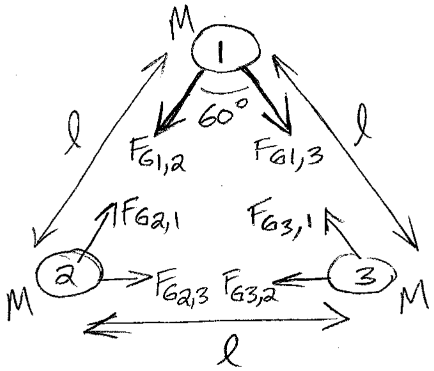
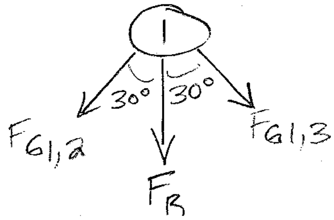
Problem 1





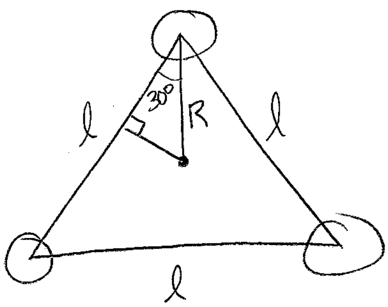
$$F_{G1,2} = G\frac{MM}{l^2} = \frac{GM^2}{l^2}$$

$$F_{G1,3} = G\frac{MM}{l^2} = \frac{GM^2}{l^2}$$

$$F_R = F_{G1,2}\cos 30° + F_{G1,3}\cos 30°$$

$$F_R = \frac{GM^2}{l^2}\left(\frac{\sqrt{3}}{2}\right) + \frac{GM^2}{2}\left(\frac{\sqrt{3}}{2}\right)$$

$$F_R = \frac{\sqrt{3}\,GM^2}{l^2}$$



$$\cos 30° = \frac{l/2}{R} \Rightarrow R = \frac{l}{2\cos 30°} = \frac{l}{\sqrt{3}}$$

$$\Sigma F_R = ma_R$$

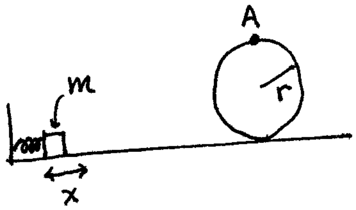
$$F_R = M\frac{v^2}{R}$$

$$\frac{\sqrt{3}\,GM^2}{l^2} = M\frac{v^2}{(l/\sqrt{3})}$$

$$v^2 = \frac{GM}{l}$$

$$\boxed{v = \sqrt{\frac{GM}{l}}}$$

2.) a.)



$$E_i = U_{spring} = \frac{1}{2}kx^2$$

$$E_A = \underbrace{\frac{1}{2}mv_A^2}_{K_A} + mg\,2r$$

Energy is conserved: $\frac{dE}{dt} = 0.$ +2

$$E_i = E_A \Rightarrow \frac{1}{2}kx^2 = K_A + mg2r$$

$$K_A = \frac{1}{2}kx^2 - mg\,2r \quad +3$$

b.) $W = \int \vec{F}\cdot d\vec{r}$ +2

In this case $\vec{F}$ is parallel to $d\vec{r}$, so the work is positive. +1

$$W = \int_0^x kx'dx' = \frac{1}{2}kx^2 \quad +2$$

c.) $a_c = \frac{v_A^2}{r}$ ↓ +1

m, ↓ mg, ↓ N +2

$$N + mg = m\frac{v_A^2}{r}$$

Given: $N = 2mg$

$$3mg = m\frac{v_A^2}{r}$$

$$v_A^2 = 3gr, \quad v_A = \sqrt{3gr} \quad +2$$

d.) $K_A = \frac{1}{2}kx^2 - mg2r = \frac{1}{2}mv_A^2$

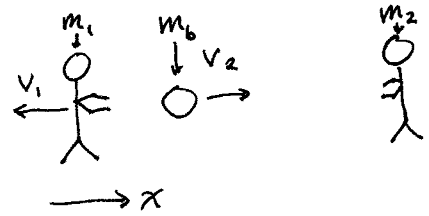
+3

$$\frac{1}{2}kx^2 = \frac{1}{2}m(3gr) + mg2r$$

$$x^2 = \frac{7mgr}{k}$$

$$x = \sqrt{\frac{7mgr}{k}} \quad +2$$

4.)



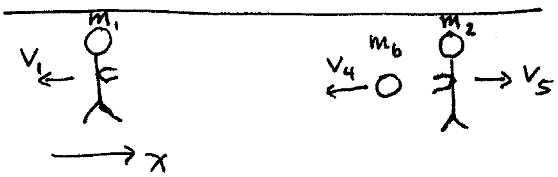
$$+5\quad\begin{cases} m_1 V_1 + m_b V_2 = 0 \\ V_2 - V_1 = V_b \end{cases}$$

$$\overset{+1}{V_1 = \frac{-m_b V_b}{m_1 + m_b}}, \quad \overset{+1}{V_2 = \frac{m_1 V_b}{m_1 + m_b}}$$

$$m_b V_2 = (m_b + m_2) V_3$$

$$V_3 = \frac{m_b}{m_b + m_2} V_2 = \frac{m_1 m_b V_b}{(m_1+m_b)(m_2+m_b)}$$

not necessary.



