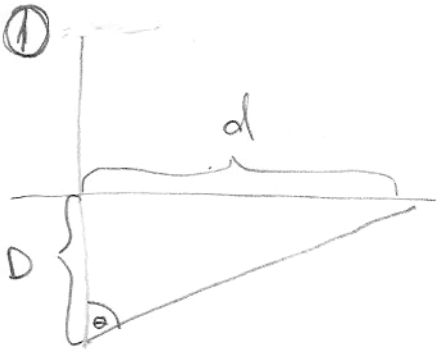
① 

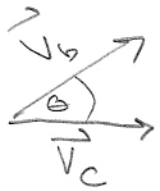
$$\tan\theta = d/D$$

$$d = v_0 t \qquad D = \tfrac{1}{2} a t^2 \qquad t = \sqrt{\frac{2D}{a}}$$

$$d = v_0 \sqrt{\frac{2D}{a}}$$

$$\theta = \arctan\left(\frac{2v_0}{at}\right) = \boxed{\arctan\left(\frac{2v_0}{\sqrt{2Da}}\right)}$$

$$\boxed{\theta = 43^\circ}$$

② 

$$v_x = v_b \cos\theta + v_c$$

$$v_y = v_b \sin\theta$$

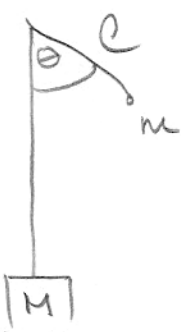
$$t = v_b \sin\theta / g \qquad D = v_x 2t = 2(v_b\cos\theta + v_c)\sin\theta \frac{v_b}{g}$$

$$D = \frac{2v_b^2}{g}\left(\cos\theta + \frac{v_c}{v_b}\right)\sin\theta \qquad \text{maximize}$$

$$\frac{g}{2v_b^2}\frac{\partial D}{\partial \theta} = \left(\cos\theta + \frac{v_c}{v_b}\right)\cos\theta - \sin^2\theta \xrightarrow{\text{set to}} 0$$

$$\frac{v_c}{v_b} \equiv 2 \qquad 2\cos^2\theta + \frac{v_c}{v_b}\cos\theta - 1 = 0$$

gives $\boxed{\cos\theta = \frac{1}{2}(\sqrt{3} - 1)}$ $\boxed{\theta = 69^\circ}$

③ 

$$T\cos\theta = mg \qquad T = Mg$$

$$T\sin\theta = mv^2/r \qquad M\cos\theta = m$$

$$r = \ell \sin\theta \qquad \boxed{\theta = \arccos(m/M)}$$

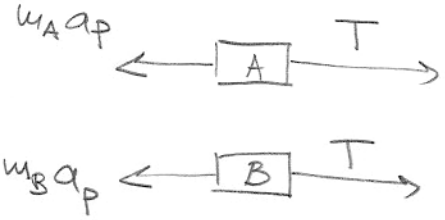
$$T\sin^2\theta = \frac{mv^2}{\ell}$$

$$Mg(1 - \cos^2\theta) = \frac{mv^2}{\ell}$$

$$\frac{M}{m} \ell g \left(1 - \left(\frac{m}{M}\right)^2\right) = v^2$$

$$\boxed{v = \sqrt{\left(\frac{M}{m} - \frac{m}{M}\right)\ell g}}$$

(4)



In noninertial frame connected to the pulley

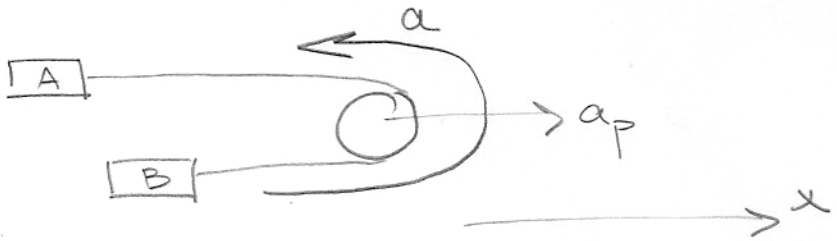
$$m_A a = m_A a_p - T$$
$$m_B a = T - m_B a_p$$
$$a(m_A + m_B) = (m_A - m_B) a_p$$
$$T = m_B (a + a_p) = m_B a_B$$

$$\boxed{T = \frac{2 m_A m_B}{m_A + m_B} a_p}$$

If $m_A = m_B$ $\quad a_A = a_B = a_p$

If one mass is larger, assume w.o.l.g.

$m_A > m_B$

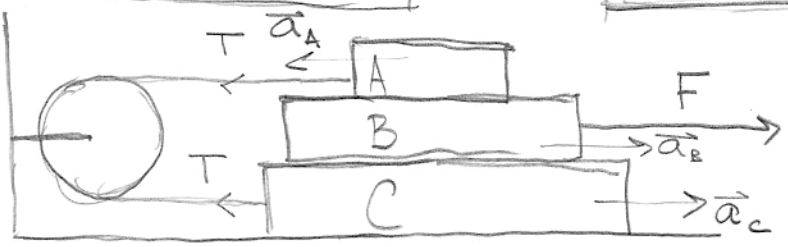


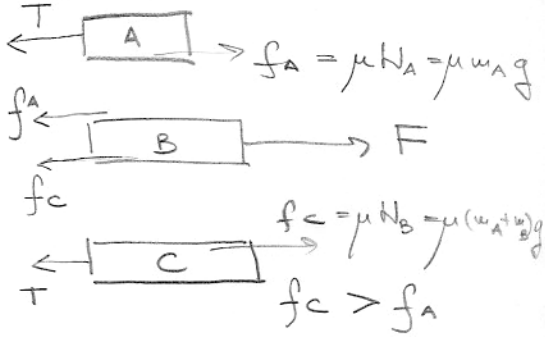
$$a_A = a_p - a$$
$$a_B = a_p + a$$
$$a = a_p \frac{m_A - m_B}{m_A + m_B}$$

$$a_A = a_p - a = a_p \frac{2 m_B}{m_A + m_B} = a_A$$

$$\boxed{a_B = a_p \frac{2 m_A}{m_A + m_B}}$$

(5)





$f_C > f_A \Rightarrow$ C goes to RIGHT, A goes to LEFT

$$|\vec{a}_C| = |\vec{a}_A| \equiv a$$

$$m_C a = f_C - T = \mu (m_A + m_B) g - T$$
$$m_A a = T - f_A = T - \mu m_A g$$
$$+$$
$$(m_A + m_C) a = \mu m_B g$$

$$a = \boxed{\frac{\mu m_B g}{m_A + m_C} = |\vec{a}_C| = |\vec{a}_A|}$$

$$m_B a_B = F - f_A - f_C$$
$$m_B a_B = F - \mu (2 m_A + m_B) g$$

$$\boxed{a_B = \frac{F - \mu (2 m_A + m_B) g}{m_B}}$$

$$a_A = a_C = 1.47 \text{ m/s}^2$$
$$a_B = 4.12 \text{ m/s}^2$$