Final Examination

May 10, 2011
75 Questions, 170 minutes

Notes:

1. Before you begin, please check that your exam has 28 pages (including this one).

2. Write your **name** and **student ID number** clearly on your scantron.

3. You may use a pencil, eraser, and a two-sided 8.5" by 11" sheet of notes for this exam provided you do not disturb those sitting nearby. No electronic devices are permitted in your work area except a graphing calculator. This includes, but is not limited to: cell phones, mp3 players, and laptops.

4. Record your answers on a green scantron sheet using a #2 pencil. There is one correct answer for each question. Multiple bubbles, incomplete bubbles, or stray marks will be marked incorrect.

5. There will be no questions regarding the exam during the examination time.

Section 1 – Programming

Question 1: E7 is a class about programming in:

a) C++
b) Java
c) Matlab
d) HTML5
e) None of the above.

Question 2: Consider the following lines of code:

```
>> f = @(p) (@(x) x.^p);
>> g = f(3);
```

Which one of the following statements is true?

a) g is the third element of f.
b) The class of g is double.
c) The first line will produce an error.
d) The second line will produce an error.
e) None of the above.

Question 3-6: Consider the following lines of code:

```
>> S(1).f = [0 1 2 3];
>> S(2).f = @(a,b) a:b;
>> S(3).f = S(1);
>> S(4).f = ‘E7’;
```

Question 3: What will be the value of x after the following line of code is executed?

```
>> x = S(1).f(1);
```

a) 0
b) 1
c) 2
d) This line of code will produce an error.
e) None of the above.

Question 4: What will be the value of x after the following line of code is executed?

```
>> x = S(2).f(1,3);
```

a) 0
b) 1
c) 2
d) This line of code will produce an error.
e) None of the above.

Question 5: What will be the value of x after the following line of code is executed?

```
>> x = S(3).f.f(3);
```

a) 0
b) 1
c) 2
d) This line of code will produce an error.
e) None of the above.

Question 6: What will be the value of `x` after the following line of code is executed?

```
>> x = strcmp(S(4).f, 'E7')
```

a) 0
b) 1
c) 'char'
d) This line of code will produce an error.
e) None of the above.

Question 7: Let `P` and `Q` be logicals, i.e. variables that are either 0 or 1. Which of the following logical expression is equivalent to `~(P && Q)`?

a) `P || Q`
b) `~P && Q`
c) `~P && ~Q`
d) `~P || ~Q`
e) None of the above.

Question 8: Consider the following function:

```
function [] = exam1(n)

if n <= 1
    disp('a')
elseif rem(n,2) == 0
    disp('b')
    exam1(n/2)
    disp('c')
    exam1(n-2)
else
    disp('d')
    exam1(n-1)
end

end % end exam1
```

What will be the order in which the letters are displayed to the screen when the following line of code is executed?

```
>> exam1(3)
```

a) b, a, c, a
b) b, a, c, a, d
c) d, b, a, c, a
d) d, b, a, a, c
e) d, c, a, b, a

Question 9: Consider the following function:

```
function [out] = exam2(A,N)
% you may assume that A is a matrix and N is a strictly positive integer.

[m,n] = size(A);

out = zeros(m*N, n);

for i = 1:m
    for j = 1:n
        out(N*(i-1)+1:i*N,j) = A(i,j)*ones(N,1);
    end
end

end % end exam2
```

What will be the value of out after the following lines of code are executed?

```
>> A = [5 1; 2 3];
>> out = exam2(A,2);
```

a) out =

10	2
4	6

b) out =

0	0
0	0
0	0
0	0

c) out =

5	5	2	2
1	1	3	3

d) These lines of code will produce an error.
e) None of the above.

Section 2 - Representation of Numbers

For this section, we use the notation x_y to mean that x is a number in y representation. For example 5_{10} means "5 in decimal". You may assume that numbers without a subscript are in decimal.

Question 10: Which of the following numbers is the binary representation of 97_{10}?

a) 01100100_2
b) 00101101_2
c) 10101100_2
d) 01100001_2
e) None of the above.

