Lecture 2 $\underline{P1}$

a) [5pts] Assuming air is an ideal gas:

$PV = NkT$

$$\boxed{N = \frac{P a^2 h}{kT}} \quad (4\text{ pts})$$

where $P = 1\,atm$
$T = 273\,K$

• Assumption: air is an ideal gas (1 pt)

Errors:

- Dropping units (eg: $P=1$ instead of $1\,atm$): (−1)
- calculating # of moles instead of # of molecules: (−1)

note:
it's ok to drop units if you specify that "a and h are given in m", for instance.

b) [5 pts]

* method 1:

$\Delta V = \beta V \Delta T$.

$$\boxed{\frac{\Delta V}{V} = 3\alpha \Delta T}$$

(4 pts)

but $\beta \approx 3\alpha$ if we are on the regime of linear expansion, i.e., if ΔT is small (1 pt)

* method 2:

• at $T = 0°C$
$V = a^2 h$

• at $T' > T$, assuming linear expansion:

$$V' = [a(1+\alpha\Delta T)]^2 [h(1+\alpha\Delta T)] = a^2 h (1+\alpha\Delta T)^3$$

$$\approx (1+3\alpha\Delta T)\, a^2 h,$$ assuming $\alpha\Delta T$ is small and neglecting $(\alpha\Delta T)^2$ and $(\alpha\Delta T)^3$

$$\Delta V = V' - V \approx a^2 h (3\alpha\Delta T)$$

$$\therefore \boxed{\frac{\Delta V}{V} = 3\alpha\Delta T} \quad \text{or} \quad \boxed{\frac{\Delta V}{V} = (1+\alpha\Delta T)^3 - 1}$$

Errors:

- not justifying why $\beta = 3\alpha$ (−1)
- calculating a change other than $\frac{\Delta V}{V}$ (−1)

c) [10 pts]

Final number density

$$n' = \frac{N'}{V'} = \frac{P}{kT'}$$

Initial:

$$n = \frac{N}{V} = \frac{P}{kT}$$

note:
P is constant!
N, V, T are changing!

Fractional change:

$$\frac{\Delta n}{n} = \frac{n'-n}{n} = \frac{n'}{n} - 1 = \frac{T}{T'} - 1 \quad , \quad \text{where } T' = T + \Delta T$$

$$\frac{\Delta n}{n} = \frac{1}{1+\frac{\Delta T}{T}} - 1 \approx \left(1 - \frac{\Delta T}{T}\right) - 1 \quad \text{if } \frac{\Delta T}{T} \ll 1$$

$$\therefore \boxed{\frac{\Delta n}{n} \approx -\frac{\Delta T}{T}}$$

Errors / grading scheme:

- Almost everything ok:
  - used $\frac{n'}{n}$ instead of $\frac{\Delta n}{n}$: (−2)
  - used numerical value P=1atm but didn't explicitly write that P is const. (−1)
  - simplification : (~1)
  - used N = const. (−5)
- Half ways calculation:
  - identified (clearly) that only P is constant + basic set-up of the problem : (+4)
  - identified that $\frac{N}{V} = \frac{P}{k_bT}$, and $\Delta\left(\frac{N}{V}\right) \neq \frac{\Delta N}{V}$ (+2)
- Heavily Flawed solutions:
  - up to 3 pts

Huang Midterm 1

Problem 2

a. Steady state assumption implies

$$H_1 = H_2 + H_3 \qquad (6 \text{ points})$$

$$\Rightarrow \frac{KA}{L}(T_1 - T_j) = 2\frac{KA}{L}(T_j - T_0)$$

$$\frac{T_1 + 2T_0}{3} = T_j \qquad (4 \text{ points})$$

b.

$$\frac{\Delta Q}{\Delta T} = \frac{KA}{L}\left(T_1 - \left(\frac{T_1 + 2T_0}{3}\right)\right) \qquad (4 \text{ points})$$

$$= \frac{KA}{L}\left(\frac{2T_1 - 2T_0}{3}\right) \qquad (1 \text{ point})$$

c.

$$H_2 = H_3 = \frac{H_1}{2} \qquad (4 \text{ points})$$

$$= \frac{KA}{L}\left(\frac{T_1 - T_0}{3}\right) \qquad (1 \text{ point})$$

3.) (a) Heat is extracted from the water in cooling and in freezing,
(4pts) so the two heat losses have the same sign.

