Q1

2011Sp PHYS 7A
MT2

momentum conservation :

$$\begin{cases} \hookrightarrow m v_A = m u_A + m u_B \\ P = \dfrac{E_f - E_i}{E_i} = \dfrac{\frac{1}{2} m u_A^2 + \frac{1}{2} m u_B^2 - \frac{1}{2} m v_A^2}{\frac{1}{2} m v_A^2} \end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{u_A}{v_A} + \dfrac{u_B}{v_A} = 1 \\ \left(\dfrac{u_A}{v_A}\right)^2 + \left(\dfrac{u_B}{v_A}\right)^2 - 1 = P \end{cases}$$

$$\Rightarrow \left(\frac{u_A}{v_A}\right)^2 + \left(1 - \frac{u_A}{v_A}\right)^2 - 1 = P$$

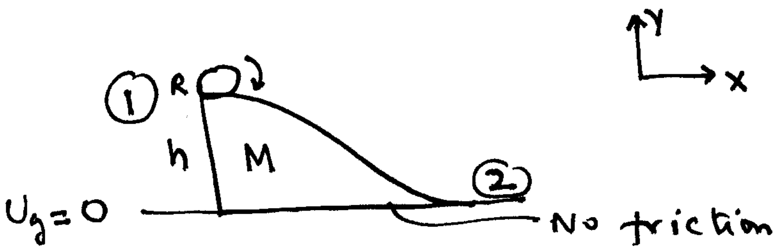
$$2\left(\frac{u_A}{v_A}\right)^2 - 2\left(\frac{u_A}{v_A}\right) - P = 0$$

take − because $u_A < u_B$

$$\frac{u_A}{v_A} = \frac{2 \pm \sqrt{4 + 8P}}{4} = \frac{1 - \sqrt{1 + 2P}}{2}$$

$$\frac{u_B}{v_A} = \frac{1 + \sqrt{1 + 2P}}{2} \quad //$$

#2



$\boxed{\text{ENERGY CONSERVED}}$ :: frictionless table and rolling w/o slipping

$\boxed{E_1 = E_2}$ $\quad M_s g(h+R) = M_s g R + \frac{1}{2} M_s V_s^2 + \frac{1}{2} I\omega^2 + \frac{1}{2} M V_M^2$

$$\Rightarrow \boxed{M_s g h = \frac{1}{2} M_s V_s^2 + \frac{1}{2} I\omega^2 + \frac{1}{2} M V_M^2} \qquad (1)$$

$\boxed{\text{ROLLIN' W/O SLIPPING}}$

$$V_P = V_s - \omega R = 0 \Rightarrow \boxed{\omega = V_s/R} \qquad (2)$$

$\Sigma F_x = 0 \Rightarrow \boxed{P_x \text{ CONSERVED}} \Rightarrow -MV_M + M_s V_s = 0 \quad (P_1 = 0)$

$\boxed{\vec{V}_M \equiv -V_M \hat{x}}$ $\boxed{\vec{V}_s \equiv V_s \hat{x}}$ $\Rightarrow \boxed{V_M = \frac{M_s}{M} \cdot V_s} \qquad (3)$

Plug 2, 3 into 1

$$M_s g h = \frac{1}{2} M_s V_s^2 + \frac{1}{2} I \cdot \frac{V_s^2}{R^2} + \frac{1}{2} M \frac{M_s^2}{M^2} V_s^2 \Rightarrow M_s g h = \frac{1}{2} M_s V_s^2 \left(1 + \frac{I}{M_s R^2} + \frac{M_s}{M}\right)$$

$$\Rightarrow \boxed{V_s = \frac{2gh}{1 + \frac{I}{M_s R^2} + \frac{M_s}{M}}} \qquad \boxed{I_{Ball} = \frac{2}{5} M_s R^2} \qquad \boxed{V_s = \sqrt{\frac{2gh}{\frac{7}{5} + \frac{M_s}{M}}}\ \hat{x}} \qquad (4)$$

$\boxed{\text{Check: if } I = 0,\ M \to \infty \quad V_s = \sqrt{2gh}\ ✓}$

Plug into 4

$$V_M = \frac{M_s}{M} \cdot \frac{2gh}{\frac{7}{5} + \frac{M_s}{M}} \qquad \boxed{\vec{V}_M = -\frac{M_s}{M} \sqrt{\frac{2gh}{\frac{7}{5} + \frac{M_s}{M}}}\ \hat{x}}$$

※3.

a) $dU = (dm)\, g y = \lambda\, dy\, g y$

infinitesimal potential energy of the cord btwn y and $y+dy$, and $\lambda = M/L$

then

$$U(Y) = \int dU = M/_L\, g \int_0^{L-Y} y\, dy$$

$$= Mg \cdot \frac{1}{L} \cdot \frac{(L-Y)^2}{2} = \frac{Mg}{2L}(L-Y)^2$$

or using center of mass for $(L-Y)$ cord.

b) We use energy conservation,

$$E_i = U(Y=0) + \frac{1}{2} M [V(Y=0)] = U(Y=0)$$

as $V(Y=0) = 0$.

