EXAM 1 (100 Points, Show All Work)

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Phy 7B
7-16-02

12 Points Each

1. During each cycle, a Carnot engine removes 100 J of energy from a reservoir at 400 K, does work and exhausts heat to a reservoir at 300 K. Compute the entropy change of each reservoir for each cycle and show that the entropy change of the universe is zero for this reversible process.

2. If 1 Kg of ice at -20° C is heated at a pressure of 1 atm until all of the ice has been changed to steam, how much heat is required?

3. 100 g of CO_2 occupies a volume of 55 L at a pressure of 1 atm.
   a) Find the temperature of the CO_2.
   b) If the volume is increased to 80 L at constant temperature, what is the new pressure?

4. A quantity of air ($\gamma = 1.4$) expands adiabatically from an initial pressure of 2 atm, a volume of 2 L and a temperature of 20° C to twice its original volume.
   a) Calculate the final pressure.
   b) Calculate the final temperature.
   c) Calculate the work done by the gas.

16 Points Each

5. Calculate the magnitude and direction of the electric field at any point P a distance y from a very (∞) long wire of uniformly distributed charge, see Figure 1. Assume that y is much smaller than the length of the wire and let λ be the charge per unit length (C/m). Hint: Show that the electric field has a magnitude, $E = 2k\lambda/y$.

6. An infinite line charge of linear density $\lambda = 0.6\ \mu C/m$ lies along the z-axis, and a point charge $q = +8\ \mu C$ lies on the y-axis at y = 3m as shown in Figure 2. Find the electric field at the point P on the x-axis at x = 4 m.

20 Points

7. Two equal positive point charges of magnitude +5 nC are on the x-axis. One is at the origin and the other is at x = 8 cm as shown in Figure 3.
   a) Find the electric potential, V, at point P_1 on the x-axis at x = 4 cm.
   b) Find the potential, V, at point P_2 on the y-axis at y = 6 cm.

Possibly Useful Constants

R = 8.31 J/mole-K = 0.0821 L-atm/mole-K

Latent Heat of Water

Heat of Fusion	3.33×10^5 J/Kg
Heat of Vaporization	2.26×10^6 J/Kg

Specific Heat of Water

Water Ice	2100 J/Kg-K
Liquid Water	4186 J/Kg-K
Steam Water	2010 J/Kg-K

$\gamma = C_p/C_v = 1.67$ (Monoatomic gas)

$\gamma = C_p/C_v = 1.4$ (Diatomic gas)

$k = 9 \times 10^9\ Nm^2/C^2$

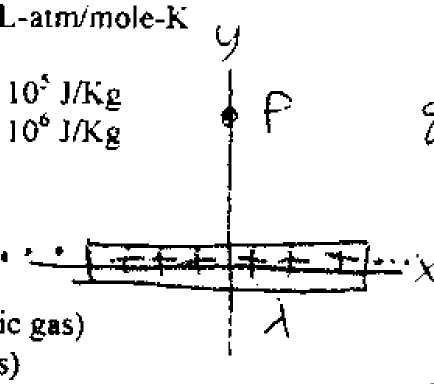


Figure 1

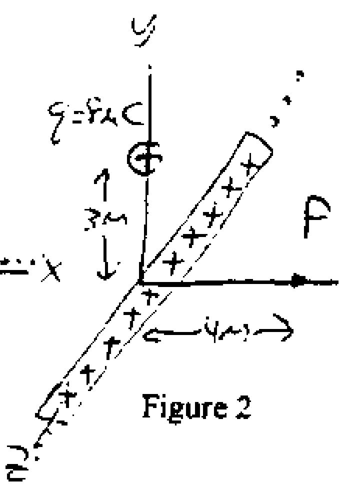


Figure 2

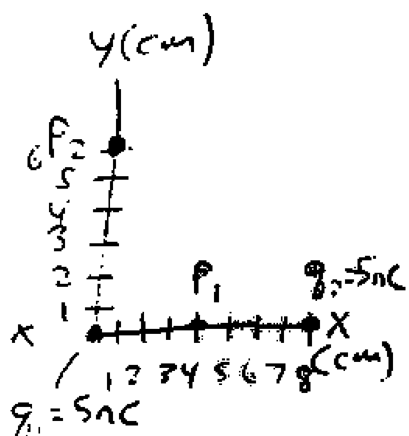


Figure 3

Exam 1 Solutions

① Carnot Heat Engine

$$e = 1 - \frac{T_l}{T_h} = 1 - \frac{300\,K}{400\,K} = 0.25 \quad , \quad e = 1 - \frac{|Q_l|}{|Q_h|}$$

$$\Rightarrow |Q_l| = |Q_h|(1-e) = (100\,J)(1-0.25) = 75\,J$$

$$\Delta S_{high} = \frac{-|Q_h|}{T_h} = \frac{-100\,J}{400\,K} = \boxed{-0.25\,\frac{J}{K} = \Delta S_{high}}$$

$$\Delta S_{low} = \frac{|Q_l|}{T_l} = \frac{75\,J}{300\,K} = \boxed{0.25\,\frac{J}{K} = \Delta S_{low}}$$

$$\Delta S_{tot} = \Delta S_{high} + \Delta S_{low} = -0.25\,\frac{J}{K} + 0.25\,\frac{J}{K} = \boxed{0 = \Delta S_{tot}}$$

