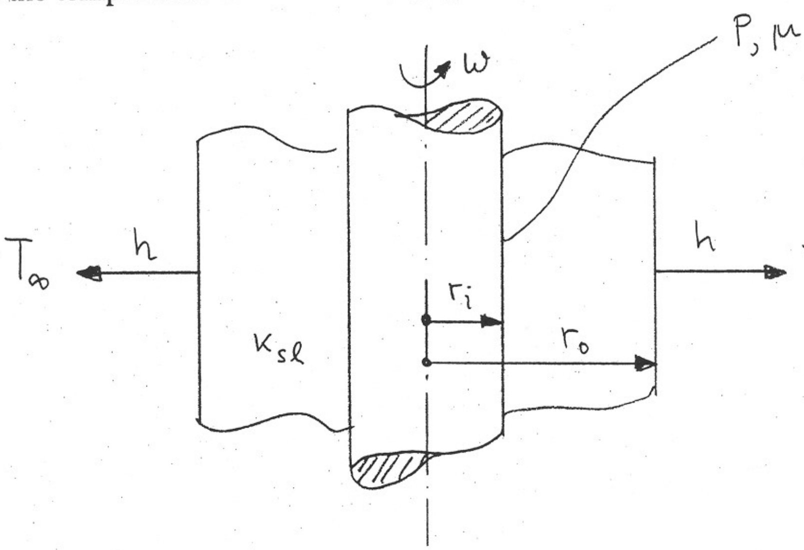
ME 109 **Midterm #1** 2/26/04

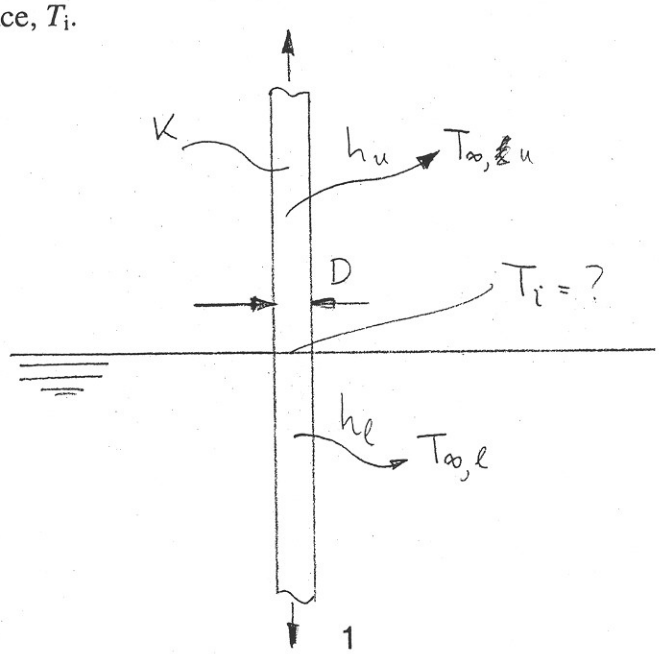
Problem 1

A long, solid shaft rotates steadily with angular velocity ω in a sleeve as shown in the Figure below. The outer sleeve surface at $r = r_o$ is exposed to a fluid of temperature T_∞ through a convective heat transfer coefficient, h. For the pressure and the coefficient of dry friction between the shaft and the sleeve P and μ respectively, the power generated at the interface $r = r_i$ per unit area is $Q'' = \mu P \omega r_i$. The thermal conductivity of the sleeve is k_{sl}. Derive an expression for the temperature of the interface, T_i.