$$+5\quad\begin{cases} (m_b + m_2) V_3 = m_b V_4 + m_2 V_5 = m_b V_2 \\ V_5 - V_4 = V_b \end{cases}$$

$$\frac{m_1 m_b}{m_1 + m_b} V_b = m_b (V_5 - V_b) + m_2 V_5$$

$$V_5 = \frac{m_b (2m_1 + m_b)}{(m_1+m_b)(m_2+m_b)} V_b, \quad \overset{+1}{V_4 = -\frac{V_b}{m_2 + m_b}\left(m_2 - \frac{m_1 m_b}{m_1 + m_b}\right)}$$

$$+3\quad m_1 V_1 + m_b V_4 = (m_1 + m_b) V_6$$

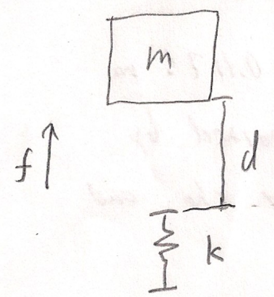
$$V_6 = \frac{m_1}{m_1 + m_b} V_1 + \frac{m_b}{m_1 + m_b} V_4$$

$$V_6 = -\frac{m_2 m_b (2m_1 + m_b)}{(m_1+m_b)^2 (m_2 + m_b)} V_b$$

Final Velocities:

Astronaut 1 : $V_6 = -\dfrac{m_2 m_b (2m_1+m_b)}{(m_1+m_b)^2(m_2+m_b)} V_b$ $+2$

Astronaut 2 : $V_5 = \dfrac{m_b (2m_1+m_b)}{(m_1+m_b)(m_2+m_b)} V_b$ $+2$



5pt a. $E_2 - E_1 = W_f$ E_2 before hitting spring, E_1 at beginning

$\left[\frac{1}{2}mv^2\right] - \left[mgd\right] = -fd$

$$v = \sqrt{\frac{2(mgd - fd)}{m}}$$

$$= \sqrt{\frac{2\left[(1800)(9.8)(3.7) - (4400)(3.7)\right]}{1800}} = \boxed{7.38\ m/s}$$

$m = 1800\ kg$

$d = 3.7\ m$

$k = 15000\ N/m$

$f = 4400 N$

6pt b. E_3 at maximum compression

$$E_3 - E_2 = W$$

$$\left[\frac{1}{2}kx^2 + mg(-x)\right] - \left[\frac{1}{2}mv^2\right] = -fx$$

$a = \frac{1}{2}k, \quad b = f - mg, \quad c = -\frac{1}{2}mv^2$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \cancel{-0.725}, \boxed{0.901\ m}$$

6pt c. E_4 after bouncing back up
use $h = 0$ at lowest position of cab

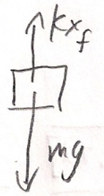
$$E_4 - E_3 = W$$

$$mgd' - \frac{1}{2}kx^2 = -fd'$$

$$d' = \left(\frac{1}{2}kx^2\right)/(mg + f) = \boxed{2.76\ m}$$

d. at rest $\Rightarrow a=0 \Rightarrow \sum F = 0$

3pt.



$$kx_f - mg = 0 \Rightarrow x_f = \frac{mg}{k} = 0.1176 \text{ m}$$

spring is compressed by 0.1176 m at the end

5pts. E_f = at the end

$$E_f - E_i = W$$

$$\frac{1}{2}kx_f^2 + mg(-x_f) - mgd = -fD$$

$$D = \frac{mg(d+x_f) - \frac{1}{2}kx_f^2}{f} = \boxed{15.07 \text{ m}}$$

Yildiz problem 5

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(a) We start with the rocket equation,

$$\sum_i \vec{F}_{i,\text{ext}} = M(t)\frac{d\vec{v}}{dt} - \underbrace{(\vec{u}-\vec{v})}_{\equiv\, v_{rel}}\frac{dm}{dt}. \tag{0.1}$$

Since there are no external forces, this gives (dropping the vector signs since the motion is horizonal)

$$\frac{dv}{dt} = \frac{v_{rel}}{M}\frac{dm}{dt}. \tag{0.2}$$

Rewriting in terms of $u = v_W$ and $v = v(t)$, this becomes

$$\boxed{\frac{dv}{dt} = \frac{v_W - v(t)}{M(t)}\frac{dm}{dt}.} \tag{0.3}$$

Note: If you didn't remember the rocket equation, you could have derived it as follows. At time t, let the cart have mass $M(t)$ and velocity $\vec{v}$, and let an infinitesimal length of water hitting the cart have mass Δm and velocity $\vec{u}$. At time $t + \Delta t$ let the combined system have mass $M(t) + \Delta m$ and velocity $\vec{v} + \Delta\vec{v}$. The change in momentum between t and $t + \Delta t$ is

$$\begin{aligned}\Delta p &= (M(t) + \Delta m)(v + \Delta v) - (\Delta m)u - M(t)v \\ &= M(t)\Delta v - (u - v)\Delta m - \Delta m \Delta v.\end{aligned} \tag{0.4}$$

To find the force we divide by Δt and take the limit as $t \to 0$. In this limit the last term, which will still be proportional to an infinitesimal quantity even after dividing by Δt, will vanish. Thus we are left with

$$\sum_i \vec{F}_{i,\text{ext}} = \frac{d\vec{p}}{dt} = \lim_{\Delta t\to 0}\frac{\Delta\vec{p}}{\Delta t} = M(t)\frac{d\vec{v}}{dt} - \underbrace{(\vec{u}-\vec{v})}_{\equiv\, v_{rel}}\frac{dm}{dt}. \tag{0.5}$$

(b) By the chain rule we know that

$$\frac{dm}{dt} = \frac{dm}{dl}\frac{dl}{dt} = \lambda v_{rel} \Rightarrow \boxed{\frac{dm}{dt} = \lambda(v_W - v(t)).} \tag{0.6}$$

Note that dl is the infinitesimal length of water hitting the cart, so that dl/dt is v_{rel} as opposed to a rest frame velocity (intuitively, if I go faster less of the water will hit me). You had to give at least a short explanation to get full credit for this.