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Math54 Midterm II, Fall 2006

This is a closed book exam; but everyone is allowed a standard one-page cheat sheet (on one-side only). You need to justify every one of your answers unless you are asked not to do so. Completely correct answers given without justification will receive little credit. Problems are not necessarily ordered according to difficulties. You need not simplify your answers unless you are specifically asked to do so. Hand in this exam before you leave.

Problem	Maximum Score	Your Score
1	5	
2	19	
3	19	
4	19	
5	19	
6	19	
Total	100	

1. (5 Points) Write your personal information below and on top of every page in the test.

   Your Name: ______________________________

   Your GSI: ______________________________

   Your SID: ______________________________

2. (19 Points) Consider the ODE

$$t^2 y'' + t y' - 4y = 0. \tag{1}$$

   (a) Show that $y_1(t) = t^2$ and $y_2(t) = 1/t^2$ are solutions to (1).

   (b) Find the solution y to (1) that further satisfies the initial conditions $y(1) = 1$ and $y'(1) = 0$.

3. (19 Points) Let matrix A be defined by

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}.$$

Find a basis and the dimension of $\mathbf{NS}(A)$.

4. (19 Points) Consider the matrix

$$A = \begin{pmatrix} 1+\epsilon & 1+\epsilon \\ -1 & -1 \end{pmatrix}.$$

(a) Diagonalize the matrix A for $\epsilon \neq 0$.

(b) Show that A is not diagonalizable if $\epsilon = 0$.

5. (19 Points) Let

$$u = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}.$$

(a) Find $\|u\|$, $\|v\|$ and $\cos\theta$, where θ is the angle between u and v.

(b) Find a unit vector perpendicular to both u and v.

6. (19 Points) Let $A \in \mathbf{R}^{m \times n}$. It is known that $\mathbf{rank}\left(A^T\right) = \mathbf{rank}\left(A^T A\right)$.

   (a) Show that $\mathbf{CS}\left(A^T A\right) = \mathbf{CS}\left(A^T\right)$.

   (b) Let $b \in \mathbf{R}^m$. Use (a) to show that the normal equation

$$\left(A^T A\right) x = A^T b$$

   always has a solution $x \in \mathbf{R}^n$.