

**ME 106.2**  
**FLUID MECHANICS**  
**EXAM 1 – open book**

**1.(25%)** Consider the steady two-dimensional flow field with Cartesian velocity components

$$(u, v) = (2ax, -ay)$$

where  $a$  is a positive constant.

- (a) [10] Determine and sketch the streamlines of the flow.
- (b) [5] Calculate the divergence of the field. What can you say about the compressibility of the flow field?
- (c) [10] Assuming that the density field is uniform (no spatial dependence), determine the density as a function of time using the differential continuity equation.

**2.(25%)** A closed rigid cylindrical container of diameter  $2a$  and height  $H$  contains equal volumes of two immiscible liquids of densities  $\rho_1$  and  $\rho_2$  such that  $\rho_1 > \rho_2$ . The container and its contents are rotating at a constant angular velocity of  $\Omega$  in the presence of gravity. Let us determine the equation for shape of the interface  $h(r)$  between the two liquids. Assume that the pressure at the top center is *zero*.

- (a) [10] Determine the pressure fields in each of the liquids. Leave the interface location  $H_o$  as a free parameter.
- (b) [5] Obtain the equation for the interface shape  $h(r)$  by using the continuity of pressure at the interface.
- (c) [5] Finally, determine  $H_o$  by using the volumetric information.
- (d) [5] What would the interface shape be if the gravity were absent?

**3.(25%)** Consider the wind coming off shore and blowing over Mount Temelpais which is 700m high where pressure is 0.92 atmosphere. The atmospheric conditions at sea level are  $1atm$ ,  $288K$  and the wind speed is 20 m/s. Let us assume that the flow is steady and there are no losses. Do not ignore the elevation change.

- (a) [10] Assuming the flow to be incompressible, determine the wind speed at the mountain peak.
- (b) [10] Allowing for the compressibility of the flow, determine the wind speed at the top.
- (c) [5] Compare the two results and comment on their difference.

**4.(25%)** Consider a *lawn sprinkler* with unequal arms  $L_1$  and  $L_2$  as shown in the figure. The water jets have identical mass flow rates  $\dot{M}$  and velocities  $U$  and discharge azimuthally. Determine the relation between the torque  $T$  on the sprinkler and its angular velocity  $\Omega$ .