M. Rieffel   Math 140   Second Midterm Exam
April 1, 2011

SHOW YOUR WORK COMPLETELY AND NEATLY. DON'T
WRITE MORE THAN YOU NEED TO.

Total points = 40.

1. Let S be a regular C^2 surface.

a) (4 points) Define precisely what is meant by the first funda-mental form of S (without mentioning parametrizations).

b) (6 points) Given a regular parametrization of a piece of S, explain what is meant by the corresponding matrix associated to its first fundamental form.

c) (5 points) For the regular parametrized surface defined by $\phi(u,v) = (u^2 v,\ u+v,\ uv^2)$ for $1 < u, v < 3$ compute the matrix corresponding to its first fundamental form at the point $\phi(2,2)$ of S.

2. Let S be a regular C^2 oriented surface with orientation n.

a) (4 points) Define what is meant by saying that n is an orien-tation for S (without mentioning parametrizations).

b) (3 points) Define what is meant by the Gauss map for (S, n).

c) (4 points) Define what it means for a parametrization of a part of S to be positively oriented with respect to n.

d) (8 points) Define what is meant by the differential Dn (de-noted by dn in our textbook) of the Gauss map.

e) (5 points) Given a positively oriented parametrization ϕ of a part of S, with its corresponding canonical basis tangent vectors ϕ_u and ϕ_v, state how to compute $Dn(\phi_u)$ and $Dn(\phi_v)$ in terms of ϕ. (No proof needed.)