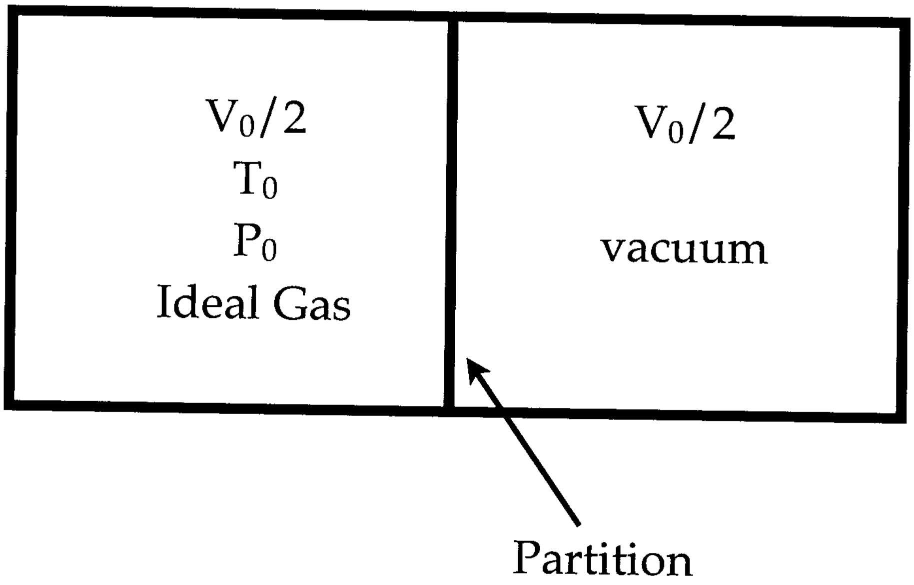
Question #1

A vessel of volume V_o is perfectly insulated from the environment, and contains a partition locked into place that divides the volume in half. On the left side of the partition is 1 mole of ideal gas at a temperature T_o and pressure P_o. On the right side of the partition there is vacuum.



Part A: The partition is released, and the gas is allowed to expand. Is this process reversible or irreversible, and why?

IRReversible, not in eq w/ other half of box during expansion

Part B: Calculate the work done by the gas during the expansion, and the final temperature of the system after the expansion is complete.

$$W = \int -p_{ext}\, dV = -\int 0 = 0$$

$$\boxed{W=0}$$

$$Q=0$$

$$\Delta U = Q + W = 0 = C_v \Delta T$$

$$\boxed{T = T_0}$$

Part C: What is the change in entropy of the system for this process?

Isothermal expansion $PV = nRT$

$dS = \cancel{\frac{dq}{T}}^{0} + \frac{P}{T}\,dV = nR\frac{dV}{V}$

$\int dS = nR\ln(V_f/V_0) = R\ln(V_0/V_{0/2}) = R\ln(2)$

$\Delta S = R\ln(2)$

Part D: Now, I interact with the system to return the partition to its initial position adiabatically. Calculate the change in internal energy and entropy for this process.

$du = dW \Rightarrow \Delta u = w$

$\gamma = \frac{C_p}{C_v} = \frac{5/2 R}{3/2 R}$

$\frac{5}{3} - \frac{3}{3} = 2/3 = M$

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$, $P = \frac{nRT}{V}$

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_0V_0^{\gamma-1} = T_2\left(\frac{V_0}{2}\right)^{\gamma-1}$

$T_2 = T_0(V_0)^{\gamma-1}\left(\frac{2}{V_0}\right)^{\gamma-1} = T_0(2^{\gamma-1})$

$T_2 = T_0(2^{2/3})$

$$\boxed{\Delta U = -\frac{3}{2}R\,(T_0 - T_0 2^{2/3}) = \frac{3}{2}RT_0(1 - 2^{2/3})}$$

$\boxed{\Delta S = 0}$ ⇒ adiabatic

$\gamma = \frac{C_p}{C_v}$

$\gamma - 1 = \frac{C_p - C_v}{C_v} = \frac{C_\gamma}{\cancel{R}}\frac{R}{C_v}$

Part E: What would be the entropy change if I returned the gas to its initial state, ($V_o/2, T_o, P_o$)?

$$0 = R\ln(2) + 0 + \Delta S_3$$

$$\boxed{\Delta S_3 = -R\ln 2}$$

Question #2

It can be shown that the internal energy per mole of a van der Waals fluid is:

$$u = \frac{3}{2}RT - a\rho$$

where R is the gas constant, T is temperature, and ρ is the molar density.

Part A: For a given volume, mole number, and temperature, which system will have a lower internal energy, an ideal gas or a van der Waals fluid? Why?

VdW, ~~pos~~ favorable attractive forces

Part B: Calculate the constant volume heat capacity of a van der Waals fluid, and compare it to the constant volume heat capacity of an ideal gas.

$C_V = \left(\frac{du}{dT}\right)_V$ $u = \frac{3}{2}RT - \frac{an}{V}$

$C_V = 3/2\, R$

– same as ideal gas

Part C: Calculate the dependence of a van der Waals fluid's internal energy on molar volume, $\left(\frac{\partial U}{\partial V}\right)_T$, and compare it to the same for an ideal gas.

$$U = \tfrac{3}{2}RT - \frac{an}{V}$$

$$\left(\frac{dU}{dV}\right)_T = +\frac{an}{V^2} = \frac{a}{V\rho}$$

i.g., $\left(\frac{dU}{dV}\right)_T = 0$, caused by molecular interactions