Mathematics 54.1
Midterm 2, 17 March 2011
75 minutes, 75 points

NAME: _____ **ID:** _____

GSI: _____

INSTRUCTIONS:

You must justify your answers, except when told otherwise.

All the work for a question should be on the respective sheet.

This is a CLOSED BOOK examination, NO NOTES and NO CALCULATORS are allowed.

NO CELL PHONE or EARPHONE use is permitted.

Please turn in your finished examination to your GSI before leaving the room.

Question 2. (15 pts, 5+5+5)
(a) Define what is meant by an **eigenvector** and an **eigenvalue** of a matrix.
(b) Find 2×2 matrices S and D, with S invertible and D diagonal, such that $A = SDS^{-1}$, if

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}$$

(c) Find a number λ such that the sequence of matrices $\lambda^{-n} D^n$ has a non-zero limit as $n \to \infty$, and determine this limit. Do the same for the matrix A.

Question 3. (14 pts, 5+4+5)

(a) By running the Gram-Schmidt process on the vectors $[-1, 2, 2]^T$ and $[0, 1, 2]^T$, in this order, find an orthonormal basis $\mathbf{q}_1, \mathbf{q}_2$ of the plane they span in $\mathbb{R}^3$.

(b) Find a third vector $\mathbf{q}_3$ such that the matrix Q with columns $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$ is orthogonal. How many possibilities are there for $\mathbf{q}_3$?

(c) For your choice of $\mathbf{q}_3$, find all the eigenvalues of Q, and find the eigenvectors for each real eigenvalue.

Hint: If you find any square roots in (a) or (b), you made a mistake.

If you can't solve the characteristic equation right away, try some small integers. There may be square roots somewhere in (c).

Question 4. (10 pts)
Find the function of the form $y = ax^3 + bx^2$ which give the best approximation, in the sense of least squares, to the data points $(x_i, y_i) = (-1, 1), (0, 0), (1, 2), (2, 4)$.

THIS PAGE IS FOR ROUGH WORK (not graded)