Question 11: Which of the following numbers is the decimal representation of 10010.101_2? (Note the decimal point)

a) 18.125_{10}
b) 18.625_{10}
c) 36.125_{10}
d) 36.625_{10}
e) None of the above.

Question 12: Which of the following numbers is the product of the binary numbers 11101_2 and 1001_2?

a) 111011001_2
b) 100111101_2
c) 100110_2
d) 100000101_2
e) None of the above.

Question 13: How many unique numbers can be represented by a 100 bit floating point number?

a) 100
b) 2^{100}
c) 100^2
d) 200
e) It depends on how the sign bit, exponent bits, and fraction bits are allocated.

Question 14: Which of the following numbers is the decimal representation of the single precision floating point number $1\ 10001010\ 01110101001100011011110_{IEEE}$?

a) 2.985554199218750e+003
b) 1.492777099609375e+003
c) 5.079657247404517e+041
d) 27.5
e) None of the above.

Question 15: Which of the following numbers is the single precision float representation of the decimal number 19?

a) 0 10000011 00110000000000000000000
b) 1 10000011 00110000000000000000000
c) 0 00000100 00110000000000000000000
d) 0 10000011 10110000000000000000000
e) None of the above.

Section 4 - Complexity

Question 16: Let v_1 and v_2 be two n-dimensional vectors. What is the Big-O complexity of computing the dot product of v_1 and v_2 in terms of the dimension, n? You may assume that

$$v_1 \cdot v_2 = \sum_{i=1}^{n} v_1(i) \times v_2(i)$$

a) O(log(n))
b) O(n)
c) O(n^2)
d) O(n^3)
e) None of the above.

Question 17: What is the Big-O complexity of the following function in terms of the input, n?

```
function out = exam3(n)
% you may assume that n is a positive integer
out = n;
for i = 1:n-1
    out = out + exam3(i);
end
end % end exam3
```

a) O(log(n))
b) O(n)
c) O(n^2)
d) O(n^3)
e) None of the above.

Question 18: Assume that f is a Matlab function that can run in O(log(n)) time. What is the Big-O complexity of the following code segment in terms of n?

```
out = 0;
for i = 1:n
      out = out + f(i);
end
```

a) O(n!)
b) O(n)
c) O(n + log(n))
d) O(n log(n))
e) None of the above.

Question 19: In Big-O notation, which of the following is equivalent to O($2n^2$log(n) + 3n)?

a) O(log(n))
b) O(n)
c) O(n^2)
d) O(n^2log(n))
e) None of the above.

Section 5 - Linear Algebra

Question 20: What is the L_2 norm of the vector v = $[1\ 0\ 0\ -1\ 1]^T$?

a) $\sqrt{2}$
b) $\sqrt{3}$
c) 3
d) 5
e) None of the above.

Question 21: Let $v_1 = [1\ 0\ 2]^T$ and $v_2 = [4\ -1\ \alpha]^T$. What value of α will make v_1 and v_2 orthogonal?

a) 0
b) -2
c) 2
d) No value of α will make v_1 and v_2 orthogonal.
e) There are an infinite number of α's that will make v_1 and v_2 orthogonal.

Question 22: Let v_1, v_2, and v_3 be vectors in R^4. Which of the following expressions is NOT a linear combination of v_1, v_2, and v_3?

a) $3^2v_1 - 2^2v_2$
b) The zero-vector in R^4.
c) $2v_1 + 3v_2 - 3v_3$
d) $(v_1 + v_2)/v_3$.
e) All of the above are linear combinations of v_1, v_2, and v_3.

Question 23: Let `v1`, `v2`, `v3`, and `v4` be 5x1 column vectors in Matlab. Which of the following Matlab commands will return true (i.e. 1) if `v1`, `v2`, `v3`, and `v4` are linearly independent?

a) `rank([v1, v2, v3, v4]) == 0`
b) `rank([v1, v2, v3, v4]) == 1`
c) `rank([v1, v2, v3, v4]) == 4`
d) `rank([v1, v2, v3, v4]) == 5`
e) None of the above.

