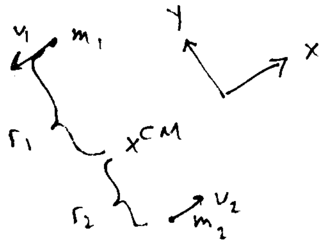
1) Setting CM at the origin, we know



$$0 = CM = \frac{r_1 m_1 - r_2 m_2}{m_1+m_2} \Rightarrow r_1 m_1 = r_2 m_2 .$$

While $d = r_1 + r_2$ so

$$d = r_1 + r_1 \frac{m_1}{m_2} = r_1\left(\frac{m_1+m_2}{m_2}\right) \Rightarrow r_1 = \frac{m_2}{m_1+m_2} d$$

$$= \frac{m_2}{m_1} r_2 + r_2 = \left(\frac{m_1+m_2}{m_1}\right) r_2 \Rightarrow r_2 = \frac{m_1}{m_1+m_2} d$$

The force on each ~~plane~~ star (by the other) is $F = \frac{G m_1 m_2}{d^2}$, so

$$a_1 = \frac{G m_2}{d^2}$$

$$a_2 = \frac{G m_1}{a^2}$$

Then, as circular motion requires centripetal acceleration,

$$a_1 = \frac{v_1^2}{r_1} \Big/ a_2 = \frac{v_2^2}{r_1},$$

and the period is given by

$$\frac{2\pi r_1}{v_1} \Big/ \frac{2\pi r_2}{v_2}$$

so

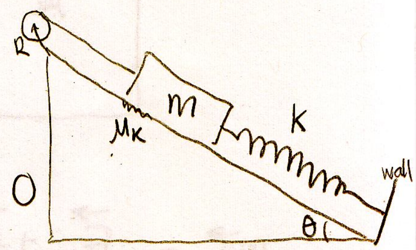
$$T_1 = \frac{2\pi r_1}{v_1} = \frac{2\pi r_1}{\sqrt{a_1 r_1}} = \frac{2\pi r_1 d}{\sqrt{G m_2 r_1}} = \frac{2\pi \frac{\cancel{m_2}}{m_1+m_2} d^2}{\sqrt{G \frac{\cancel{m_2^2}}{m_1+m_2} d}} = 2\pi \sqrt{\frac{d^3}{G(m_1+m_2)}}$$

$$T_2 = \frac{2\pi r_2}{v_2} = \frac{2\pi r_2}{\sqrt{a_2 r_2}} = \frac{2\pi r_2 d}{\sqrt{G m_1 r_2}} = \frac{2\pi \frac{m_1}{m_1+m_2} d^2}{\sqrt{G \frac{m_1^2}{m_1+m_2} d}} = 2\pi \sqrt{\frac{d^3}{G(m_1+m_2)}}$$

As expected, the periods of the 2 stars are the same, as they are really orbitting around each other.

Problem 2 :

Ref : potential energy = 0 at equilibrium position



Initially : $E_i = E_{reel_i} + E_{mass_i}$

$$= 0 + mgd\sin\theta + \frac{1}{2}kd^2 + 0$$

Finally : $E_f = E_{reel_f} + E_{mass_f} + W_{\text{done by friction}}$

$$= \frac{1}{2}I\omega^2 + 0 + \frac{1}{2}mv_m^2 + \mu_k mg\cos\theta\, d$$

Assume massless & unstretchable string $\rightarrow$ $v_{reel} = v_m$

thus $v_m = \omega R$

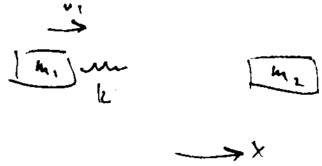
Therefore $E_i = E_f$

$$\Leftrightarrow mgd\sin\theta + \frac{1}{2}kd^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}m\omega^2R^2 + \mu_k mg\cos\theta\, d$$

$$\hookrightarrow \omega = \sqrt{\frac{2mgd(\sin\theta - \mu_k\cos\theta) + kd^2}{I + mR^2}} \; \#$$

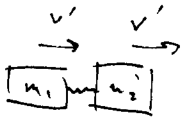
izzy

3) a) Let us solve this via energy methods. We know the spring will compress until the velocities of the 2 gliders are the same. Then