② Heat Capacity & Heat of Transformation

$Q_1 = mc\Delta T = (1\,kg)(2100\,J/kg\,K)(20\,K) = 4.2\times10^4\,J$ ($-20°C$ to $0°C$)

$Q_2 = mL_f = (1\,kg)(3.33\times10^5\,J/kg) = 3.33\times10^5\,J$ (fusion)

$Q_3 = mc\Delta T = (1\,kg)(4186\,J/kg\,K)(100\,K) = 4.19\times10^5\,J$ ($0°C$ to $100°C$)

$Q_4 = mL_v = (1\,kg)(2.26\times10^6\,J/kg) = 2.26\times10^6\,J$ (vaporization)

$$Q_{tot} = Q_1 + Q_2 + Q_3 + Q_4 = 4.2\times10^4\,J + 3.33\times10^5\,J + 4.19\times10^5\,J + 2.26\times10^6\,J$$

$$\Rightarrow \boxed{Q_{tot} = 3.05\times10^6\,J}$$

③ Expansion of an Ideal Gas

a) $n = \frac{m}{M} = \frac{100g}{44g/mole} = 2.27\ mole$

$PV = nRT \Rightarrow T = \frac{PV}{nR} = \frac{(1atm)(55L)}{(2.27mole)(0.0821\frac{L\text{-}atm}{mole\text{-}K})}$

$\Rightarrow \boxed{T = 295K}$

b) $P_iV_i = P_fV_f \Rightarrow P_f = \frac{V_i}{V_f}P_i = \frac{55L}{80L}(1atm) = \boxed{0.69atm = P_f}$

④ Adiabatic Expansion of Air (Ideal Gas)

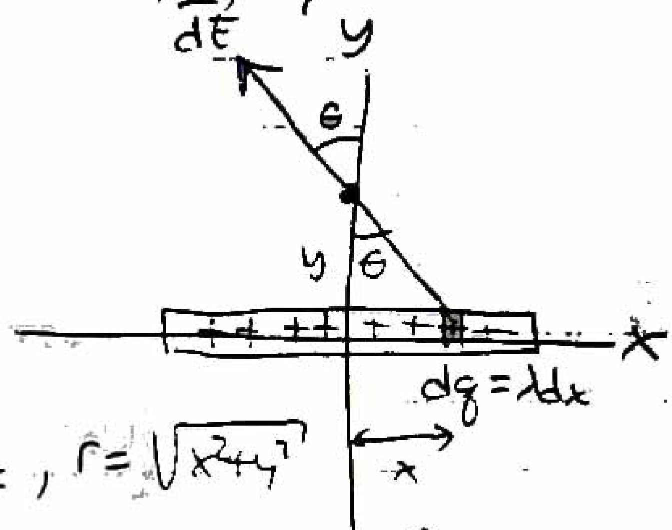
a) $P_iV_i^{\gamma} = P_fV_f^{\gamma} \Rightarrow P_f = P_i\left(\frac{V_i}{V_f}\right)^{\gamma} = (2atm)\left(\frac{2L}{4L}\right)^{1.4} = \boxed{0.76atm = P_f}$

b) $T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1} \Rightarrow T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = (293K)\left(\frac{2L}{4L}\right)^{0.4} = \boxed{222K = T_f}$

c) $W = \frac{1}{\gamma-1}\left[P_iV_i - P_fV_f\right] = \frac{(2atm)(2L)-(0.76atm)(4L)}{1.4-1}$

$\boxed{W = 2.42\ L\text{-}atm}$

⑤ Electric Field due to Very Long Wire



$$|d\vec{E}| = \frac{k\,dq}{r^2} = \frac{k\lambda\,dx}{r^2}$$

From Symmetry

$$E_x = 0 \Rightarrow \vec{E} = E_y\,\hat{j}$$

$$dE_y = \frac{k\lambda\,dx}{r^2}\cos\theta \quad , \cos\theta = \frac{y}{r} \;, \; r = \sqrt{x^2+y^2}$$

$$E_y = \int dE_y = \int_{x=-\infty}^{x=\infty} dE_y = \int_{-\infty}^{\infty} \frac{k\lambda\,dx\cos\theta}{r^2} = \int_{-\infty}^{\infty} \frac{k\lambda\,dx\,y}{(x^2+y^2)^{3/2}}$$

$$E_y = k\lambda y \int_{-\infty}^{\infty} \frac{dx}{(x^2+y^2)^{3/2}} \qquad \text{let } x = y\tan\theta$$
$$dx = y\sec^2\theta\,d\theta$$

$$\Rightarrow E_y = k\lambda y \int \frac{y\sec^2\theta\,d\theta}{(y^2\tan^2\theta + y^2)^{3/2}} = \frac{k\lambda}{y}\int \cos\theta\,d\theta$$

$$E_y = \frac{k\lambda}{y}\sin\theta\Big|_{-\pi/2}^{\pi/2} = \frac{k\lambda}{y}\left(1-(-1)\right) = \boxed{\frac{2k\lambda}{y} = E_y}$$