Question 24: Which of the following matrix equations is the matrix form of the given system of equations? Note that x, y, and z are variables and A – H are known constants.

$$Ax - By + Cz = D$$

$$Ey - 4Fz = G$$

$$2x + H = -y$$

a) $\begin{bmatrix} A & -B & C \\ E & -4F & 0 \\ 2 & H & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} D \\ G \\ -y \end{bmatrix}$

b) $\begin{bmatrix} x & -y & z \\ Ey & 4Fz & 0 \\ 2 & 0 & H \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} D \\ G \\ 0 \end{bmatrix}$

c) $\begin{bmatrix} A & -B & C \\ E & -4F & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} D \\ G \\ H \end{bmatrix}$

d) $\begin{bmatrix} A & -B & C \\ 0 & E & -4F \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} D \\ G \\ -H \end{bmatrix}$

e) $\begin{bmatrix} A & -B & C \\ 0 & E & -4F \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} D \\ G \\ -H \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Questions 25 – 27: Consider the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 1 & 2 & 3 & 4 & 0 \\ 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 25: What is rank(A)?

a) 1
b) 2
c) 3
d) 4
e) 5

Question 26: Consider the vector

$$b = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

How many solutions are there for the matrix equation Ax = b?

a) 0
b) 1
c) 2
d) 5
e) infinity

Question 27: Consider the vector

$$b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}$$

How many solutions, x, are there for the matrix equation Ax = b?

a) 0
b) 1
c) 2
d) 5
e) infinity

Question 28: Consider the following matrix and vector:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

How many solutions are there to the matrix equation Ax = b?

a) 0
b) 1
c) 2
d) 5
e) infinity

Question 29: Consider the following matrix and vector:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 3 \\ 1 \\ 2 \\ 7 \end{bmatrix}$$

How many solutions are there to the matrix equation Ax = b?

a) 0
b) 1
c) 2
d) 5
e) infinity

Question 30: Let P be a 3rd order polynomial of the form $P(x) = ax^3 + bx^2 + cx + d$. How many combinations of a, b, c, and d satisfy the following conditions:

P(11) = -62
P(-17) = 123
P′(23) = 27
P′′(-5) = -32

You may assume the four equations resulting from the conditions produce a system,

$$A \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -62 \\ 123 \\ 27 \\ -32 \end{bmatrix},$$

where A is invertible.

a) 0
b) 1
c) Infinity
d) None of the above.
e) The solution cannot be determined with the given information.

Section 6 - Regression

Question 31: Assume the function $\hat{y}(x) = 2x^2 - 3$ estimates the 3 data points: (0,-2), (1,0), and (2, 4). What is the total squared error of $\hat{y}(x)$ to the given data?

a) 0
b) 1
c) -1
d) 2
e) None of the above

Question 32: Consider the following data set: x = [0 1 2 3 4] and y = [1 -1 2 3 5]. Let the estimation function for the given data be $\hat{y}(x) = \alpha$. What is the value of α computed using least squares regression?

a) 0
b) 1
c) 2
d) 5
e) None of the above.

Question 33: Let x and y be column vectors of the same size and let $\hat{y}(x) = \alpha\sin(2x) + 2\beta\sqrt{x}$ be an estimation function for the data. Assuming that p is a column vector where $p(1) = \alpha$ and $p(2) = \beta$, which of the following lines of code will solve for the parameters of $\hat{y}(x)$ using least squares regression?

a) `>> A = [sin(2*x), 2*sqrt(x)];        p = inv(A'*A)*A'*y;`
b) `>> A = [sin(2*x), sqrt(x)];          p = inv(A'*A)*A'*y;     p(2) = p(2)/2;`
c) `>> A = [sin(2*x), 2*sqrt(x)];        p = A\y;`
d) `>> A = [sin(2*x), 2*sqrt(x)];        p = pinv(A)*y;`
e) All of the above will find the correct parameters.