Problem 2

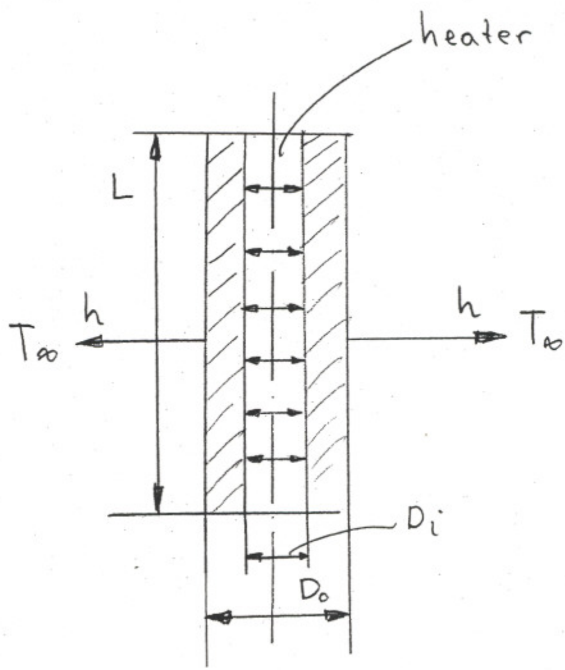
Consider a long vertical rod of thermal conductivity k = 60 W/(mK), circular cross section and diameter D=1cm, partially immersed in a liquid. The part exposed to air of temperature $T_{\infty;u}$ = 20°C is subjected to a heat transfer coefficient h_u = 10 W/(m^{2}K). The part immersed into the liquid transfers heat to the liquid of temperature $T_{\infty;l}$ = 50°C through a convective heat transfer coefficient h_l = 100 W/(m^{2}K). Estimate the steady rod temperature right across the liquid/air interface, T_i.



Problem 3

An electric heater is fitted snugly into a hollow aluminum cylinder, of $D_i = 0.5$ cm inner diameter, $D_o = 1$ cm outer diameter and length $L = 30$ cm as shown in the figure below. Air of temperature, $T_\infty = 20°C$ flows over the outer cylindrical surface, giving rise to a heat transfer coefficient $h = 20$ W/(m²K). The heat transfer from the cylinder ends may be neglected. The power supply to the heater is 50 W. Initially the cylinder is at $T_i = T_\infty = 20°C$. At time $t = 0$, the heater power is turned on. Find the temperature of the cylinder at $t = 3$ min.

The aluminum density, $\rho = 2700$ kg/m³, specific heat, $C = 900$ J/(kgK), and thermal conductivity, $k = 200$ W/(mK).



ME 109 Midterm #1 Solutions

Problem 1

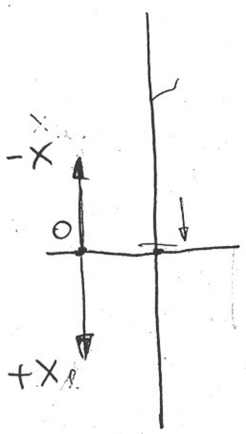
$$Q'' \cdot A_i = q''_{cond} \cdot A_i$$

$$2\pi \mu P \omega_i r_i^2 L = \frac{T_i - T_\infty}{\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k_{si} L} + \frac{1}{2\pi h r_o}} \Longrightarrow$$

$$T_i - T_\infty = \mu P \omega r_i^2 \left[\frac{\ln\left(\frac{r_o}{r_i}\right)}{k_{si}} + \frac{1}{h r_o} \right]$$

Solid shaft temperature is at T_i

Problem 2



$$\left.\begin{aligned} T_u(x) - T_{\infty;u} &= (T_i - T_{\infty;u})\, e^{m_u x_u} \\ T_\ell(x) - T_{\infty;\ell} &= (T_i - T_{\infty;\ell})\, e^{-m_\ell x} \end{aligned}\right\}$$

$$x = 0 \Longrightarrow T_u(0) = T_\ell(0) = T_i$$

$$-\cancel{k} \left.\frac{\partial T_u}{\partial x}\right|_{x=0} = -\cancel{k} \left.\frac{\partial T_\ell}{\partial x}\right|_{x=0} \Longrightarrow$$

$$m_u \cdot (T_i - T_{\infty;u}) = -m_\ell (T_i - T_{\infty;\ell}) \Longrightarrow$$

$$\sqrt{\frac{h_u \cancel{P}}{\cancel{k} \cancel{A_c}}}\,(T_i - T_{\infty;u}) = -\sqrt{\frac{h_\ell \cancel{P}}{\cancel{k} \cancel{A_c}}}\,(T_i - T_{\infty;\ell}) \Longrightarrow$$

$$\sqrt{\frac{h_u}{h_e}}\,(T_i - T_{\infty,u}) = -T_i + T_{\infty,e} \Rightarrow$$