6. Electric Field due to Line Charge & Point Charge

$$\vec{E}_L = \frac{2k\lambda}{x}\hat{i} = \frac{2(9\times10^9 Nm^2/C^2)(0.6\times10^{-6} C/m)}{4m}\hat{i}$$

$$\vec{E}_L = 2.7\times10^3 \frac{N}{C}\hat{i}$$, $r = \sqrt{x^2+y^2} = \sqrt{(4m)^2+(3m)^2}$

$r = 5m$

$$\vec{E}_P = \frac{kq}{r^2}\hat{r} = \frac{(9\times10^9 Nm^2/C^2)(8\times10^{-6} C)}{(5m)^2}\hat{r} = 2.88\times10^3 \frac{N}{C}\hat{r}$$

$$E_{Px} = E_P\cos\theta = (2.88\times10^3 \tfrac{N}{C})(\tfrac{4}{5}) = 2.3\times10^3 N/C$$

$$E_{Py} = -E_P\sin\theta = -2.88\times10^3 N/C(\tfrac{3}{5}) = -1.73\times10^3 N/C$$

$$E_x = E_{Lx} + E_{Px} = 2.7\times10^3 \tfrac{N}{C} + 2.3\times10^3 \tfrac{N}{C} = 5.0\times10^3 \tfrac{N}{C}$$

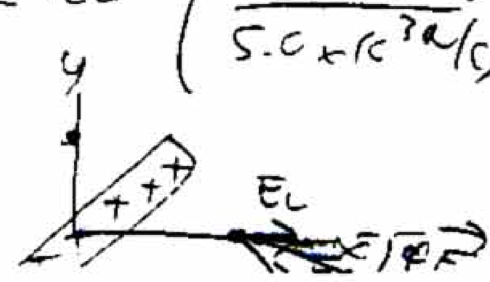
$$E_y = E_{Ly} + E_{Py} = 0 + (-1.73\times10^3 N/C) = -1.73\times10^3 N/C$$

$$E = |\vec{E}| = \sqrt{E_x^2+E_y^2} = \sqrt{(5.0\times10^3 \tfrac{N}{C})^2 + (-1.73\times10^3 N/C)^2}$$

$$\boxed{E = 5.29\times10^3 N/C}$$

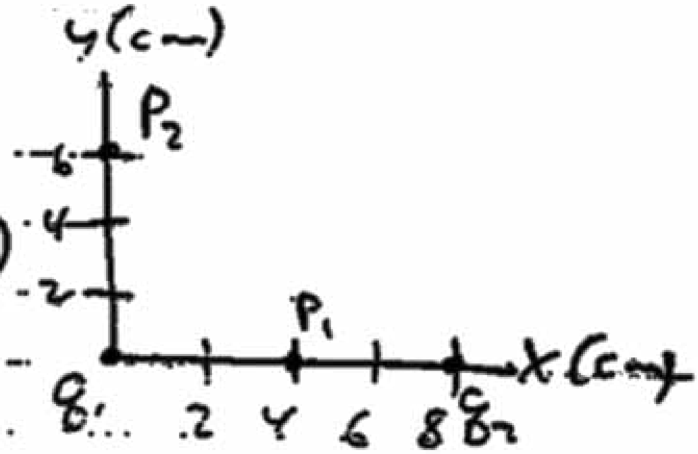
$$\tan\phi = \frac{|E_y|}{|E_x|} \Rightarrow \phi = \tan^{-1}\left(\frac{|E_y|}{|E_x|}\right) = \tan^{-1}\left(\frac{1.73\times10^3 N/C}{5.0\times10^3 N/C}\right)$$

$\boxed{\phi = 19.1^\circ \text{ below the x-axis}}$



7) Potential due to Two Point Charges

a) $V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$



$$V = \frac{2kq}{r} = \frac{2\left(9\times10^{9}\,\frac{Nm^2}{C^2}\right)\left(5\times10^{-9}\,C\right)}{0.04\,m}$$

$$\boxed{V = 2250\,V}$$

b) $V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$ $\qquad r_2 = \sqrt{x^2+y^2} = \sqrt{(8cm)^2+(6cm)^2}$

$r_2 = 10\,cm$

$$\Rightarrow V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = \frac{\left(9\times10^{9}\,Nm^2/C^2\right)\left(5\times10^{-9}\,C\right)}{0.06\,m} + \frac{\left(9\times10^{9}\,\frac{Nm^2}{C^2}\right)\left(5\times10^{-9}\,C\right)}{0.10\,m}$$

$$\boxed{V = 1200\,V}$$