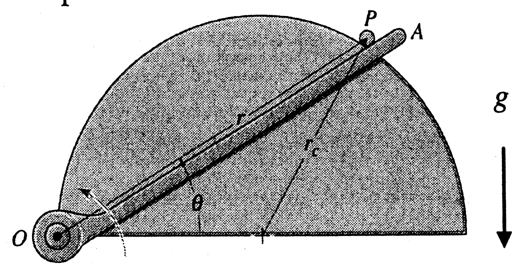
Department of Mechanical Engineering
University of California at Berkeley
ME 104 Engineering Mechanics II
Spring Semester 2007

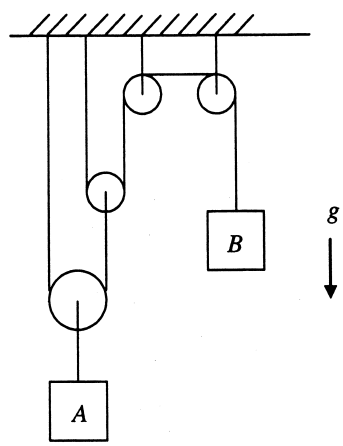
Instructor: F. Ma
Midterm Examination No. 1 Feb 23, 2007

The examination has a duration of 50 minutes.
Answer ALL questions.
All questions carry the same weight.

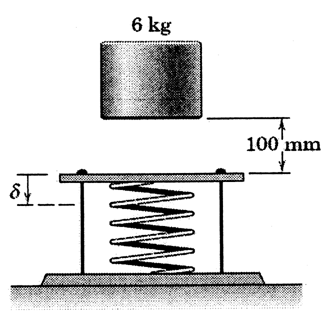
1. A particle of P mass m is guided along a smooth circular path of radius r_c by the rotating arm OA. If the arm has a constant angular velocity ω, determine the angle $\theta \leq 45^\circ$ at which the particle leaves the circular path.



2. The 50-kg block A shown is released from rest. If the masses of the pulleys and the cords are neglected, determine the velocity of the 20-kg block B in 3 seconds.



3. A 6-kg cylinder is released from rest in the position shown and falls onto a spring, which has been initially precompressed 50 mm by a light strap and restraining wires. If the stiffness of the spring is 4 kN/m, compute the additional deflection δ of the spring produced by the falling cylinder before it rebounds.



1. Suppose the particle leaves the circular path at $\beta \le 45°$. Before the particle leaves the path, it travels in a circle of radius r_c. At any position $\theta < \beta$, the polar coordinates of the particle satisfy

$$\dot{\theta} = \omega \quad \Rightarrow \quad \ddot{\theta} = 0$$

$$r = 2r_c \cos\theta \quad \Rightarrow \quad \dot{r} = -2r_c\dot{\theta}\sin\theta = -2r_c\omega\sin\theta$$

$$\Rightarrow \quad \ddot{r} = -2r_c\omega^2\cos\theta$$

Since the force F exerted by the arm OA on m is perpendicular to OA while the reaction N is normal to the circular path,

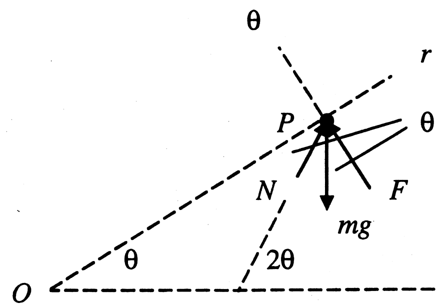
$$\sum F_r = ma_r \quad \Rightarrow \quad -mg\sin\theta + N\cos\theta = m(\ddot{r} - r\dot{\theta}^2)$$

$$\Rightarrow \quad -mg\sin\theta + N\cos\theta = m(-4r_c\cos\theta\omega^2)$$

When m leaves the path at $\theta = \beta$, $N = 0$. Thus

$$-mg\sin\beta = -4mr_c\omega^2\cos\beta$$

$$\Rightarrow \quad \beta = \tan^{-1}\left(\frac{4r_c\omega^2}{g}\right)$$



2. Blocks A and B perform rectilinear motion. From a horizontal reference line through the centers of the upper pulleys, measure the positions of A, B, and the pulley C by y_A, y_B, and y_C. There are two constraints between the coordinates:

$$2y_A - y_C = l_1 = \text{constant}$$

$$2y_C + y_B = l_2 = \text{constant}$$

where l_1 and l_2 are overall vertical lengths of the cords measured from the horizontal reference line. By differentiation, one obtains

$$2v_A - v_C = 0 \qquad (1)$$

$$2v_C + v_B = 0 \qquad (2)$$

Combine equations (1) and (2) to eliminate v_C,

$$4a_A + a_B = 0 \qquad (3)$$

Since the cords and pulleys have negligible weight, a force balance shows that if block B is subjected to a tension of T, then block A is acted upon by $4T$. For block A,

$$\sum F_y = ma_y \quad \Rightarrow \quad m_A g - 4T = m_A a_A \quad \downarrow \qquad (4)$$

For block B,

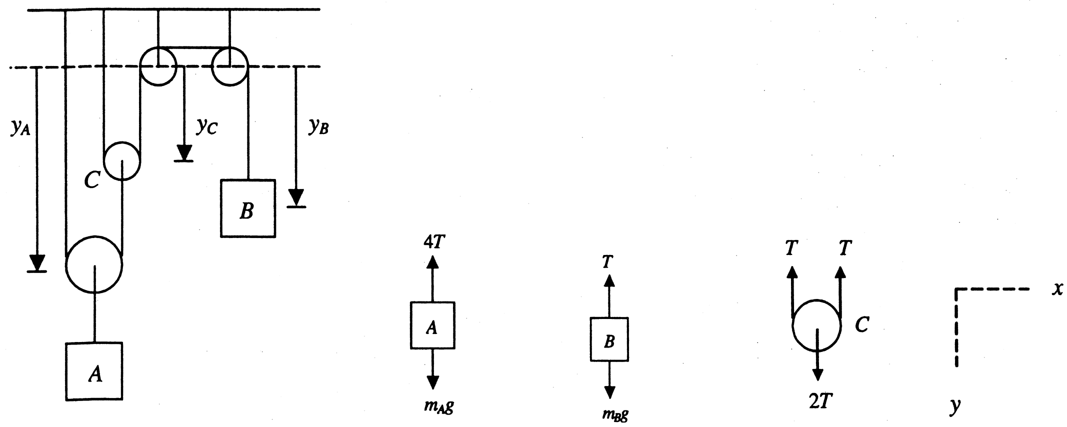
$$\sum F_y = ma_y \quad \Rightarrow \quad m_B g - T = m_B a_B \quad \downarrow \qquad (5)$$

There are three unknowns a_A, a_B, and T in three equations. Upon solution,

$$a_B = \frac{12g}{37} = 3.1784 \qquad \downarrow$$

Note that block B has a downward acceleration. Velocity of block B after 3s is equal to

$$v_B = v_0 + a_B t = 3a_B = 9.535\,\text{m/s} \qquad \downarrow$$



3. Let position 1 of the cylinder be its initial position at 100 mm above the spring. Suppose position 2 corresponds to deflection δ in the spring. Using the equation of work and energy between positions 1 and 2,

$$U = \Delta T + \Delta V_g + \Delta V_e$$

where the work done by forces other than gravitational and spring forces is

$$U = 0$$

$$\Delta T = T_2 - T_1 = 0$$

Define the reference level for measuring potential energy as the level associated with the precompressed spring before impact. Then

$$\Delta V_g = mgh_2 - mgh_1 = mg(-\delta) - mg(0.1) = -mg(\delta + 0.1)$$

$$\Delta V_e = \left(\frac{1}{2}kx_2^2\right) - \left(\frac{1}{2}kx_1^2\right) = \left(\frac{1}{2}k(0.05+\delta)^2\right) - \left(\frac{1}{2}k(0.05)^2\right)$$

Thus

$$U = \Delta T + \Delta V_g + \Delta V_e$$

$$\Rightarrow \quad -6g(\delta + 0.1) + 2000((0.05+\delta)^2 - 0.05^2) = 0$$

$$\Rightarrow \quad 2000\delta^2 + 141.14\delta - 5.886 = 0$$

$$\Rightarrow \quad \delta = 0.0294 \text{ or } -0.1$$

The additional deflection is $\delta = 29.4\,\text{mm}$.