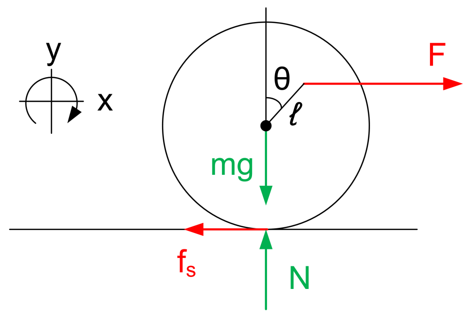
$$E_i = E(Y) = U(Y) + \frac{1}{2} M V(Y)^2$$

then $V(Y) = \sqrt{\dfrac{2\,(U(Y=0) - U(Y))}{M}}$

$$= \sqrt{2/_M \left(\frac{Mg}{2L} L^2 - \frac{Mg}{2L}(L-Y)^2 \right)} = \sqrt{g/_L \left(L^2 - (L-Y)^2 \right)}.$$

Midterm 2 Solution: Problem 4

Physics 7A, UC Berkeley, Spring 2011, Prof. A. Speliotopoulos
Grader: Aaron Alpert



Linear Dynamics. We draw the free body diagram as shown, making careful note of the fact that we have defined *clockwise* as positive rotation. We write down the linear dynamics equation,

$$\sum F_x = F - f_s = ma. \tag{1}$$

Rotational Dynamics. The magnitude of the torque about the CM due to F is $rF\sin\phi$, as usual, but we note that in this definition, ϕ is the angle between vectors $\boldsymbol{\ell}$ and $\mathbf{F}$. As shown in the diagram, θ is the angle between $\boldsymbol{\ell}$ and the vertical. Thus, we must use $\ell\cos\theta$, which is the perpendicular length between the disk's center and $\mathbf{F}$. For the frictional force, the corresponding torque's magnitude is $f_s R$. Both forces tend to cause clockwise rotation, so,

$$\sum \tau = F\ell\cos\theta + f_s R = I\alpha. \tag{2}$$

[If you defined counter-clockwise as positive, the left hand side would have a negative.] For a disk, $I = \frac{1}{2}MR^2$, so

$$\sum \tau = F\ell\cos\theta + f_s R = \frac{1}{2}MR^2\alpha. \tag{3}$$

Kinematic Relations. The disk rolls without slipping. We note that if counter-clockwise were defined as positive, the following equation would have a negative sign on one side.

$$a = R\alpha \tag{4}$$

Solution. We solve equations 1, 3, and 4 for the unknowns f_s, α, and a. Begin by using 4 to eliminate a from 1.

$$F - f_s = ma = m(R\alpha) \quad \rightarrow \quad \alpha = \frac{F - f_s}{mR} \tag{5}$$

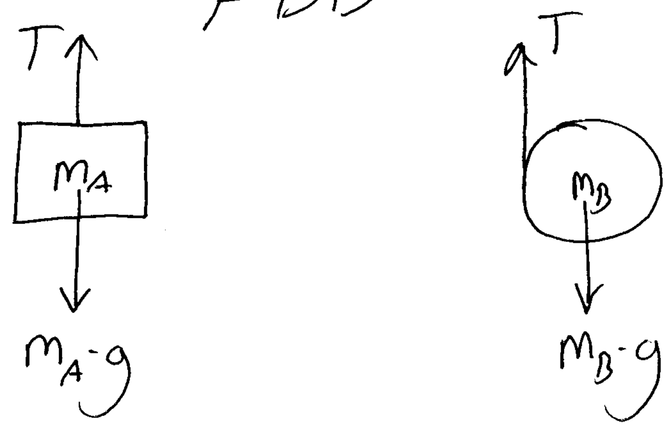
Substitute the expression for α in 5 into 3.

$$F\ell\cos\theta + f_s R = \frac{1}{2}mR^2\left(\frac{F - f_s}{mR}\right) = \frac{1}{2}R(F - f_s) \tag{6}$$

$$\frac{3}{2}f_s R = F\left(\frac{1}{2}R - \ell\cos\theta\right) \quad \rightarrow \quad f_s = \boxed{\frac{2}{3}F\left(\frac{1}{2} - \frac{\ell}{R}\cos\theta\right)} \tag{7}$$

51 Solution

FBD



Net force & net torque equations

m_A: 1) $T - m_A g = m_A a_A$

m_B 2) $T - m_B g = m_B a_B$

3) $-T \cdot R = \left(\frac{1}{2} m_B R^2\right) \cdot \alpha_B$ net torque clockwise is negative

constraint for rolling without slipping

4) $a_B - \alpha_B \cdot R = -a_A$ absolute acceleration of point on disk in contact with rope is accelerating at $-a_A$

4 unknowns T, α_B, a_A, a_B

4 equations! solve for a_B and a_A.

$1)+2)\quad 2T-(m_A+m_B)g = m_A a_A + m_B a_B$

$1)-2)\quad (m_B-m_A)\,g = m_A a_A - m_B a_B$

$4)\rightarrow 3)\quad -T\cdot R = \left(\frac{1}{2}m_B R^2\right)\frac{(a_B+a_A)}{R}$

$5)\quad -T = \frac{1}{2}m_B(a_B+a_A)$

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$-2\cdot 5)\rightarrow \boxed{1)+2)}$

$$\ast\quad -(m_A+m_B)g = (m_A+m_B)a_A + 2m_B a_B$$

$1)-2)$
$$\ast\quad (m_B-m_A)g = m_A a_A - m_B a_B$$

2 unknowns, a_A & a_B, 2 equations!

$$a_A = \frac{m_B-3m_A}{3m_A+m_B}\cdot g$$

$$a_B = \frac{-(m_A+m_B)}{3m_A+m_B}\cdot g$$