Physics 7C Summer 2010

Midterm 1

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① We want our lens to put an image at the near point; otherwise the eye will not be able to focus it. Thus, it will be <u>virtual</u>.

The image distance is measured from the lens but the near point is measured from the eye; so we must subtract off the lens-eye distance.

The object is at a distance L <u>from the eye</u> so we must subtract off the lenseye distance.

all in all: $d_i = -(N-S)$, $d_o = L-S$, so

$$\frac{1}{f} = \frac{-1}{N-S} + \frac{1}{L-S} = \frac{N-S-(L-S)}{(N-S)(L-S)} = \frac{N-L}{(N-S)(L-S)}$$

where d_i is negative because it's a virtual image.

So $\boxed{f = \frac{(N-S)(L-S)}{N-L}}$

Note: $f > 0$ so converging lens

$m = \frac{N-S}{L-S} > 0$ so image is magnified.

② $\bar{E} = E_0 \cos(-Kx + \omega t + \phi)\hat{y}$.

ⓐ We can write this function as:

$\bar{E} = E_0 \cos(-[Kx - \omega t - \phi])\hat{y}$ and since $\cos(-\theta) = \cos\theta$,

$\bar{E} = E_0 \cos(Kx - \omega t - \phi)\hat{y}$. We can also factor this to get

$\bar{E} = E_0 \cos(K(x - \frac{\omega}{K}t) - \phi)\hat{y}$. Now it's looking like our

general solution to the wave equation, $\bar{E} = f(x - vt)\hat{y}$

So this wave is moving to the positive x-direction as time goes on.

ⓑ From the argument above, it is clear that

$v = \frac{\omega}{K}$. Alternatively, we could plug this solution into the wave equation and solve for V:

$$\frac{d^2\bar{E}}{dt^2} = v^2 \frac{d^2\bar{E}}{dx^2}.$$

Left hand side: $\frac{d\bar{E}}{dt} = -E_0\omega \sin(-Kx + \omega t + \phi)\hat{y}$

$$\frac{d^2\bar{E}}{dt^2} = -E_0\omega^2 \cos(-Kx + \omega t + \phi)\hat{y}.$$

Right hand Side: $$\frac{d\vec{E}}{dx} = -E_0(-k)\sin(-kx+\omega t+\phi)\,\hat{y}$$

$$\frac{d^2\vec{E}}{dx^2} = -E_0(-k)^2\cos(-kx+\omega t+\phi)\,\hat{y}$$

So using these in the wave equation, we have

$$\cancel{-E_0}\,\omega^2\cancel{\cos(-kx+\omega t+\phi)\hat{y}} = \cancel{-E_0}\,k^2\cancel{\cos(-kx+\omega t+\phi)}\,v^2\,\hat{y}$$

So $$\omega^2 = k^2 v^2$$

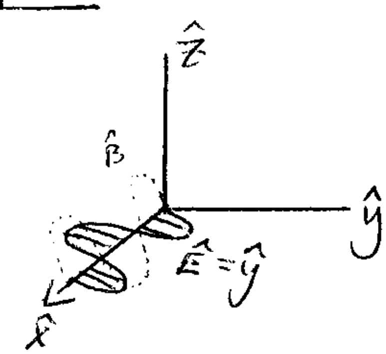
or $$v^2 = \frac{\omega^2}{k^2} \Rightarrow \boxed{v = \frac{\omega}{k}}$$ where we have

chosen $v > 0$ for a wave propagating in the positive x-direction.

(c) We know that (a) $E_0 = B_0 c$

(b) $\hat{E}\times\hat{B} = \hat{x}$ is the direction of propagation

(c) $\vec{E}$ and $\vec{B}$ are in <u>phase</u>



So $\vec{B} = B_0\cos(-kx+\omega t+\phi)\,\hat{B}$

From the right-hand rule, we see that

$\hat{B} = +\hat{z}$

So $$\boxed{\vec{B} = \frac{E_0}{c}\cos(-kx+\omega t+\phi)\,\hat{z}}$$

(d) The poynting vector is defined as $\bar{S} = \frac{1}{\mu_0}(\bar{E} \times \bar{B})$

so $\vec{S} = \frac{1}{\mu_0}\left(E_0 \cos(-kx+\omega t+\phi) \cdot \frac{E_0}{c} \cos(-kx+\omega t+\phi)\right) \hat{y} \times \hat{z}$

$$\boxed{\vec{S} = \frac{E_0^2}{\mu_0 c} \cos^2(-kx+\omega t+\phi)\, \hat{x}}$$

Lemma: The time average of $\cos^2\theta$ is $\frac{1}{2}$:

Proof $\sin^2\theta + \cos^2\theta = 1$

and $\langle \sin^2\theta \rangle + \langle \cos^2\theta \rangle = 1$, where $\langle g \rangle = \frac{1}{T}\int_0^T g \, dt$

$T =$ Period

so since $\sin^2\theta = \cos^2(\theta + \frac{\pi}{2})$

$\langle \sin^2\theta \rangle = \langle \cos^2(\theta + \frac{\pi}{2}) \rangle$ which doesn't

affect the average over a period, so

$\langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle$

so $\langle \sin^2\theta \rangle + \langle \cos^2\theta \rangle = 1$ which we write as

either $2\langle \sin^2\theta \rangle = 1$ or $2\langle \cos^2\theta \rangle = 1$

so $\langle \cos^2\theta \rangle = \frac{1}{2}$ ∎

Thus $\langle S \rangle = \frac{E_0^2}{\mu_0 c} \langle \cos^2(-kx + \omega t + \phi) \rangle = \boxed{\frac{1}{2} \frac{E_0^2}{\mu_0 c}}$

(e) For a fully-reflected wave, $P = \frac{2 \langle S \rangle}{c}$

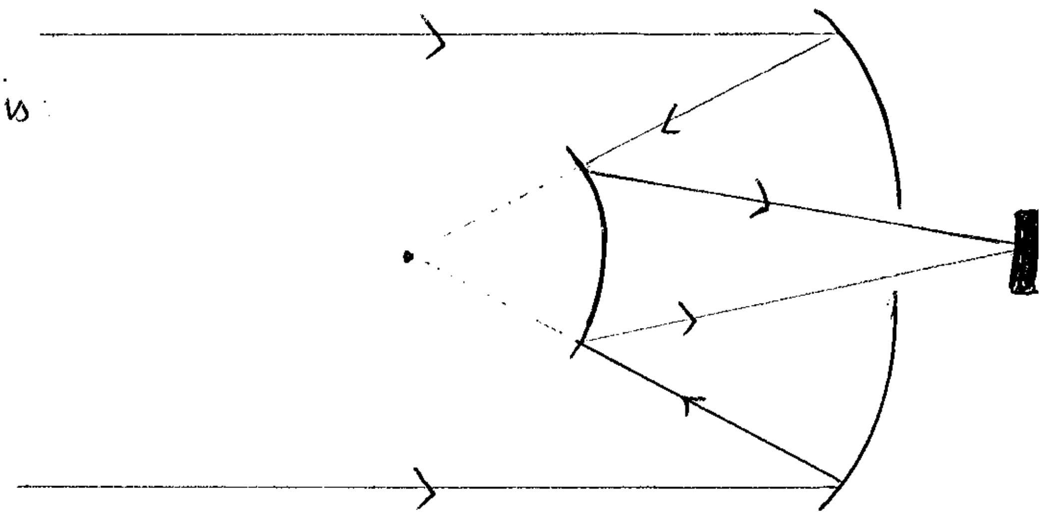
so $P = \frac{E_0^2}{\mu_0 c^2}$ and we know $c^2 = \frac{1}{\mu_0 \epsilon_0}$ so

$\boxed{P = \epsilon_0 E_0^2}$

③ Here we have a reflecting telescope, whose primary mirror has a radius of curvature R_1 and secondary mirror has a radius of curvature $-\frac{R_1}{2}$.

ⓐ The ray diagram is:

assuming $\underline{D < \frac{R_1}{2}}$.



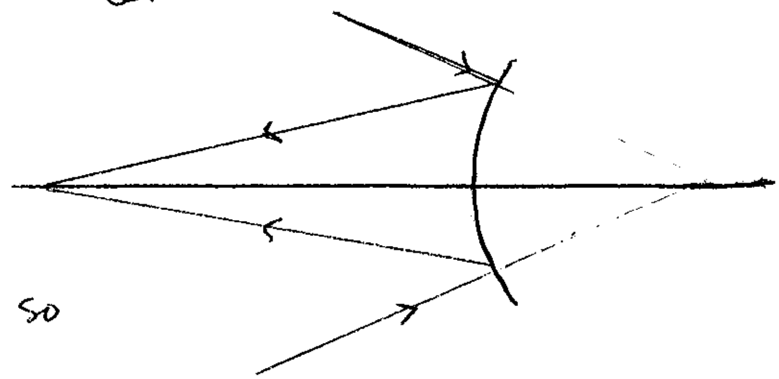
ⓑ We are asked to calculate $d_{i,s}$, the image distance of the secondary mirror, since that is where a star will be in focus. Let's split this task into two parts; first we calculate $d_{i,p}$, the image distance of the primary mirror.

