CE93 Engineering Data Analysis

Spring 2011 Midterm

11:10am-12:00pm, Mar15, 2011

You have 50 minutes to complete the midterm exam.
There are 4 questions; points are indicated for each question part. There are a total of 100 points.
Please be sure to write your name and student ID number on each and every page that you turn in.
Put your answers on this exam sheet. To receive full credit, clearly show your steps.
You may use a calculator and a cheat sheet (one side of one 8.5"x11").

Good luck!

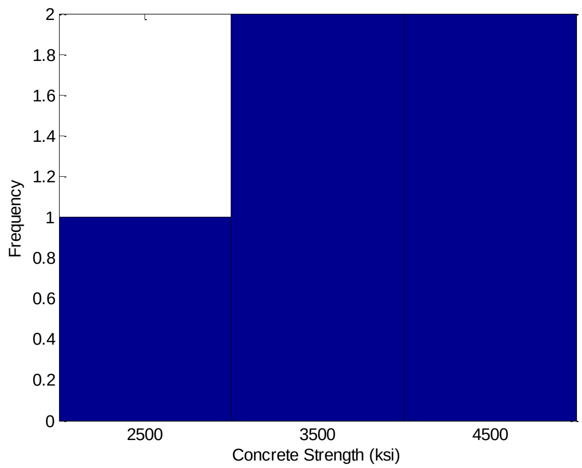
Name: ______________________

Lab Section: ____________________

1. In a sample of concrete core specimens from similar concrete mixes, you observe the following:

Specimen Number	Strength (ksi)
1	3500
2	3200
3	4500
4	4100
5	2500

a) (9 points) Plot a histogram of these results. Use 3 bins: [2000-3000), [3000-4000), and [4000-5000).



b) Calculate the:

i. (8 points) Sample Mean

ii. (8 points) Sample Variance

a)

b) i)

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{2500 + 3200 + 3500 + 4100 + 4500}{5} = 3560$$

ii)

$$\sigma_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1060^2 + 360^2 + 60^2 + 540^2 + 940^2}{4} = 608000$$

2. A city commuter can take one of three possible routes A, B, or C to work. On a given weekday morning, during rush hours, the chances that routes A, B, and C will have heavy traffic congestion, H, are respectively, 60%, 60%, and 40%. Routes A and B are close to each other such that if there is traffic congestion in one route, the chance of congestion in the other route is increased to 85%, whereas the condition in route C is unaffected (i.e. independent) by the traffic conditions in routes A and B. Also, if the traffic in all the three routes are congested, the chance that the commuter will be late, L, for work is 90%, otherwise it will be 30%.
   a) (8 points) What is the probability that only route A will be congested on a given weekday morning?
   b) (8 points) What is the probability that the commuter will be late for work on a given weekday morning?
   c) (9 points) On one particular weekday morning, the city commuter hears on the radio of heavy traffic congestion on route A. With this new information, what is the probability that the commuter will be late for work that day?

a) $P(AB^cC^c)=P(C^c)P(AB^c)$

$=(1-P(C))*(P(A)-P(A|B)P(B))$

$=0.6*(0.6-0.85*0.6)=0.6*0.09=0.054$

b) $P(L)=P(L|ABC)P(ABC)+P(L|(ABC)^c)P((ABC)^c)$

$P(ABC)=P(C)P(AB)=0.4*0.85*0.6=0.204$

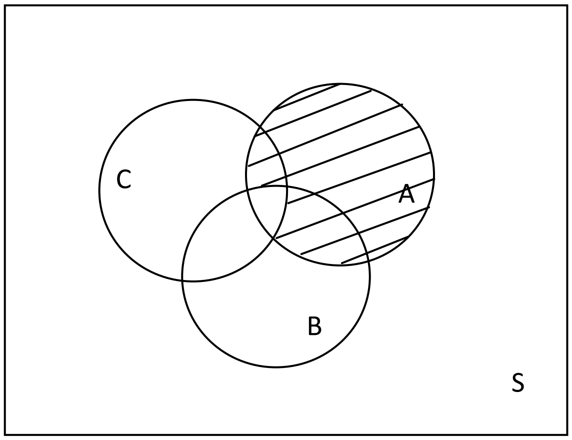
$P((ABC)^c)=1- P(ABC)=0.796$

$P(L)=0.9*0.204+0.3*0.796=0.4224$

c) $P(L|A)=P(L|ABC)P(BC|A)+P(L|A(BC)^c)P((BC)^c|A)=0.9*P(ABC)/P(A)+0.3*P((BC)^cA)/P(A)$

There are two methods to solve for $P((BC)^cA)$:

Method 1:
Look at the Venn diagram:



$(BC)^cA$ is represented by the shaded area.
Obviously, $P((BC)^cA)=P(A)-P(ABC)=0.6-0.204=0.396$

