Ö. Savaş NAME: ____________________

11:10-12:30, Tuesday, March 11, 2003

ME-163 ENGINEERING AERODYNAMICS
MIDTERM EXAM - SOLUTIONS

1.(20%) Consider the two-dimensional flow field described by the potential function

$$\phi = ar^2 + b\,\theta$$

written in polar coordinates (r, θ), where a and b are positive constants.

(a) Determine the velocity field, its divergence and curl. Comment about the kinematic properties of the flow field.

(b) Determine and sketch the streamline passing through $(r, \theta) = (1, 0)$

$grad\,\phi = \left(\frac{\partial \phi}{\partial r}, \frac{1}{r}\frac{\partial \phi}{\partial \theta}\right)$, $div\,\mathbf{u} = \left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right)$, $curl\,\mathbf{u} = \left(\frac{\partial u_\theta}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{u_\theta}{r}\right)\mathbf{e}_z$

(a) $\mathbf{u} = (u_r, u_\theta) = (2ar, b/r)$, $div\ \mathbf{u} = 4a$, $curl\ \mathbf{u} = 0$ except at $r = 0$; uniform expansion and a point vortex.

(b) $d\mathbf{x} \times \mathbf{u} = (dr, rd\theta) \times (2ar, b/r) = 0 \Longrightarrow \frac{b}{2a}\frac{dr}{r^3} = d\theta \Longrightarrow \theta + \frac{b}{4a}\left(\frac{1}{r^2} - 1\right) = 0$

2.(20%) Consider the wind over a hill-side shown in the sketch. The wind over this topography may be descried as the potential flow of a combination of a uniform flow and a source. A Pitot-static tube at point A on the ridge is indicating a maximum reading of 0.5 torr (mmHg). Estimate the wind speed at point B on the flat land.

$U_A = \sqrt{2\Delta p/\rho} = \sqrt{2 \times 0.5 \times 133.32/1.21} = 10.5 m/s$

(§4.4) $U_A = (u^2 + v^2)^{1/2}_{(x,y)=(0,h/2)} = \left[U_\infty^2 + \left(\frac{2U_\infty}{\pi}\right)^2\right]^{1/2} = 1.19U_\infty \Longrightarrow U_\infty = 0.844U_A = 8.86\ m/s$

$U_B = (u^2 + v^2)^{1/2}_{(x,y)=(-5h,0)} = \left[\left(U_\infty - \frac{U_\infty}{5\pi}\right)^2_\infty + 0\right]^{1/2} = 0.968U_\infty = 0.817U_A = 8.58 m/s$

3.(20%) NASA's Helios prototype solar-powered *atmospheric satellite* is essentially a wing of span of 75.3 m and chord of 2.44 m. It is a two-dimensional 12%-thick asymmetric Selig S6078 airfoil designed to fly at 270 km/hr at about 25 km altitude where the air density is 0.039 kg/m^3 and temperature $220K$. Its maximum gross mass is 930 kg. Determine its design lift coefficient. If the mean camber line were a parabola, what would be the maximum camber when flying at zero angle of attack.

$U = 270 km/hr = 75 m/s$ $C_L = \frac{W}{\frac{1}{2}\rho U^2 A} = \frac{930 \times 9.81}{\frac{1}{2} \times 0.039 \times 75^2 \times 75.3 \times 2.44} = 0.543$

$C_L = 2\pi(\alpha + 2z_m/c); \alpha = 0, \Longrightarrow z_m/c = C_L/2\pi = 0.036 \Longrightarrow z_m = 8.8cm$

4.(20%) Determine the reading of a Pitot-static tube mounted on Helios described above in question #3.

$p_\infty = \rho_\infty R T_\infty = 0.039 \times 287 \times 220 = 2460\ Pa$

$\frac{p_o}{p_\infty} = \left(1 + \frac{\gamma - 1}{\gamma}\frac{\rho_\infty}{p_\infty}\frac{U^2}{2}\right)^{\gamma/(\gamma-1)} = \left(1 + \frac{1.4-1}{1.4} \times \frac{0.039}{2460} \times \frac{75^2}{2}\right)^{1.4/(1.4-1)} = 1.0453$

$\Longrightarrow \Delta p = p_o - p_\infty = 0.0453 p_\infty = 111\ Pa$

5.(20%) Consider two identical thin vortex rings as sketched in the figure. Describe the motion of the vortex ring-pair. *Hint: Recall the motion of two identical parallel line vortices.*

The rings leap frog through each other.