Second Midterm Examination
Tuesday November 1 2005
Closed Books and Closed Notes

Question 1
A Single Particle
25 Points

$r = re_r$
$v = \dot{r}e_r + r\dot{\theta}e_\theta$
$G = mv$
$H_0 =$

As shown in Figure 1, a particle of mass m is attached to a fixed point O by a linear spring of stiffness K and unstretched length L. A gravitational force $-mg\mathbf{E}_z$ also acts on the particle. The particle is free to move on the surface of a disk which is spinning about $\mathbf{E}_z$ with a speed $\Omega = \Omega(t)$.

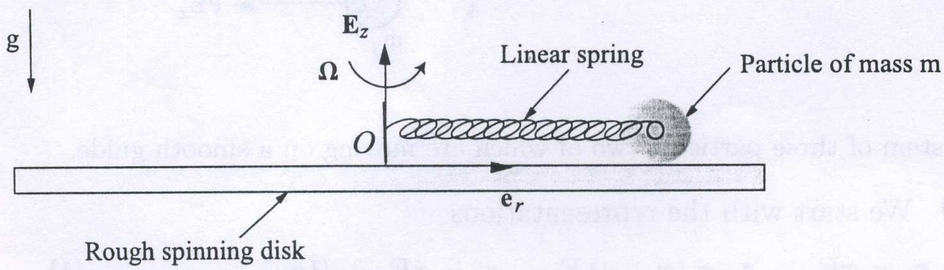

Figure 1: A particle moving on the rough surface of a spinning disk.

(a) *(5 Points)* Starting from the representation $\mathbf{r} = r\mathbf{e}_r$, establish expressions for $\mathbf{G}$ and $\mathbf{H}_O$ of the particle.

(b) *(5 Points)* For the case where the particle is moving relative to the disk, draw a free–body diagram of the particle. Accompanying your free–body diagram, give clear expressions for the spring force, friction force, and $\mathbf{v}_{rel}$.

(c) *(5 Points)* Starting from the angular momentum theorem for a single particle, prove that the angular momentum of the particle relative to O in the $\mathbf{E}_z$ direction is not conserved.

$H_0 = \sum r_i \times F_i$

(d) *(5 Points)* Starting from the work-energy theorem for a single particle $\dot{T} = \mathbf{F} \cdot \mathbf{v}$, prove that the total energy of the particle is not conserved.

(e) *(5 Points)* Suppose that the particle is stuck on the surface of the disk. What modifications do you need to make to **(b)**, and why are $\mathbf{H}_O \cdot \mathbf{E}_z$ and the total energy E still not constant?

Question 2

A System of Three Particles

25 Points

As shown in Figure 2, two particles are connected by a linear spring of stiffness K and unstretched length L_0 to each other, and each of them by a rod of length H to a particle of mass m_3. The motion of this system of particles lies in the $\mathbf{E}_x - \mathbf{E}_y$ plane, and a constant force $P\mathbf{E}_x$ acts on m_3.

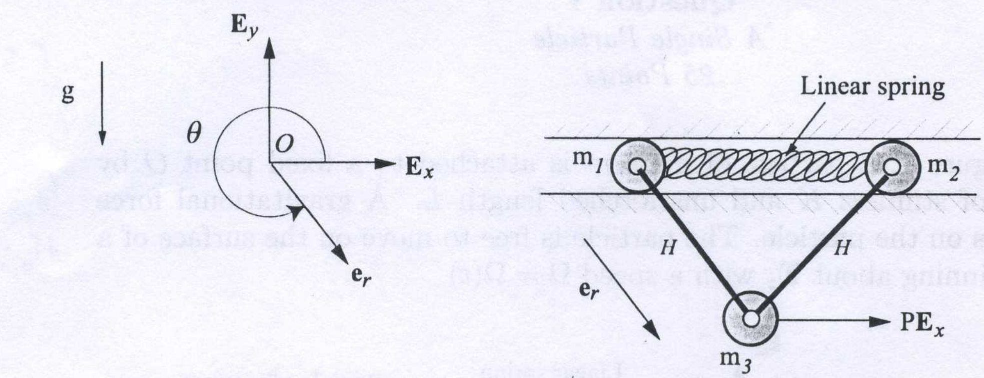

Figure 2: A system of three particles, two of which are moving on a smooth guide.

(a) *(4 Points)* We start with the representations

$$\mathbf{r}_1 = x\mathbf{E}_x, \quad \mathbf{r}_2 = (x+u)\,\mathbf{E}_x, \quad \mathbf{r}_3 = x\mathbf{E}_x + H\mathbf{e}_r, \tag{1}$$

where $u = 2H\cos(\theta)$. Using (1), show that

$$\dot{\mathbf{r}} = \dot{x}\mathbf{E}_x + \frac{1}{m}\left(m_2\dot{u}\mathbf{E}_x + m_3 H\dot{\theta}\mathbf{e}_\theta\right), \tag{2}$$

where $m = m_1 + m_2 + m_3$ and $\mathbf{r}$ is the position vector of the center of mass.

(b) *(6 Points)* Show that the kinetic energy T and angular momentum $\mathbf{H}_O$ of the system are

$$\begin{aligned} T &= \frac{1}{2}m\dot{x}^2 + \frac{m_2}{2}\left(\dot{u}^2 + 2\dot{u}\dot{x}\right) + \frac{m_3}{2}\left(H^2\dot{\theta}^2 - 2\dot{x}H\dot{\theta}\sin(\theta)\right), \\ \mathbf{H}_O &= m_3\left(H^2\dot{\theta} + Hx\dot{\theta}\cos(\theta) - H\dot{x}\sin(\theta)\right)\mathbf{E}_z. \end{aligned} \tag{3}$$

(c) *(5 Points)* Draw free–body diagrams for each of the particles. Give clear expressions for the spring force and the tension forces in the rods.

(d) *(7 Points)* Starting from the work-energy theorem for a system of particles, show that the total energy E of the system is constant. Clearly indicate any intermediate results that you use.

$E = \frac{1}{2}ke^2$
$+T + m_3 g \mathbf{E}_y \cdot \mathbf{r}_3$
$- P\mathbf{E}_x \cdot \mathbf{r}_3$

(e) *(3 Points)* The system is at rest at time $t = 0$ and the spring is unstretched. Then the force $P\mathbf{E}_x$ is applied. How would you determine $\dot{\theta}$ at a later instant $t = t_1$ when x and u are known?