.1 & 0 \\ 34.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The maximum shear stress is:

$$\tau_{max} = 34.1\ \text{ksi}$$

$$\tau_{max} < \frac{\sigma_y}{2} = 50\ \text{ksi}$$

Will not yield according to Tresca.

Von Mises:

$$\begin{aligned} \bar{\sigma} &= \sqrt{\frac{(\sigma_{zz}-\sigma_{rr})^2 + (\sigma_{zz}-\sigma_{\theta\theta})^2 + (\sigma_{\theta\theta}-\sigma_{rr})^2}{2} + 3\left(\sigma_{z\theta}^2 + \sigma_{zr}^2 + \sigma_{r\theta}^2\right)} \\ &= \sqrt{3}\sigma_{\theta z} \\ &= 59.1\ \text{ksi} < \sigma_y \end{aligned}$$

Yielding will not occur.

b) The bending moment will act on the z plane, the tensile stress and bending moment relationship is given as:

$$\sigma_{zz} = \frac{My}{I}$$

Where $M = 1000\ \text{ft} \cdot \text{lbf}$, and y is the distance from neutral axis. The moment of inertia can be calculated as:

$$I = \frac{\pi}{4}\left[R^4 - (R-t)^4\right]$$
$$= \frac{J}{2} = 0.66\ \text{in}^4$$

The maximum shear occurs when y is furthest away from the neutral axis. The neutral axis in this problem, by symmetry, is the middle of the tube. Therefore $y = R$, thus:

$$\sigma_{zz} = \frac{1000(12)(1.25)}{0.66}$$
$$= 22.8\ \text{ksi}$$

Thus the stress tensor:

$$\sigma = \begin{bmatrix} 22.8 & 34.1 & 0 \\ 34.1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Radius of the θz Mhor circle is:

$$R = \sqrt{\left(\frac{\sigma_{zz}}{2}\right)^2 + \sigma_{\theta z}^2} = 35.97\ \text{ksi}$$

Center of the θz Mhor circle is:

$$C = \frac{\sigma_{zz}}{2} = 11.37\ \text{ksi}$$

The principal stresses:

$$\sigma_I = C + R = 47.34\ \text{ksi}$$
$$\sigma_{II} = 0\ \text{ksi}$$
$$\sigma_{III} = C - R = -24.59\ \text{ksi}$$

Maximum shear stress will be:

$$\tau_{max} = \frac{\sigma_I - \sigma_{III}}{2} = 35.97\ \text{ksi} < 50\ \text{ksi}$$

Will not yield.

Von Mises:

$$\bar{\sigma} = \sqrt{\frac{(\sigma_I - \sigma_{II})^2 + (\sigma_I - \sigma_{III})^2 + (\sigma_{II} - \sigma_{III})^2}{2}} = 45.49\ \text{ksi} < 100\ \text{ksi}$$

Will not yield.

c) Check for thin wall conditions:

$$\frac{R}{t} = \frac{1.25}{0.125} = 10 \gg 1$$

We can proceed with using the thin wall solutions. First:

$$\sigma_{\theta\theta} = \frac{PR}{t}$$

where $P = 8$ ksi:

$$\sigma_{\theta\theta} = \frac{8(1.25)}{.125} = 80 \text{ ksi}$$

$$\sigma_{zz} = \frac{\sigma_{\theta\theta}}{2} = 40 \text{ ksi}$$

$$\sigma_{rr} \approx 0$$

Stress tensor:

$$\sigma = \begin{bmatrix} 62.8 & 34.1 & 0 \\ 34.1 & 80 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

There are many ways to compute the principal stresses, the easiest is find the eigenvalue: http://www.wolframalpha.com/input/?i=eigenvalue%5B%7B%7B62.8%2C34.1%2C0%7D%2C%7B34.1%2C80%2C0%7D%2C%7B0%2C0%2C0%7D%7D%5D

$$\sigma_I = 106.6 \text{ ksi}$$

$$\sigma_{II} = 36.2 \text{ ksi}$$

$$\sigma_{III} = 0 \text{ ksi}$$

The maximum shear stress:

$$\tau_{max} = \frac{\sigma_I - \sigma_{III}}{2} = 53.3 \text{ ksi} > 50 \text{ ksi}$$

Yielding will occur according to Tresca!