I_0

I_1



$p_0 = m_1 v_1 + m_2 v_2$ $\qquad\qquad$ $p_1 = (m_1 + m_2) v'$

$E_0 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$ $\qquad\qquad$ $E_1 = \frac{1}{2} m_1 v'^2 + \frac{1}{2} m_2 v'^2 + \frac{1}{2} k x_{max}^2$

$$p_0 = p_1 \Rightarrow v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$E_0 = E_1 \Rightarrow \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 \left(\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\right)^2 + \frac{1}{2} m_2 \left(\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\right)^2 + \frac{1}{2} k x_{max}^2$$

$$= \frac{1}{2}\left(\frac{m_1^2}{(m_1+m_2)} v_1^2 + \frac{2 m_1 m_2}{(m_1+m_2)} v_1 v_2 + \frac{m_2^2}{(m_1+m_2)} v_2^2\right) + \frac{1}{2} k x_{max}^2$$

$$\Rightarrow k x_{max}^2 = \left(\frac{m_1 m_2}{m_1 + m_2} v_1^2 - \frac{2 m_1 m_2}{m_1 + m_2} v_1 v_2 + \frac{m_1 m_2}{m_1 + m_2} v_2^2\right)$$

$$= \frac{m_1 m_2}{m_1 + m_2} (v_1 - v_2)^2$$

$$\Rightarrow x_{max}^2 = \frac{1}{k} \frac{m_1 m_2}{m_1 + m_2} (v_1 - v_2)^2$$

b) We found the speed of each mass at x_{max} in part a)

$$v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

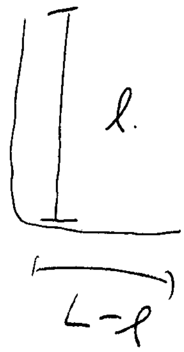
c) As there are no dissipative forces & momentum is conserved, the collision is elastic.

4) Velocity at length l:

$$\frac{1}{2}\delta m v^2 = \delta m g \Delta h$$

$$v^2 = 2g\Delta h$$

$$\boxed{V = \sqrt{2g(L-l)}} \quad \left(V = \sqrt{gL} \text{ for } l = \frac{L}{2}\right)$$



Normal reaction:

$$N = N_i + N_w, \quad N_i \text{ is the impulse to stop the rope.}$$

N_w is to support the weight of the remaining rope!

$$N_w = \frac{L-l}{L} M g$$

$$= \left(1 - \frac{l}{L}\right) M g \;//$$

$$N_i = \frac{\delta m v}{\delta t}, \quad \delta t = \frac{\delta l}{V}, \quad \delta m = \frac{\delta l}{L} M$$

$$= \frac{\frac{\delta l}{L} M}{\delta l / V} V$$

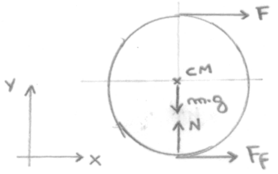
$$= \frac{M V^2}{L} \;//$$

$$N = \frac{M\, 2g(L-l)}{L} + \left(1 - \frac{l}{L}\right) M g$$

$$\boxed{N = 3\left(1 - \frac{l}{L}\right) M g} \quad \left(= \frac{3}{2} M g \text{ for } l = \frac{L}{2}\right)$$

P5

a) FBD



- $\sum F_y = 0 \rightarrow N = m \cdot g$ (1)
- $\sum F_x = m \cdot a_{cm} \rightarrow F + F_f = m \cdot a_{cm}$ (2)
- $\sum \tau_{cm} = I_{cm} \cdot \alpha \rightarrow F_f \cdot R - F \cdot R = I_{cm} \cdot \alpha$ (3)
- Rolls without slipping $\rightarrow a_{cm} = -\alpha \cdot R$ (4)

$$(2) \rightarrow a_{cm} = \frac{F + F_f}{m} \quad (5)$$

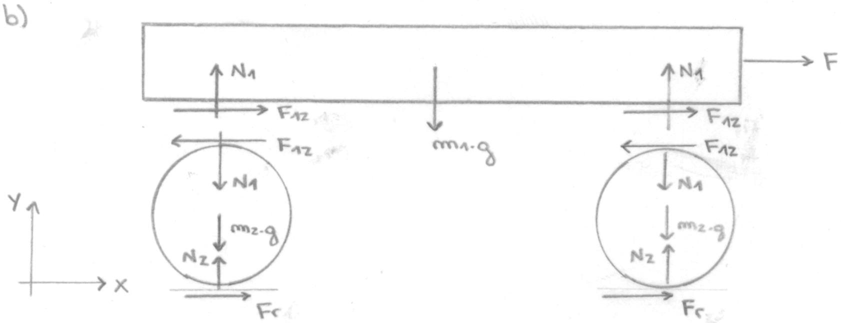
$$(5) \text{ and } (4) \text{ in } (3) \rightarrow F_f \cdot R - F \cdot R = I_{cm} \cdot \left(-\frac{F+F_f}{m \cdot R}\right) = \frac{1}{2} \cancel{m} R^{\cancel{2}} \cdot \left(-\frac{F+F_f}{\cancel{m} \cdot \cancel{R}}\right)$$

