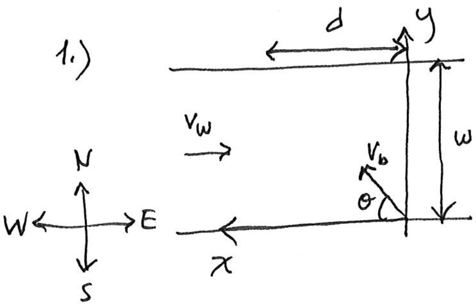
1.)



$V_w = 1.1\ m/s$

$V_b = 4.0\ m/s$

$d = 82\ m$

$w = 200\ m$

$$\bar{V} = \bar{V_b} + \bar{V_w}$$

$$V_x = V_b \cos\theta - V_w$$

$$V_y = V_b \sin\theta$$

$$\frac{V_x}{V_y} = \frac{d}{w}$$

$$\frac{V_b\cos\theta - V_w}{V_b \sin\theta} = \frac{d}{w}$$

$$V_b \cos\theta - V_w = \frac{d}{w} V_b \sin\theta$$

$$V_b^2\cos^2\theta - 2V_bV_w\cos\theta + V_w^2 = \frac{d^2}{w^2}V_b^2\sin^2\theta = \frac{d^2}{w^2}V_b^2\left(1-\cos^2\theta\right)$$

$$V_b^2\left(1+\frac{d^2}{w^2}\right)\cos^2\theta - 2V_bV_w\cos\theta + V_w^2 - \frac{d^2}{w^2}V_b^2 = 0$$

Define: $A \equiv V_b^2\left(1+\frac{d^2}{w^2}\right)$ $\qquad A \approx 18.69\ m^2/s^2$

$B \equiv -2V_bV_w$ $\qquad B \approx -8.8\ m^2/s^2$

$C \equiv V_w^2 - \frac{d^2}{w^2}V_b^2$ $\qquad C \approx -1.48\ m^2/s^2$

$$\cos\theta = -\frac{B}{2A} \pm \frac{\sqrt{B^2-4AC}}{2A}, \quad \text{but } \cos\theta > 0$$

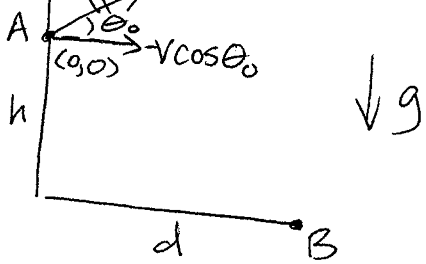
$$\cos\theta = -\frac{B}{2A} + \frac{\sqrt{B^2-4AC}}{2A}$$

$$\cos\theta \approx 0.602$$

$\theta \approx 53°$ N of W $\quad$ or $\quad$ 37° W of N

$$T = \frac{w}{V_y} = \frac{w}{V_b\sin\theta}$$

$T \approx 63$ sec



a. $$x_f = \cancel{x_0}^{\,0} + v_{0x}t + \frac{1}{2}\cancel{a_x}^{\,0}t^2$$

$$d = v\cos\theta_0 t$$

$$t = \frac{d}{v\cos\theta_0}$$

$$y_f = \cancel{y_0}^{\,0} + v_{0y}t + \frac{1}{2}a_y t^2$$

$$-h = v\sin\theta_0 t - \frac{1}{2}gt^2$$

$$-h = v\sin\theta_0\left(\frac{d}{v\cos\theta_0}\right) - \frac{1}{2}g\left(\frac{d}{v\cos\theta_0}\right)^2$$

$$-h = d\tan\theta_0 - \frac{gd^2}{2v^2\cos^2\theta_0}$$

$$\frac{gd^2}{2v^2\cos^2\theta_0} = d\tan\theta_0 + h$$

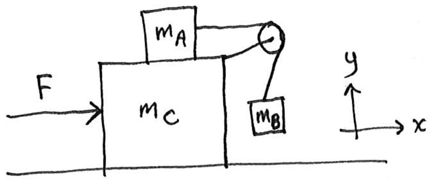
$$v = \sqrt{\frac{gd^2}{2\cos^2\theta_0\,(d\tan\theta_0 + h)}}$$