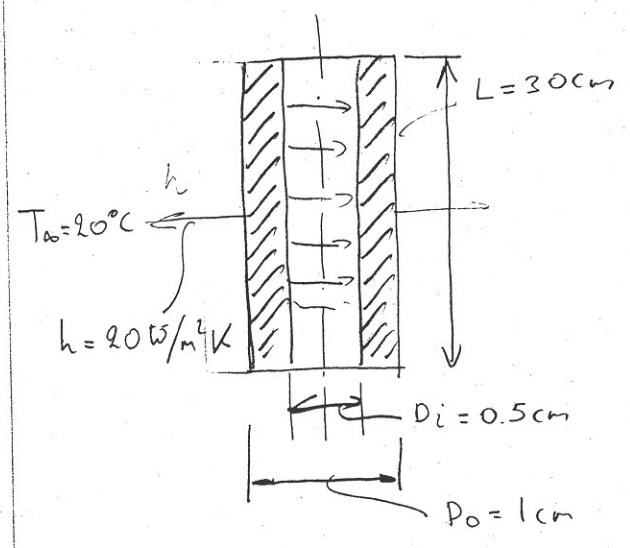
$$T_i\left(1+\sqrt{\frac{h_u}{h_e}}\right) = \sqrt{\frac{h_u}{h_e}}\,T_{\infty,u} + T_{\infty,e} \Rightarrow$$

$$T_i = \frac{T_{\infty,e} + \sqrt{\frac{h_u}{h_e}}\,T_{\infty,u}}{1+\sqrt{\frac{h_u}{h_e}}} \Rightarrow$$

$$T_i = \frac{50 + \sqrt{\frac{10}{100}}\,20}{1+\sqrt{\frac{10}{100}}} \Rightarrow$$

$$T_i = 39.5^\circ C$$

Problem 3



$q = 50W$

$A_s = \pi D_o L = \pi \cdot 0.01 \cdot 0.3 \Rightarrow$

$A_s = 9.42 \; 10^{-3} \; m^2$

$V = \frac{\pi (D_o^2 - D_i^2)}{4} L = \frac{\pi (0.01^2 - 0.005^2)}{4} \cdot 0.3$

$V = 1.767 \cdot 10^{-5} \; m^3$

$L_c = \frac{V}{A_s} = 1.876 \cdot 10^{-3} \; m$

$B_i = \frac{hL}{K} = \frac{20 \cdot 1.876 \cdot 10^{-3}}{200} = 1.8 \, 10^{-4} < 0.1$

Lumped capacitance appropriate.

$\tau_t = \frac{\rho c V}{h A_s} = \frac{2700 \cdot 900 \cdot 1.767 \cdot 10^{-5}}{20 \cdot 9.42 \cdot 10^{-3}} = 227 \; s$

$(5.24) \Rightarrow \frac{T(t) - T_\infty - \frac{q}{hA_s}}{T_i - T_\infty - \frac{q}{hA_s}} = \exp\left(-\frac{t}{\tau_t}\right) \Rightarrow$

$\frac{q}{hA_s} = \frac{50}{20 \cdot 9.42 \cdot 10^{-3}} = 265°C$

$0 < t \leq 3 min$

$T(t) = T_\infty + \frac{q}{hA_s} + \left(T_i - T_\infty - \frac{q}{hA_s}\right) \exp\left(-\frac{t}{\tau_t}\right)$

$T(180) = 20 + 265 + (20 - 20 - 265) \exp\left(-\frac{180}{227}\right)$

$T(180s) = 165 °C$

$t > 3 min \Rightarrow \frac{T(t) - T_\infty}{T(180) - T_\infty} = \exp\left(-\frac{(t - 180s)}{\tau_t}\right) \Rightarrow$

$T(6min) = 20 + (165 - 20) \exp\left(-\frac{180}{227}\right) \Rightarrow T(6min) = 85.6 °C$