Method 2:

$$\begin{aligned}
P((BC)^cA) = P(AB^cUAC^c) &= P(AB^c) + P(AC^c) - P(AB^cC^c) \\
&= P(A) - P(AB) + P(A) - P(AC) - P(AB^c)P(C^c) \\
&= P(A) - P(AB) + P(A) - P(AC) - [P(A) - P(AB)]P(C^c)\frac{P(A(BC)^c)}{P(A)} \\
&= 0.6 - 0.85 * 0.6 + 0.6 - 0.4 * 0.6 - (0.6 - 0.85 * 0.6) * 0.6 = 0.396
\end{aligned}$$

Therefore

$P(L|A)=0.9*0.204/0.6+0.3*0.396/0.6=0.504$

3. Ten construction assistants in a company are candidates for promotion to project managers. Six are women and four are men. Assume that they are equally qualified. For this question, please arrive at a numerical value (not an expression with fractions or factorials).

   a) (12 points) If the company intends to promote four of the ten at random, what is the probability that exactly two of the four are men?

   b) (13 points) If the ten assistants randomly line up for their favorite Cheeseboard pizza, how many different formations are there where at least three men are in a row?

a) $\frac{\binom{6}{2}\binom{4}{2}}{\binom{10}{4}} = 0.4286$

b) Here we need to consider two possible situations: exactly three men in a row and exactly four men in a row.

For the first case, we need to choose three men from the four $\binom{4}{3}$, and do a permutation (3!). On the women side we have 6! ways to arrange them. Give a sequence of women, we need to choose 2 out of 7 positions to insert the three-men (as a group) and the remaining man between women. So there are $\binom{7}{2}$ ways. Finally, we can swap the positions of three-men group and the single man (2!). Therefore, in this case there are a total of $\binom{4}{3} * 3! * 6! * \binom{7}{2} * 2!$ formations.

If the four men are grouped together, there are 4! ways to arrange them within this group. Similarly, we have 6! ways to sequence women. Then we need to insert that four-men (as a group) to one of the 7 possible positions between women. So in this case we have $4! * 6! * 7$ formations.

The total number of formations satisfying b) is therefore

$$\binom{4}{3} * 3! * 6! * \binom{7}{2} * 2! + 4! * 6! * 7 = 846720$$

4. The maximum load S (in tons) on a structure is modeled by a continuous random variable S whose CDF is given as follows:

$$F_S(s) = \begin{cases} 0 & s \le 0 \\ -\frac{s^3}{864} + \frac{s^2}{48} & 0 < s \le 12 \\ 1 & s > 12 \end{cases}$$

   a) (8 points) What is the probability density function for S?

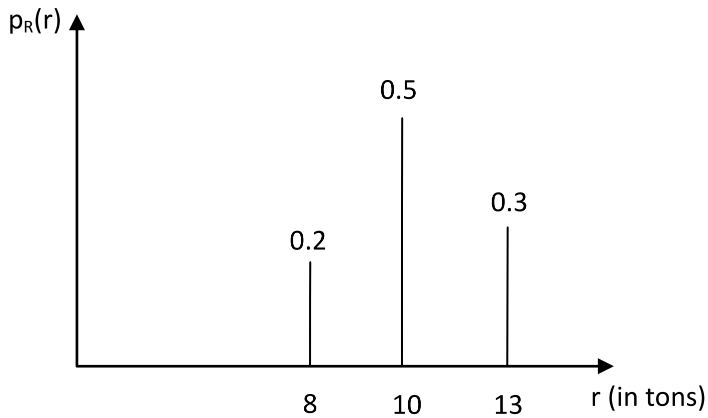
   b) (8 points) Determine the expected value of S

(part "c" is on the next page)

a) $f_S(s) = \frac{dF_S(s)}{ds} = \begin{cases} \frac{-s^2}{288} + \frac{s}{24} & 0 < s \le 12 \\ 0 & \text{otherwise} \end{cases}$

b) $E(S) = \int_0^{12} (\frac{-s^3}{288} + \frac{s^2}{24}) ds = 6$

c) (9 points) The strength R of the structure can be modeled by a discrete random variable with the following probability mass function:



Assume that the strength of the structure and load are statistically independent. What is the probability of failure, i.e. the probability that loading S is greater than the strength R?

c)

$P(S>R)=P(S>R|R=8)P(R=8)+ P(S>R|R=10)P(R=10)+ P(S>R|R=13)P(R=13)$

$P(S>R|R=13)=0$ since S has non-zero probability only between 0 and 12.

$P(S>R|R=8)= P(S>8)=1-F_S(8)=1-0.74074=0.25926$

$P(S>R|R=10)=P(S>10)= 1-F_S(8)=0.074074$

Therefore

$P(S>R)=0.2*0.25926+0.5*0.074074=0.088889$