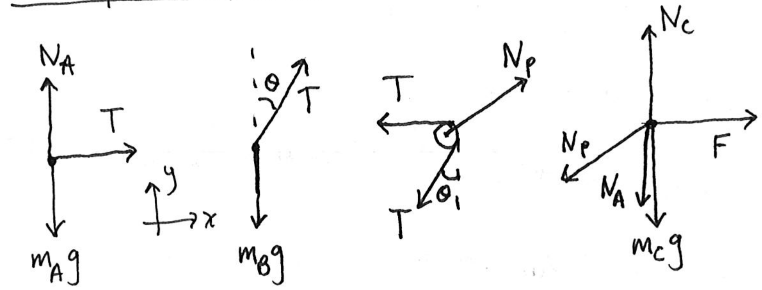
$$= \sqrt{\frac{(9.8\,\text{m/s}^2)(9.40\times10^3\,\text{m})^2}{2\cos^2(35^\circ)(9.40\times10^3\,\text{m}\tan 35^\circ + 3.30\times10^3\,\text{m})}}$$

$$= \boxed{256\,\text{m/s}}$$

$$t = \frac{d}{v\cos\theta_0} = \frac{9.40\times10^3\,\text{m}}{(256\,\text{m/s})\cos 35^\circ} = \boxed{44.9\,\text{s}}$$

3.)





$N_A - m_A g = 0 \qquad T\cos\theta - m_B g = 0 \qquad N_{px} - T - T\sin\theta = 0 \qquad N_c - N_A - m_c g - N_{py} = 0$

$T = m_A a \qquad T\sin\theta = m_B a \qquad N_{py} - T\cos\theta = 0 \qquad F - N_{px} = m_c a$

$$m_A a \sin\theta = m_B a$$

$$\sin\theta = \frac{m_B}{m_A}$$

$$T = \frac{m_B g}{\cos\theta}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{m_B}{m_A}\right)^2}$$

$$T = \frac{m_B g}{\sqrt{1-\left(\frac{m_B}{m_A}\right)^2}} = \frac{m_A g}{\sqrt{\left(\frac{m_A}{m_B}\right)^2 - 1}}$$

$$N_{px} = T(1+\sin\theta) = T\left(1+\frac{m_B}{m_A}\right)$$

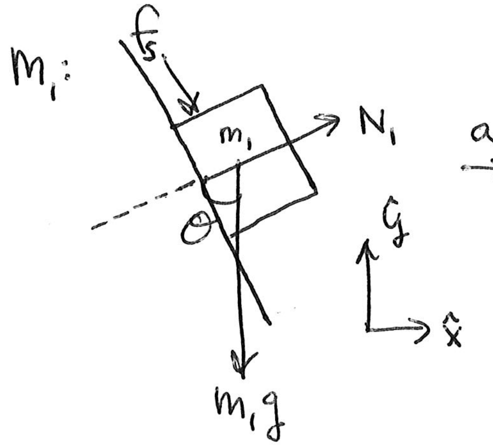
$$F = N_{px} + m_c a = N_{px} + T\frac{m_c}{m_A} = T\left(1+\frac{m_B}{m_A}+\frac{m_c}{m_A}\right) = \frac{T}{m_A}(m_A+m_B+m_c)$$

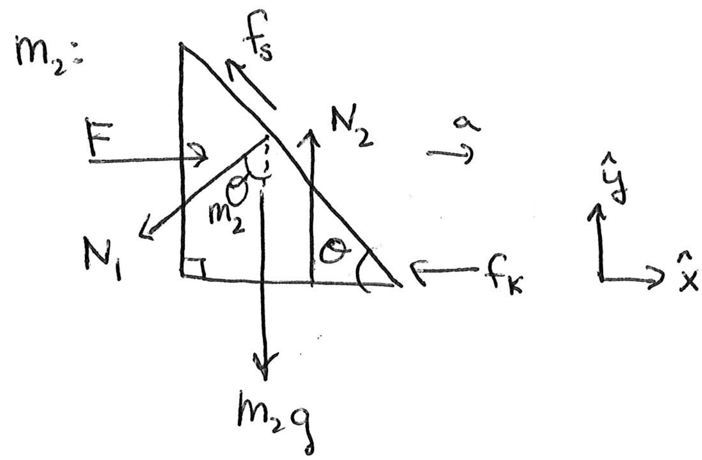
$$F = \frac{m_A g}{m_A\sqrt{\left(\frac{m_A}{m_B}\right)^2-1}}(m_A+m_B+m_c)$$