Question 34: Let x and y be column vectors of the same size and let $\hat{y}(x) = \alpha x^2 + \beta x^3 + 1$ be an estimation function for the data. Assuming that p is a column vector where $p(1) = \alpha$ and $p(2) = \beta$, which of the following lines of code will solve for the parameters of $\hat{y}(x)$ using least squares regression?

a) `>> A = [x.^3, x.^2, x.^0];           p = inv(A'*A)*A'*y;`
b) `>> A = [x.^2, x.^3, 1];              p = inv(A'*A)*A'*y;`
c) `>> A = [x.^2, x.^3];                 p = inv(A'*A)*A'*y;`
d) `>> A = [x.^2, x.^3];                 p = inv(A'*A)*A'*(y-1);`
e) Least squares regression cannot be used for this estimation function.

Question 35: Let x and y be column vectors of the same size and let $\hat{y}(x) = e^{\alpha x + \beta}$ be an estimation function for the data. Assuming that p is a column vector where $p(1) = \alpha$ and $p(2) = \beta$, which of the following lines of code will solve for the parameters of $\hat{y}(x)$ using least squares regression?

a) `>> A = [x.^1, x.^0];                 p = inv(A'*A)*A'*log(y);`
b) `>> A = [x.^1, x.^0];                 p = inv(A'*A)*A'*log(y);      p = exp(p);`
c) `>> A = [log(x.^1), log(x.^0)];       p = inv(A'*A)*A'*log(y);`
d) `>> A = [log(x.^1), log(x.^0)];       p = inv(A'*A)*A'*y;`
e) Least squares regression cannot be used for this estimation function.

Section 7 - Interpolation

Question 36: Given the data x = [0, 1, 2] and y = [3, 1, 2], what is the linear interpolation at X = 1.5?

a) 0.5
b) 1
c) 1.5
d) 2
e) None of the above.

Question 37: Given the data x = [0, 1, 2] and y = [3, 1, 2], what is the cubic spline interpolation at X = 2?

a) 0.5
b) 1
c) 1.5
d) 2
e) None of the above.

Question 38: Given the data x = [0, 1, 2] and y = [3, 1, 2], what is the Lagrange interpolation at X = 0.5?

a) 0.625
b) 1.625
c) 1.675
d) 2
e) None of the above.

Question 39: Consider the problem of finding the parameters of n-1 *quintic* splines between n data points, i.e. $S_i(x) = a_ix^5 + b_ix^4 + c_ix^3 + d_ix^2 + e_ix + f_i$ for $x_i < x \leq x_{i+1}$. To find the parameters of the quintic splines, we insist that the splines intersect the data points and that the first, second, third, and fourth derivatives are equal at the data point between adjacent splines. In addition to the aforementioned constraints, how many more constraints are required so that there will be a unique set of splines satisfying the constraints?

a) 1
b) 2
c) 3
d) 4
e) 5(n-1)

Section 8 - Series

Question 40: Consider the statement "Any mathematical function has a Taylor series and the Taylor series always converges to the function as the number of terms in the series goes to infinity".

a) The statement is true.
b) The statement is false
c) The statement is probably true
d) No one knows if the statement is true or false.
e) Only Oski knows if the statement is true or false.

Question 41: Which of the following expressions is equivalent to the Taylor series expansion for $f(x) = e^{3x}$ taken around the point a = 0.

a) $3\sum_{n=0}^{\infty}\frac{x^n}{n!}$ b) $\sum_{n=0}^{\infty}\frac{3x^n}{n!}$ c) $\sum_{n=0}^{\infty}\frac{x^{3n}}{n!}$ d) $\sum_{n=0}^{\infty}\frac{(3x)^n}{n!}$ e) None of these.

