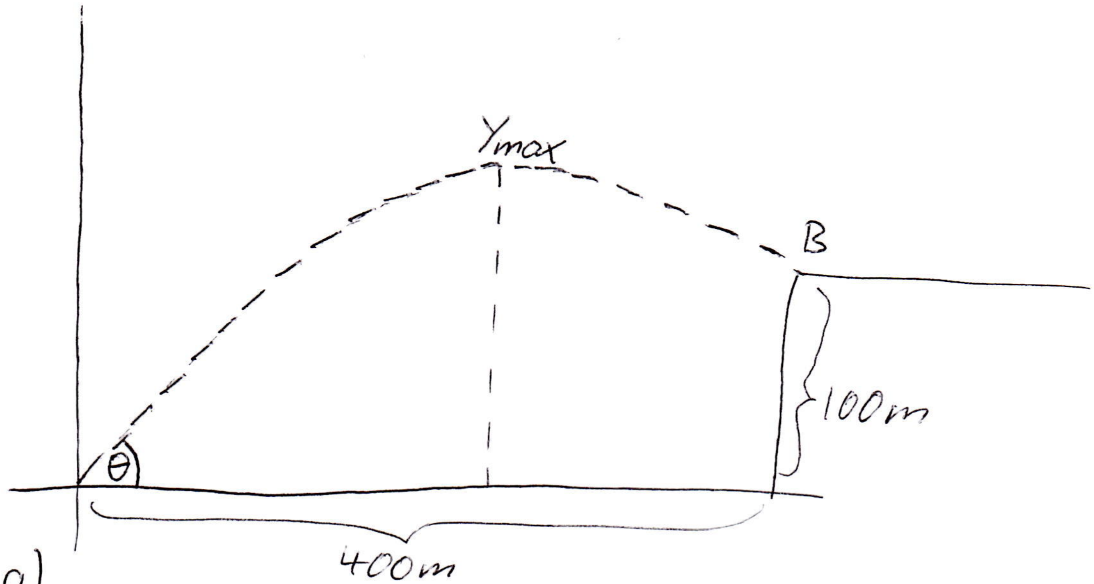
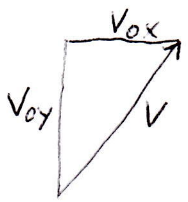
Problem 1



a)

$x = V_{ox} t = 400m \Rightarrow V_{ox} = 40 \, m/s$

$y = V_{oy} t - \frac{1}{2} g t^2 = 100m \Rightarrow V_{oy} = 59 \, m/s.$



$V_0^2 = V_{ox}^2 + V_{oy}^2 = 71.3 \, m/s.$

$\tan\theta = \frac{V_{oy}}{V_{ox}} \Rightarrow \theta = 55.9°.$

b) The velocity in the y-direction at the maximum height is zero. Using

$V_{y,max}^2 = 0 = V_{oy}^2 - 2g y_{max}$, we find

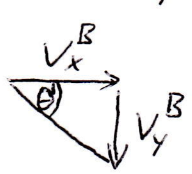
$y_{max} = \frac{V_{oy}^2}{2g} = 178 \, m.$

c) The velocity in the x-direction is constant:

$V_x^B = V_{ox} = 40 \, m/s.$

The velocity in the y-direction is given as

$V_y^B = V_{oy} - gt = -39 \, m/s.$



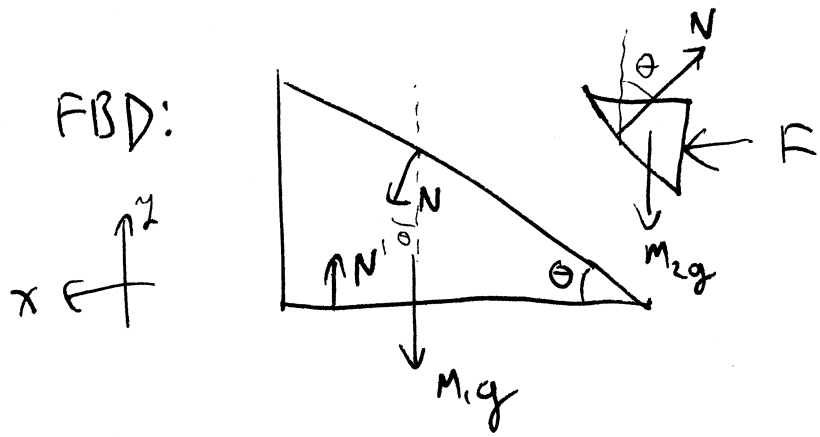
$(V^B)^2 = (V_x^B)^2 + (V_y^B)^2 \Rightarrow V^B = 55.9 \, m/s$

$\tan\theta' = \frac{|V_y^B|}{V_x^B} = 0.975 \Rightarrow \theta' = 44.3°.$

PHYS 7A Sp11 MT1 L1 Sol

Q 2

FBD:



2nd Law for M_1: x) $N\sin\theta = m_1 a$ — ①

M_2 x) $F - N\sin\theta = m_2 a$ — ②

y) $N\cos\theta - m_2 g = 0$ — ③

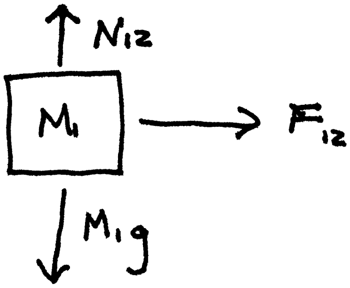
①, ② ⇒ $N\sin\theta = \dfrac{m_1 F}{m_1 + m_2}$ — ④

③, ④ ⇒ $\tan\theta = \dfrac{m_1 F}{(m_1+m_2) m_2 g}$

$$F = \frac{(m_1+m_2)}{m_1} m_2 g \tan\theta$$

3

1)



Concepts:

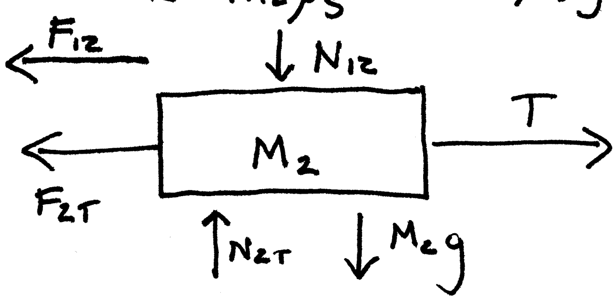
- Set all 3 'a' equal
- Assume force due to static friction is at max, $N_{12}\mu_s = F_{12}$

$$F_y = 0 = N_{12} - M_1 g \implies \underline{N_{12} = M_1 g}$$

$$F_x = \underbrace{F_{12}} = M_1 a = M_1 \mu_s g \implies \underline{a = \mu_s g}$$

$$F_{12} = N_{12}\mu_s = M_1 \mu_s g$$

2)



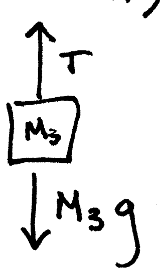
$$F_y = 0 = N_{2T} - N_{12} - M_2 g = N_{2T} - (M_1 + M_2)g$$

$$\implies \underline{N_{2T} = (M_1 + M_2)g}$$

$$F_x = T - F_{12} - F_{2T} = \underline{T - \mu_s M_1 g - \mu_k (M_1 + M_2) g = M_2 a}$$

$$F_{2T} = N_{2T}\mu_k = \mu_k (M_1 + M_2) g$$

3)



$$M_3 a = M_3 g - T$$

$$1) + 2) + 3) \implies (M_1 + M_2 + M_3)a = M_3 g - \mu_K (M_1 + M_2) g$$

$$1) \implies a = \mu_s g$$

$$(M_1 + M_2 + M_3)\mu_s g = M_3 g - \mu_K (M_1 + M_2) g$$

gs cancel,

$$M_3 (\mu_s - 1) = -(\mu_K + \mu_s)(M_1 + M_2)$$

$$\boxed{M_3 = \frac{\mu_K + \mu_s}{1 - \mu_s}(M_1 + M_2)}$$

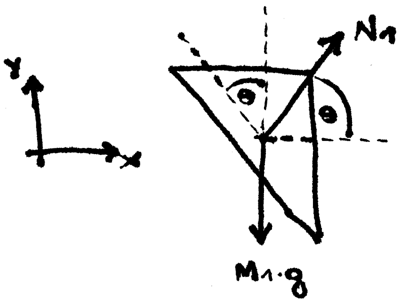
$$M_3 = \frac{0.3 + 0.5}{1 - 0.5}(1 + 2)\ \text{kg}$$

$$= \frac{0.8}{0.5} \times 3\ \text{kg} = 4.8\ \text{kg}$$

Problem 4

Part a)

FBD



$$\Sigma F_x : \quad N_1 \cdot \cos(\theta) = M_1 \cdot \frac{v^2}{r} \quad (1)$$

$$\Sigma F_y : \quad N_1 \cdot \sin(\theta) - M_1 \cdot g = M_1 \cdot \cancel{a_y}^{\,0} \quad (2)$$

$$a_y = 0 \Rightarrow \boxed{N_1 = \frac{M_1 \cdot g}{\sin(\theta)}} \quad (3)$$

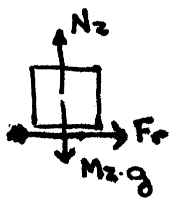
(3) in (1):

$$\frac{\cancel{M_1} \cdot g \cdot \cos(\theta)}{\sin(\theta)} = \cancel{M_1} \cdot \frac{v^2}{r} \longrightarrow \boxed{r = \frac{v^2 \cdot \tan(\theta)}{g}}$$

(Note that r does not depend on the mass)

Part b)