$$F = \frac{(m_A+m_B+m_c)g}{\sqrt{\left(\frac{m_A}{m_B}\right)^2-1}}$$

Problem 4:

(a)





m_1: $\sum F_y = N_1\cos\theta - f_s\sin\theta - m_1 g = 0$ ①

$\sum F_x = N_1\sin\theta + f_s\cos\theta = m_1 a$ ②

Friction:

$f_s = \mu_s N_1$ @ $F = F_{max}$

$f_k = \mu_k N_2$

m_2: $\sum F_y = N_2 + f_s\sin\theta - N_1\cos\theta - m_2 g = 0$ ③

$\sum F_x = F_{max} - N_1\sin\theta - f_s\cos\theta - f_k = m_2 a$ ④

From ①, $N_1(\cos\theta - \mu_s\sin\theta) = m_1 g \Rightarrow N_1 = \dfrac{m_1 g}{\cos\theta - \mu_s\sin\theta}$

From ②, $N_1(\sin\theta + \mu_s\cos\theta) = m_1 a \Rightarrow a = \dfrac{N_1}{m_1}(\sin\theta + \mu_s\cos\theta)$

From ③, $N_2 = N_1(\cos\theta - \mu_s\sin\theta) + m_2 g = (m_1+m_2)g$

From ④, $F_{max} = N_1(\sin\theta + \mu_s\cos\theta) + m_2 a + \mu_K N_2$

$= N_1\left[\dfrac{m_2}{m_1}(\sin\theta + \mu_s\cos\theta) + (\sin\theta + \mu_s\cos\theta)\right] + \mu_K N_2$

Plugging in N_1 and N_2 we have

$$\boxed{F_{max} = (m_1+m_2)g\left[\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} + \mu_K\right]}$$

We can take a limit to check that our answer makes sense:

As $\theta \to 0$: $F_{max} \to (m_1+m_2)g(\mu_s + \mu_k)$ ✓

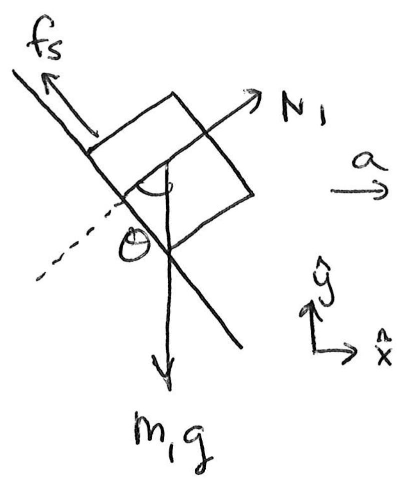
(what we expect with no slope)

You can also check the limit $\theta \to \frac{\pi}{2}$.

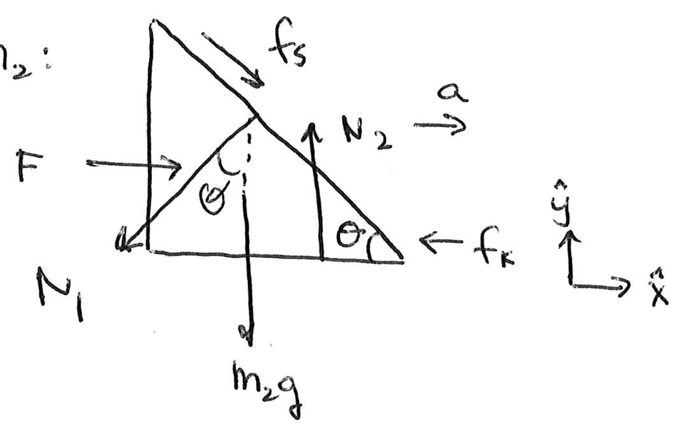
Note that it is a bad idea to pick coordinates that are tilted along the wedge for m_1 because then you will have to break a into components, and the algebra will get very complicated.