Question 42: Let P be the polynomial $P(x) = 9x^5 - 3x^3 - 5x^2 + 97$. What is the absolute difference between the 10^{th} order Taylor series approximation of P taken around a = 1 and the true value of P, both evaluated at x = 17.

a) 0.000012934
b) 0.290384
c) 1.5834
d) 1439.1934
e) None of the above

Section 9 - Root Finding

Question 43: Let

$$f(x) = e^{\frac{x}{\pi} - \sqrt{x}} - 1.$$

Which of the following is a root of f?

a) 1
b) 2
c) π
d) π^2
e) None of the above.

Question 44: Let f(x) be a function with a root at x* and let a = 0 and b = 1 be scalars that bound x* at the first iteration of the Bisection method. Let the error at the i^{th} iteration be defined as |b – a|, where a and b are the updated bounds at each iteration of the Bisection method. Which of the following is the closest to the number of iterations of the Bisection method required to achieve an error of less than 10^{-5}?

a) 5
b) 10
c) 15
d) 20
e) There is not enough information to answer the question.

Question 45: Let the function $f(x) = e^x – 2$ and let a = 0 and b = 2 bound a root of f. What will be the value of a and b after a single iteration of the bisection method, assuming that the current error (however it may be defined) is still larger than the tolerance?

a) a = 0, b = 2
b) a = 0, b = 1
c) a = 1, b = 2
d) a = 1, b = 1
e) None of the above.

Question 46: Let the function f(x) = 2x – 6 and let x_0 = -1 be an initial guess for the root of f. If the error at the i^{th} iteration is defined as $e_i = |f(x_i)|$, how many Newton steps will be required to achieve an error less than 10^{-7}?

a) 0
b) 1
c) 2
d) 3
e) None of the above.

Question 47: Let P be the polynomial $P(x) = -3.5x^4 + 7.5x^2 - 2$ and let x_i be an approximation of a root of P after i Newton steps. Starting at $x_1 = -1$, what will be the value of x_{10}?

a) -1
b) 0
c) 1
d) 1.353042756497228
e) None of the above.

Section 10 - Numerical Differentiation

Question 48 – 50: Consider the following data set:

X = [1.0, 1.2, 1.4, 1.6, 1.8, 2.0]
Y = [0.8415, 0.9320, 0.9854, 0.9996, 0.9738, 0.9093]

Question 48: What is the value of the forward difference at x = 1.6?

a) 0
b) 0.0011
c) -0.0290
d) 0.0710
e) -0.1290

Question 49: What is the value of the backward difference at x = 1.6?

a) 0
b) 0.0011
c) -0.0290
d) 0.0710
e) -0.1290

Question 50: What is the value of the central difference at x = 1.6?

a) 0
b) 0.0011
c) -0.0290
d) 0.0710
e) -0.1290

Question 51: For which of the following functions will the forward, backward, and central difference method produce the same approximation to f'(x)?

a) $f(x) = x$
b) $f(x) = x^2$
c) $f(x) = x^3$
d) $f(x) = x^4$
e) None of the above.

Question 52: Let P be the n^{th} order polynomial $P(x) = p_0 + p_1x + p_2x^2 + p_3x^3 + p_4x^4 + p_5x^5$ and p be the column vector $p = [p_0, p_1, p_2, p_3, p_4, p_5]^T$. Additionally, let D be the n-1^{th} order polynomial $D(x) = d_0 + d_1x + d_2x^2 + d_3x^3 + d_4x^4$ and d be the column vector $d = [d_0, d_1, d_2\ d_3, d_4]^T$. If A is a matrix and d = Ap, which A will make d the coefficients of P'(x)?