$$\rightarrow (F_f - F) \cdot \cancel{R} = -\frac{F+F_f}{2} \cdot \cancel{R} \rightarrow 2 \cdot F_f - 2 \cdot F = -F - F_f$$

$$\rightarrow \boxed{F_f = 1/3 \cdot F}$$ the direction is the x-positive direction ($\longrightarrow$)

$$(2) \rightarrow F + \frac{1}{3} \cdot F = m \cdot a_{cM} \rightarrow \boxed{a_{cM} = \frac{4 \cdot F}{3\,m}}$$

b) FBD's



Equations for the plank:

- $\sum F_x = m_1 \cdot a_{cm1}$

$\rightarrow F + 2 \cdot F_{12} = m_1 \cdot a_{cm1}$ (1)

- $\sum F_y = 0 \rightarrow N_1 = \frac{m_1 \cdot g}{2}$ (2)

Equations for each cylinder

- $\sum F_x = m_2 \cdot a_{cm2}$

$\rightarrow F_r - F_{12} = m_2 \cdot a_{cm2}$ (3)

- $\sum F_y = 0 \rightarrow N_2 = N_1 + m_2 \cdot g$ (4)
- $\sum \tau_{cm} = I_{cm} \cdot \alpha$

$\rightarrow F_{12} \cdot R + F_r \cdot R = I_{cm} \cdot \alpha$ (5)

5 equations ; 7 unknows ($a_{cm1}, a_{cm2}, N_1, N_2, F_r, F_{12}, \alpha$)

Kinematic equations

- Roll without slipping on a flat surface $\rightarrow a_{cm2} = -\alpha \cdot R$ (6)
- Roll without slipping between cylinder and plank $\rightarrow a_{cm1} = 2 \cdot \alpha \cdot R$ (7)

$\boxed{a_{cm1} = 2 \cdot a_{cm2}}$

$$(1) \rightarrow a_{cm1} = \frac{F + 2 \cdot F_{12}}{m_1} \quad (8)$$

$$(3) \rightarrow a_{cm2} = \frac{F_r - F_{12}}{m_2} \quad (9)$$

(9) and (6) in (5)

$$\rightarrow F_{12} \cdot R + F_r \cdot R = I_{cm} \cdot \left(-\frac{F_r - F_{12}}{m_2 \cdot R}\right) = \frac{1}{2} \cancel{m_2} \cdot R^{\cancel{2}} \cdot \frac{F_{12} - F_r}{\cancel{m_2} \cdot \cancel{R}}$$

$$\cancel{R}(F_{12} + F_r) = \frac{F_{12} - F_r}{2} \cdot \cancel{R} \rightarrow 2 \cdot F_{12} + 2 \cdot F_r = F_{12} - F_r$$

$$\rightarrow F_{12} = -3 \cdot F_r \quad (10)$$

$\hookrightarrow$ F_{12} goes in the other direction

(10) in (9)

$$\rightarrow a_{cm2} = \frac{F_r + 3 \cdot F_r}{m_2}$$

$$\rightarrow a_{cm2} = +4 \cdot \frac{F_r}{m_2} \quad (11)$$

(11) and (8) ($a_{cm1} = 2 a_{cm2}$)

$$\rightarrow \frac{F + 2 \cdot F_{12}}{m_1} = \frac{-8 \cdot F_{12}}{3 \cdot m_2}$$

$$\boxed{F_{12} = \frac{+3 \cdot m_2}{8 \cdot m_1 + 6 \cdot m_2} \cdot F} \rightarrow$$

$$\boxed{F_r = \frac{m_2}{8 \cdot m_1 + 6 \cdot m_2} \cdot F} \rightarrow$$

$$\boxed{a_{cm1} = \frac{F}{m_1 + \frac{3}{4} m_2}}$$