(b)

m_1:



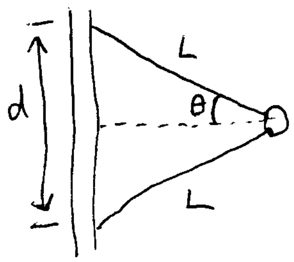
m_2:



This is just the same but with the sign of static friction flipped, $f_s \to -f_s$, or equivalently $\mu_s \to -\mu_s$, so the answer is

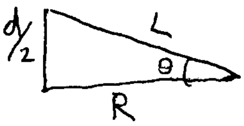
$$\boxed{F_{min} = (m_1+m_2)g\left[\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta} + \mu_k\right]}$$

Problem 5



$L = 1.7\text{ m}$
$d = 1.7\text{ m}$
$T_1 = 35\text{ N}$
$m = 1.34\text{ Kg}$

so, ⇒

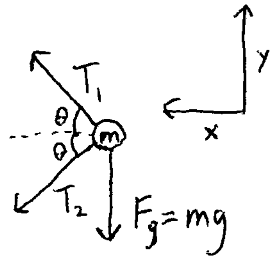


$$\sin\theta = \frac{d/2}{L} = \frac{d}{2L} \underset{(b/c\ L=d)}{=} \frac{1}{2}$$

$$\Rightarrow \theta = \sin^{-1}\left(\tfrac{1}{2}\right) = \underline{30^\circ}$$

$$\Rightarrow R = L\cos\theta = \underline{1.47\text{m}}$$

FBD



a) $\sum F_y = T_1 \sin\theta - T_2 \sin\theta - mg = m\cancel{a_y}^{\,0} = 0$ (b/c no acceleration in the y-direction)

$$\Rightarrow T_2 \sin\theta = T_1 \sin\theta - mg$$

$$\Rightarrow T_2 = \frac{T_1 \sin\theta - mg}{\sin\theta} = T_1 - \frac{mg}{\sin\theta} = 35\text{N} - \frac{(1.34\text{ Kg})(9.8\text{ m/s}^2)}{\sin(30^\circ)} = \boxed{8.74\text{ N}}$$

b) $|\vec{F}_{net}| = \sum F_x = T_1 \cos\theta + T_2 \cos\theta = m a_x$

$$\Rightarrow |\vec{F}_{net}| = (T_1 + T_2)\cos\theta = (35\text{ N} + 8.74\text{ N})\cos(30^\circ) = \boxed{37.9\text{ N}}$$

c) $\underbrace{\sum F_x}_{|\vec{F}_{net}|} = m a_x = \frac{mv^2}{R}$ (centripetal acceleration $a_x = \frac{v^2}{R}$)

$$\Rightarrow |\vec{F}_{net}| = \frac{mv^2}{R} \quad \text{(solve for v)}$$

$$\Rightarrow v^2 = \frac{R|\vec{F}_{net}|}{m}$$

$$\Rightarrow v = \sqrt{\frac{R|\vec{F}_{net}|}{m}} = \sqrt{\frac{L\cos\theta\,|\vec{F}_{net}|}{m}} = \sqrt{\frac{(1.7\text{m})\cos(30^\circ)\,\overset{\text{from part b)}}{(37.9\text{ N})}}{1.34\text{ Kg}}} = \boxed{6.45\text{ m/s}}$$

d) $\vec{F}_{net}$ is directed from the ball ~~[illegible]~~ radially inward toward the rod, intersecting the rod halfway between the two points where the strings are anchored. It is perpendicular to the direction of the rod. This is consistent with uniform circular motion, where the acceleration and net force vectors point radially inward toward the center of the circle defined by the path of the ball's motion.