a) $$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

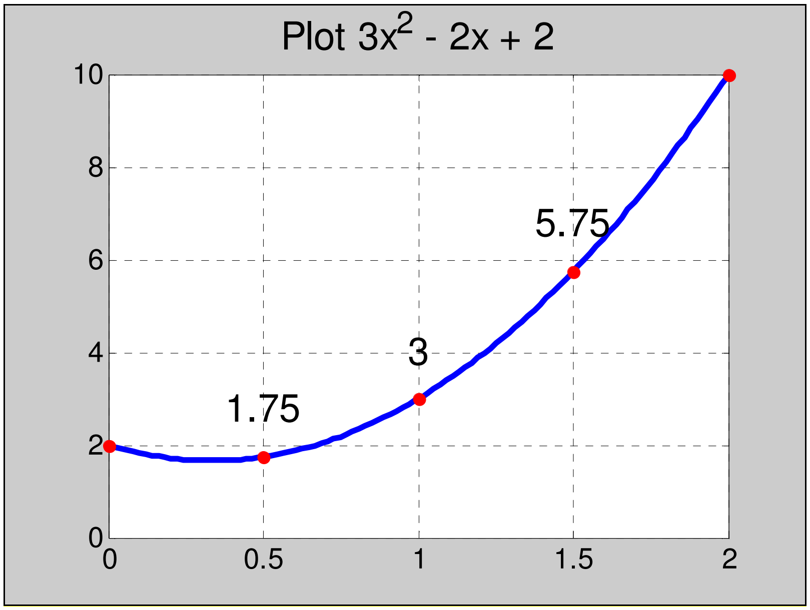
c) $$A = \begin{bmatrix} 5 & 4 & 3 & 3 & 1 \\ 0 & 4 & 3 & 2 & 1 \\ 0 & 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

d) $$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix}$$

e) none of the above

Section 11 - Numerical Integration

Questions 53 - 55: Let $f(x) = 3x^2 - 2x + 2$ and let the set of subintervals between 0 and 2 be defined by the points at x = 0, 0.5, 1, 1.5, 2.



Question 53: What is the Riemann approximation to the integral of f from 0 to 2 using the right endpoints of each subinterval?

a) 6.25
b) 8
c) 8.25
d) 10.25
e) 11.25

Question 54: What is the trapezoid rule approximation to the integral of f from 0 to 2 with the given subintervals?

a) 6.25
b) 8
c) 8.25
d) 10.25
e) 11.25

Question 55: What is the Simpson's rule approximation to the integral of f from 0 to 2 with the given subintervals?

a) 6.25
b) 8
c) 8.25
d) 10.25
e) 11.25

Question 56: Consider the function f(x) = |x| and assume that the domain of f between -0.5 and 0.5 has been discretized in increments of 0.5. Which of the following statements is true?

a) The trapezoidal approximation and Simpson's rule approximation of the integral of f from -0.5 to 0.5 will be equivalent.
b) The trapezoidal approximation of the integral of f from -0.5 to 0.5 will be closer to the true integral of f than the Simpson's rule approximation.
c) The Simpson's rule approximation of the integral of f from -0.5 to 0.5 will be closer to the true integral of f than the trapezoidal approximation.
d) The function f cannot be numerically integrated using trapezoid rule or Simson's rule.
e) All of the above statements are false.

Section 12 - Numerical Solutions to ODEs

Question 57: Which of the following expressions is an ordinary differential equation?

a) $ax_1 + bx_2 + cx_3 = d$

b) $\ddot{y} = \sqrt{t + \dot{y}}$

c) $c_1 x^3 + c_2 x^2 + c_3 x = 17$

d) $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

e) All of the above are ODE's.

Question 58: Which function satisfies the following ordinary differential equation?

$$\frac{dS}{dt} = 2te^{-S(t)}$$

a) $S(t) = e^{2t}$ b) $S(t) = t$ c) $S(t) = \log(t)$ d) $S(t) = \log(1 + t^2)$

e) None of the above.

Question 59: Which of the following matrix equations reduces the third order ODE, $y'''(t) + 2y'(t) - ty(t) = 0$ to a first order ODE in S? You may assume that $S(t) = \begin{bmatrix} y(t) \\ y'(t) \\ y''(t) \end{bmatrix}$.

a) $S'(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ t & -2 & 0 \end{bmatrix} S(t)$

b) $S'(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & t & -2 \end{bmatrix} S(t)$

c) $S'(t) = S(t) + \begin{bmatrix} t \\ -2 \\ 0 \end{bmatrix}$

d) $S'(t) = 2S(t) - t$

e) None of the above

Question 60 – 62: Consider the following first order ODE and initial condition.

$$\frac{dS}{dt} = \sin\left(S(t)\right)\cos\left(2S(t)\right)$$

$$S(0) = \frac{\pi}{2}$$

Question 60: What is the approximate value of S(π) computed using a single step from the Explicit Euler method?

a) 0
b) π/2
c) -π/2
d) 3π /2
e) None of the above.