Since $T_1 > T_2$, $\boxed{|Q_1| = \underbrace{m c_w (T_1 - T_2)}_{>0} + \underbrace{m L_f}_{>0}}$

where c_w is the specific heat of water (per unit mass) and L_f is the latent heat of fusion per unit mass of water

(b) (6pts)
$$|\Delta S_1| = \left| \int_{T_1}^{T_2} \frac{dQ_{cooling}}{T} \right| + \frac{Q_{latent}}{T_2}$$

(note the integration necessary for cooling, but not freezing)

$$= \left| m c_w \ln \frac{T_2}{T_1} \right| + \frac{m L_f}{T_2}$$

Since $T_1 > T_2$,

$$\boxed{|\Delta S_1| = m c_w \ln \frac{T_1}{T_2} + \frac{m L_f}{T_2}}$$

(c) As in part (a), the heat gain from heating and vaporizing have
(4pts) same sign. Note only n' moles of water get vaporized.

$$\boxed{|Q_2| = m c_w (T_3 - T_1) + m n' L_v}$$

where L_v is the latent heat of vaporization per unit mass of water and mn' is the mass of water that gets vaporized

(d) As in part (b), we need to integrate for heating, but not
(6pts) for vaporizing.

$$\boxed{|\Delta S_2| = m c_w \ln \frac{T_3}{T_1} + \frac{m n' L_v}{T_3}}$$

(e) Notice that if we consider the fridge as the system,
(4pts) ΔS_1 and ΔS_2 are entropy changes of the <u>environment</u>.

$\Delta S_{universe} = \Delta S_1 + \Delta S_2 + \Delta S_{fridge}$. $\Delta S_{fridge} = 0$ because it completes a cyclic process

$\Delta S_{universe} = 0$ because all processes are reversible

Thus $\boxed{\Delta S_1 + \Delta S_2 = 0}$

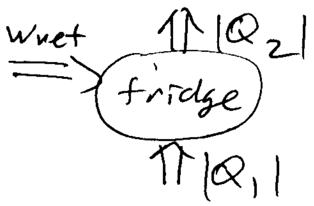
(f) Using part (e), we have $|\Delta S_1| = |\Delta S_2|$

So from parts (b), (d), we have

$$m c_w \ln \frac{T_1}{T_2} + \frac{m L_f}{T_2} = m c_w \ln \frac{T_3}{T_1} + \frac{m n' L_v}{T_3}$$

Solving for n', we have $\boxed{n' = \frac{T_3}{L_v}\left(c_w \ln \frac{T_1^2}{T_2 T_3} + \frac{L_f}{T_2}\right)}$

(g) The fridge has the following schematic



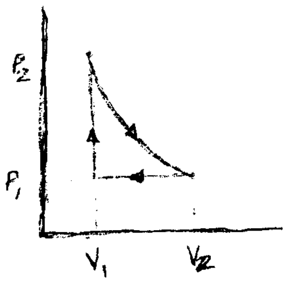
So $|W_{net}| + |Q_1| = |Q_2|$

$$|W| = |Q_2| - |Q_1|$$
$$= m c_w (T_3 - T_1) + m n' L_v - m c_w (T_1 - T_2) - m L_f$$

Plugging in for n',

$$\boxed{|W| = m c_w \left(T_3 + T_2 - 2T_1 + T_3 \ln \frac{T_1^2}{T_2 T_3}\right) + m L_f \left(\frac{T_3}{T_2} - 1\right)}$$