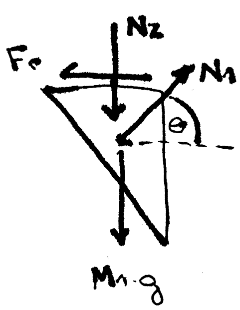
FBD



$$\Sigma F_x : \quad F_f = M_2 \cdot \frac{v^2}{r} \quad (4)$$

(it is not moving relative to M_1 → same velocity)

$$\Sigma F_y : \quad N_2 - M_2 \cdot g = M_2 \cdot \cancel{a_y}^{\,0} \longrightarrow N_2 = M_2 \cdot g \quad (5)$$



$$\Sigma F_x : \quad N_1 \cdot \cos(\theta) - F_f = M_1 \cdot \frac{v^2}{r} \quad (5)$$

$$\Sigma F_y : \quad N_1 \cdot \sin(\theta) - N_2 - M_1 \cdot g = M_2 \cdot \cancel{a_y}^{\,0} \longrightarrow N_1 = g \frac{(M_1 + M_2)}{\sin(\theta)} \quad (6)$$

(4) and (6) into (5) $\longrightarrow$

$$g \cdot \frac{(M_1 + M_2)}{\sin(\theta)} \cdot \cos(\theta) - M_2 \cdot \frac{v^2}{r} = M_1 \cdot \frac{v^2}{r}$$

$$\longrightarrow g \frac{\cancel{(M_1 + M_2)}}{\tan(\theta)} = \cancel{(M_1 + M_2)} \cdot \frac{v^2}{r} \Rightarrow \boxed{\frac{v^2 \cdot \tan(\theta)}{g} = r}$$

(Same r)

From part a) you can see that r doesn't depend on mass, therefore adding M_2 that doesn't move relative to M_1 will give you the same radius.

Part c)

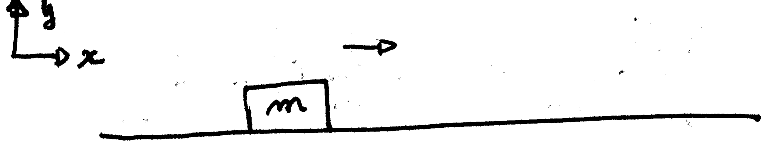
Equation 4 $\Rightarrow$ $F_r = M_2 \cdot \frac{v^2}{r}$ but $F_r = \mu_s \cdot N_2 = \mu_s \cdot M_2 \cdot g$

$$\rightarrow \mu_s \cdot \cancel{M_2} \cdot g = \cancel{M_2} \cdot \frac{v^2}{r} \Rightarrow \mu_s = \frac{v^2}{r \cdot g}$$

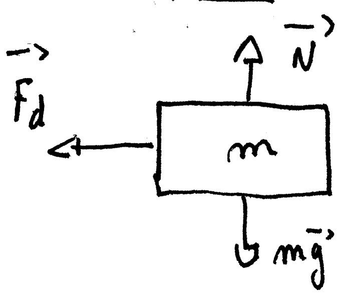
But $r = \frac{V^2 \cdot \tan(\theta)}{g}$ $\Rightarrow$ $\mu_s = \frac{V^2}{\frac{V^2 \cdot \tan(\theta)}{g} \cdot g} = \frac{1}{\tan(\theta)}$

$$\Rightarrow \boxed{\mu_s = \frac{1}{\tan(\theta)}}$$

5]



forces diagram



$$\sum F_x : F_d = m a_x$$

$$\left.\begin{array}{l}\sum F_y : N - mg = m a_y \\ a_y = 0\end{array}\right\} N = mg$$

a] $F_d = m a_x \iff -bv = m \dfrac{dv}{dt}$

$$\iff -\frac{b}{m} dt = m \frac{dv}{v}$$

$$\iff -\frac{b}{m}\int_0^t dt = m \int_0^v \frac{dv}{v}$$

$$\iff -\frac{b}{m} t = \ln\left(\frac{v}{v_0}\right)$$

$$\iff \boxed{v = v_0 \exp\left(-\frac{b}{m} t\right)}$$

b] $v = \dfrac{dx}{dt} \iff dx = v\,dt$

$$\iff \int_0^x dx = \int_0^t v_0 \exp\left(-\frac{b}{m} t\right) dt$$

$$\iff \boxed{x(t) = -\frac{m v_0}{b}\left[\exp\left(-\frac{b}{m} t\right) - 1\right]}$$

because $x = 0$ at $t = 0$

c] $\lim_{t\to\infty} x(t) = \lim_{t\to\infty} -\frac{m v_0}{b}\left[\exp\left(-\frac{b}{m}t\right)-1\right]$

$= \frac{m v_0}{b}$ because $\lim_{t\to\infty} \exp\left(-\frac{b}{m}t\right)=0$

$\Rightarrow \boxed{\text{maximum distance} = \frac{m v_0}{b}}$