Math 104: Midterm 1

1. Consider the two sets

$$A = (0,1] \cup [4,\infty), \qquad B = \left\{ \frac{1}{2n} \,:\, n \in \mathbb{N} \right\}.$$

   For each set, determine its maximum and minimum if they exist. For each set, determine its supremum and infimum. Detailed proofs are not required, but you should justify your answers.

2. Consider the following series, defined for $n \in \mathbb{N}$:

$$\sum \frac{6^n}{n^n}, \qquad \sum \frac{1}{n + 1/2}.$$

   For each series, determine whether it converges or diverges. If you make use of any of the theorems for determining series properties, you should state which one you use.

3. Let S be a non-empty bounded subset of $\mathbb{R}$. Define $T = \{|x| \,:\, x \in S\}$ to be the set of all absolute values of elements in S. Prove that $\sup T = \max\{\sup S, -\inf S\}$.

4. Let (s_n) and (t_n) be two sequences defined for $n \in \mathbb{N}$. Suppose $\lim s_n = \infty$, and $\limsup t_n < 0$. Prove that $\lim s_n t_n = -\infty$.

   Note: make sure to consider both cases when $\limsup t_n$ *is a real number, and when* $\limsup t_n$ *is* $-\infty$.