SECTION 2 - PROBLEM 4



EXPRESS IN TERMS OF n, P_1, P_2, V_1, **ONLY**

FIRST FIND V_2 IN TERMS OF P_1, P_2, V_1

ADIABATIC: $P_2 V_1^{\gamma} = P_1 V_2^{\gamma}$ $\qquad \gamma = \frac{d+2}{d} = \frac{5}{3}$

$$V_2 = \left(\frac{P_2}{P_1}\right)^{\frac{3}{5}} V_1$$

POINTS

+5 V_2 & γ

CALC Q & W

a) $Q = 0$ $\quad \leftarrow$ ADIABATIC

$W_{BY} = \frac{3}{2}\left(P_2 V_1 - P_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{3}{5}} V_1\right]\right)$ $\quad \leftarrow W_{BY} = -\Delta E$

a) + 4

b) $Q = \frac{5}{2} P_1\left[V_1 - \left(\frac{P_2}{P_1}\right)^{3/5} V_1\right]$ $\quad \leftarrow Q = W + \Delta E$

$W_{BY} = -P_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{3}{5}} V_1 - V_1\right]$ $\quad \leftarrow W = P\Delta V$

b) + 4

c) $Q = \frac{3}{2} V_1 (P_2 - P_1)$ $\quad \leftarrow Q = \Delta E$

$W_{BY} = 0$ $\quad \leftarrow$ ISOCHORIC

c) + 4

d) $Q_{NET} = \frac{3}{2} V_1 (P_2 - P_1) + \frac{5}{2} P_1\left[V_1 - \left(\frac{P_2}{P_1}\right)^{\frac{3}{5}} V_1\right]$

$W_{NET} = \frac{3}{2}\left(P_2 V_1 - P_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{3}{5}} V_1\right]\right) + -P_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{3}{5}} V_1 - V_1\right]$

d) + 2 NO DOUBLE COUNTING

e) $$\frac{W_{NET}}{Q_{IN}} = \frac{\frac{3}{2}\left(P_2 V_1 - P_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{3}{5}} V_1\right]\right) - P_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{3}{5}} V_1 - V_1\right]}{\frac{3}{2} V_1 (P_2 - P_1)}$$

e) + 2 NO DOUBLE COUNTING

f) a) ADIABATIC $\quad \Delta S = \frac{Q}{T} \quad Q = 0 \quad \therefore \Delta S = 0$

b) ISOBARIC $\quad \Delta S = \frac{3}{2} nR \ln\left(\frac{T_f}{T_i}\right) + nR\ln\left(\frac{V_f}{V_i}\right)$

$= \frac{3}{2} nR \ln\left(\frac{P_1 V_1}{P_1 V_2}\right) + nR \ln\left(\frac{V_1}{V_2}\right)$

$= \frac{5}{2} nR \ln\left(\frac{V_1}{V_2}\right) = \frac{5}{2} nR \ln\left(\frac{1}{\left(\frac{P_2}{P_1}\right)^{\frac{3}{5}}}\right) = -\frac{3}{2} nR \ln\left(\frac{P_2}{P_1}\right)$

c) ISOCHORIC $\quad \Delta S = \frac{3}{2} nR \ln\left(\frac{T_f}{T_i}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$

$= \frac{3}{2} nR \ln\left(\frac{P_2 V_1}{P_1 V_1}\right) = \frac{3}{2} nR \ln\left(\frac{P_2}{P_1}\right)$

f) + 6 (2 for each)

g) 3 +2 math, +1 explanation

g) $\Delta S_{CYCLE} = 0 = -\frac{3}{2} nR \ln\left(\frac{P_2}{P_1}\right) + \frac{3}{2} nR \ln\left(\frac{P_2}{P_1}\right) = 0$ REVERSIBLE, COMPLETE PROCESS