Question 61: What is the approximate value of S(π) computed using a single step from the 4^{th} Order Runge-Kutta method?

a) 0
b) π/6
c) -π/6
d) 3π /2
e) None of the above.

Question 62: Which of the following calls to ode45 will solve the ODE numerically between t = 0 and t = 10 with the initial condition $S(t_0) = \pi/2$?

a) `[T, S] = ode45(@(S) sin(S).*cos(2*S), [0 10], pi/2)`
b) `[T, S] = ode45(@(t, S) sin(t).*cos(2*t), [0 10], pi/2)`
c) `[T, S] = ode45(@(t, S) sin(S).*cos(2*S), [0 10], 0)`
d) `[T, S] = ode45(@(t, S) sin(S).*cos(2*S), [0 10], pi/2)`
e) None of the above.

Question 63 - 67: Consider the following system of ODE's.

$$\begin{cases} \frac{dx}{dt} = xyz \\ \frac{dy}{dt} = x(y-z) \\ \frac{dz}{dt} = xy + z \end{cases}$$

where x, y, and z are functions of t.

Also, let exam4.m be a function that is formatted to represent the previous ODE's to ode45 and consider the following line of code:

```
>> [T, S] = ode45(@exam4, linspace(0,10,100), [1 1 1]);
```

Question 63: Which of the following versions of exam4 correctly solves the system of ODE's?

a)
```
function [dS] = exam4(S)
% let S = (x,y,z)
dS = [S(1)*S(2)*S(3);
    S(2)*(S(1) - S(3));
    S(2)*S(1) + S(3)];
end
```

b)
```
function [dS] = exam4(t, S)
% let S = (x,y,z)
dS = [S(1)*S(2)*S(3);
    S(1)*(S(2) - S(3));
    S(1)*S(2) + S(3)];
end
```

c)
```
function [dS] = exam4(S, t)
% let S = (x,y,z)
dS = [S(1)*S(2)*S(3);
    S(1)*(S(2) - S(3));
    S(1)*S(2) + S(3)];
end
```

d)
```
function [dS] = exam4(S)
% let S = (x,y,z)
dS = [S(1)*S(2)*S(3);
    S(1)*(S(2) - S(3));
    S(1)*S(2) + S(3)];
end
```

e) None of the above.

Question 64: How many elements will be contained in T?

a) 0
b) 10
c) 100
d) ode45 decides how many elements T will have.
e) None of the above.

Question 65: How many rows will there be in S?

a) 0
b) 10
c) 100
d) ode45 decides how many rows there are in S.
e) None of the above.

Question 66: How many columns will there be in S?

a) 0
b) 10
c) 100
d) ode45 decides how many columns are in S.
e) None of the above.

Question 67: Which line of code will retrieve the numerically integrated values of y from S?

a) `>> y = S(2);`
b) `>> y = S(:,2);`
c) `>> y = S(2,:);`
d) `>> y = S(:,1);`
e) None of the above.

Section 13 – Code Interpretation

Question 68: The following code is an implementation of:

a) Root Finding
b) Numerical Differentiation
c) Numerical Integration
d) Interpolation
e) None of the above.

```
function [out] = exam5(p, x0, tol)
% p is a row vector denoting a polynomial. i.e. p = [1, 3, 2] is the
% polynomial P(x) = x^2 + 3x + 2.
% polyval(p,x) returns the value of the polynomial p evaluated at x
% polyder returns a row vector denoting the derivative of p. i.e. d =
% polyder(p) results in d = [2, 3].

while abs(polyval(p, x0)) > tol
    x0 = x0 - polyval(p,x0)/polyval(polyder(p),x0);
end

out = x0;

end % end exam5
```