Von Mises:

$$\bar{\sigma} = \sqrt{\frac{(\sigma_I - \sigma_{II})^2 + (\sigma_I - \sigma_{III})^2 + (\sigma_{II} - \sigma_{III})^2}{2}}$$

$$= 93.9 \text{ ksi} < 100 \text{ ksi}$$

Will not yielding according to Von Mises, but it is awfully close.

d) Tresca appears to be more conservative, however this is not always the case. Von Mises will always generate a higher stress than Tresca, but depending on the question that is asked, neither is always conservative. For example, determine yielding given fixed geometry, Tresca tends to be conservative. But determining geometry with fixed loading, Von Mises tends to be conservative.

Problem 2

The load of 200 MN will cause a tensile stress:

$$\sigma_{11} = \frac{P}{A} = \frac{200}{1} = 200 \text{ MPa}$$

This stress appears to be larger than silver alloy's yield stress, but recall that the silver braze is constrained. First, we know the 22 and 33 strain of the silver is controlled by steel strain.

$$\varepsilon_{22}^{Ag} = \varepsilon_{22}^{St}$$
$$\varepsilon_{33}^{Ag} = \varepsilon_{33}^{St}$$

The steel experiences a Poisson contraction in x_2 and x_3 due to loading in x_1 direction:

$$\varepsilon_{22}^{St} = -\frac{\nu^{St}\sigma_{11}}{E^{St}} = -\frac{0.3(200)}{200 \times 10^3} = -3 \times 10^{-4}$$
$$\varepsilon_{33}^{St} = -\frac{\nu^{St}\sigma_{11}}{E^{St}} = -3 \times 10^{-4}$$

Now we can compute the strain in the silver braze:

$$\varepsilon_{22}^{Ag} = \frac{\sigma_{22} - \nu^{Ag}(\sigma_{11} + \sigma_{33})}{E^{Ag}}$$
$$\varepsilon_{33}^{Ag} = \frac{\sigma_{33} - \nu^{Ag}(\sigma_{11} + \sigma_{22})}{E^{Ag}}$$

We know $\sigma_{11} = 200$ MPa, and we know everything to solve for σ_{22} and σ_{33}, by symmetry, $\sigma_{22} = \sigma_{33}$, thus:

$$\begin{aligned}\sigma_{22} = \sigma_{33} &= \frac{\varepsilon_{22}^{Ag}E^{Ag} + \nu^{Ag}\sigma_{11}}{1 - \nu^{Ag}} \\ &= \frac{-3 \times 10^{-4}(30 \times 10^3) + 0.37(200)}{1 - 0.37} \\ &= 103.2 \text{ MPa}\end{aligned}$$

Thus the stress state looks like this:

$$\sigma = \begin{bmatrix} 200 & 0 & 0 \\ 0 & 103.2 & 0 \\ 0 & 0 & 103.2 \end{bmatrix} \text{MPa}$$

Check for yielding, Tresca:

$$\tau_{max} = \frac{200 - 103.2}{2} = 48.4 \text{ MPa} < \frac{\sigma_y^{Ag}}{2} = 70 \text{ MPa}$$

Yielding will not occur.

Von Mises:

$$\bar{\sigma} = \sqrt{\frac{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2}{2}}$$

$$= \sqrt{\frac{(200 - 103.2)^2 + (200 - 103.2)^2 + (103.2 - 103.2)^2}{2}}$$

$$= 96.8 \text{ MPa} < \sigma_y = 140 \text{ MPa}$$

Yielding will not occur.

This is the power of constraint, where it is possible to load the material with greater than the unconstrained yielding force and yet not causing yielding.