* For distant objects, $d_o \to \infty$; so $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \approx \frac{1}{d_i}$

Thus, $d_i = f$, or $d_{i,p} = F_1$. For mirrors, $F = \frac{r}{2}$, so

$$d_{i,p} = \frac{R_1}{2}$$

Now let's flip this problem so that we are looking at the second mirror from the left:



The object is behind the mirror, so we write $d_o = -(d_{i,P} - D) = -\left(\frac{R_1}{2} - D\right) = D - \frac{R_1}{2}$

We are told that $R_2 = -\frac{R_1}{2}$ so $F_2 = \frac{-R_1}{4}$

Then $\frac{1}{F_2} = \frac{1}{d_{o,s}} + \frac{1}{d_{i,s}} \Rightarrow -\frac{4}{R_1} + \frac{1}{\frac{R_1}{2} - D} = \frac{1}{d_{i,s}}$

which we write $\frac{-2R_1 + 4D + R_1}{R_1(R_1/2 - D)} = \frac{1}{d_{i,s}}$

so $$\boxed{d_{i,s} = \frac{R_1\left(\frac{R_1}{2} - D\right)}{4D - R_1}}$$

Note: we must have $d_{i,s} > 0$ so we must have $D > \frac{R_1}{4}$

In other words, $\frac{R_1}{4} < D < \frac{R_1}{2}$.

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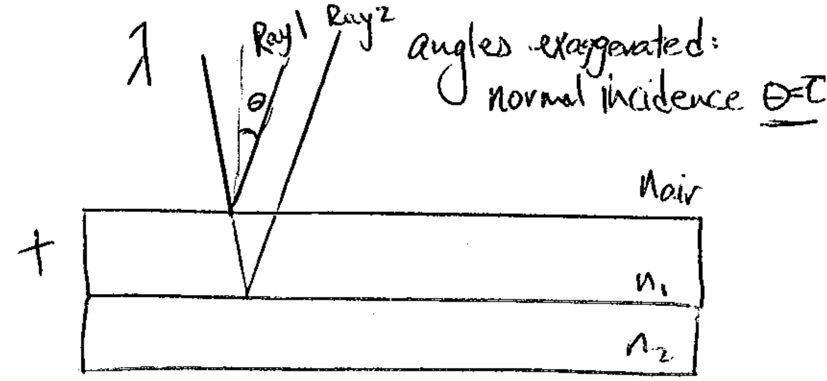

Suppose $n_2 > n_1 > n_{air}$. Then we see an interference maximum for $\lambda_1 = \lambda$, which means that compared to Ray 1, Ray 2 travels a longer path by a distance $PD = m\lambda$ for $m = 1, 2, \ldots$

* Since $n_1 > n_{air}$ and $n_2 > n_1$, each ray experiences a phase shift when reflecting, corresponding to an extra path of $\frac{\lambda}{2}$. So, This effect cancels out when we compare the two rays.

So remembering that $\theta = 0$ for normal incidence, we can write $m = \frac{2t}{\lambda_{n_1}} = \frac{2tn_1}{\lambda}$ where λ_{n_1} is the wavelength in the film.

Or $\underline{t = \frac{m\lambda}{2n_1}}$ for $m = 1, 2, \ldots$

We also see an interference minimum for $\lambda_2 = 2\lambda$. In this case, the total path difference is a half-integer multiple of λ.

So $m' + \frac{1}{2} = \frac{2t}{(2\lambda n_1)} = \frac{2tn_1}{2\lambda} = \underline{\frac{tn_1}{\lambda}}$ for $m' = 0, 1, \ldots$

Or in other words, $\underline{t = \left(m' + \frac{1}{2}\right)\frac{\lambda}{n_1}}$. Note that m and m' are not necessarily the same integer.

• To find the minimum allowed thickness based on our experimental observation, let's write out the possible values of t given by each λ:

$\underline{\lambda_1}$

$$t = \frac{m\lambda}{2n_1} = \left\{\frac{\lambda}{2n_1}, \frac{\lambda}{n_1}, \frac{3\lambda}{2n_1}, \frac{2\lambda}{n_1}, \ldots\right\}$$

$\underline{\lambda_2}$

$$t = \left(m' + \frac{1}{2}\right)\frac{\lambda}{n_1} = \left\{\frac{\lambda}{2n_1}, \frac{3\lambda}{2n_1}, \frac{5\lambda}{2n_1}, \ldots\right\}$$

So taking the smallest <u>common</u> value, we see that

$$\boxed{t_{min} = \frac{\lambda}{2n_1}}$$