Question 69: The following code is an implementation of:

a) Root Finding
b) Numerical Differentiation
c) Numerical Integration
d) Interpolation
e) None of the above.

```
function [out, X] = exam6(x,y,a,b)
% assuming x is a vector, [m, I] = min(x) returns the minimum value of x, m, and the
% index where it occurs, I. you may assume that the values in x are unique and in
ascending order.

X = linspace(a,b,100);
out = zeros(1,100);

for i = 1:100
    [m,I] = min(abs(x - X(i)));
    out(i) = y(I);
end % end exam6
```

Question 70: The following code is an implementation of:

a) Root Finding
b) Numerical Differentiation
c) Numerical Integration
d) Interpolation
e) None of the above.

```
function [out] = exam7(x,y)
% x and y are column vectors of the same size, and x is unique and in ascending order.

out = sum((y(1:end-1)+y(2:end)).*(x(2:end)-x(1:end-1)))/2;

end % end exam7
```

Question 71: The following code is an implementation of:

a) Root Finding
b) Numerical Differentiation
c) Numerical Integration
d) Interpolation
e) None of the above.

```
function out = exam8(f, x, tol)
% you may assume that f is a function handle, x is a row vector of unique numbers, and
% tol is a positive scalar value such that tol << 1.

out = zeros(size(x));

for i = 1:length(x)

    w = 1;
    temp = (f(x(i) + w) - f(x(i) - w))/(2*w);

    w = .5;
    d = (f(x(i) + w) - f(x(i) - w))/(2*w);

    while abs(d - temp) > tol
        temp = d;
        w = w/2;
        d = (f(x(i) + w) - f(x(i) - w))/(2*w);
    end

    out(i) = d;
end

end % end exam8
```

Question 72: The following code is an implementation of:

a) Root Finding
b) Numerical Differentiation
c) Numerical Integration
d) Interpolation
e) None of the above.

```
function [out] = exam9(f, a, b, tol)
% you may assume that f is a function handle, a and b are scalars such that a < b, and
that tol is small positive scalar value.

m = (a + b)/2;
I = f(m)*(b - a);

if  I < tol
    out = I;
else
    out = exam9(f, a, m, tol) + exam9(f, m, b, tol);
end

end % end exam 9
```

Question 73: The following code is an implementation of:

a) Root Finding
b) Numerical Differentiation
c) Numerical Integration
d) Interpolation
e) None of the above.

```
function [out] = exam10(x, y, n)
% you may assume that x and y are column vectors of the same size and that
% n is a strictly positive integer

X = [];
for i = 0:n
    X = [X, x.^i];
end
out = inv(X'*X)*X'*y;
end % end exam10
```

Question 74: The following code is an implementation of:

a) Root Finding
b) Numerical Differentiation
c) Numerical Integration
d) Interpolation
e) None of the above.

```
function [out, X] = exam11(x,y,a,b,n)
% you may assume that x and y are vectors of the same length, a and b are
% scalars such that a < b, and n is a strictly positive integer. you may assume that x is
% in ascending order and has unique elements.

X = linspace(a,b,n);
out = zeros(1,n);

for i = 1:n
    for j = 1:length(x)
        if X(i) < x(j)
            J = j-1;
            break
        end
    end
    out(i) = y(J) + (X(i) - x(J))*(y(J+1)-y(J))/(x(J+1) - x(J));
end
end % end exam11
```

Question 75: The following code is an implementation of:

a) Root Finding
b) Numerical Differentiation
c) Numerical Integration
d) Interpolation
e) None of the above.

```
function [out, X] = exam12(x,y,n)
% you may assume that x and y are column vectors of the same length and
% that n is a strictly positive integer. The length of x and y will always
% be much greater than n. you may assume that x is unique and in ascending order.

N = length(x);
X = x(n+1:N-n);
out = zeros(size(X));

for i = n+1:N-n
    out(i-n) = mean(y(i-n:i+n));
end

end % end exam12
```