UNIVERSITY OF CALIFORNIA, BERKELEY
College of Engineering
Department of Electrical Engineering and Computer Sciences

EE 105: Microelectronic Devices and Circuits — Fall 2010

MIDTERM EXAMINATION #2

Time allotted: 60 minutes

NAME: ____________________ Solution ____________________

STUDENT ID#: ____________________

INSTRUCTIONS:

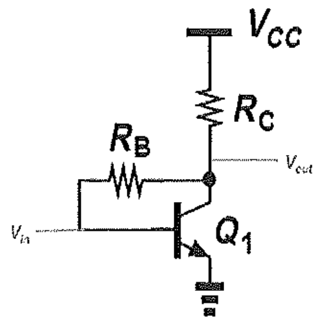
1. **SHOW YOUR WORK. (Make your methods clear to the grader!)**
   Specially, while using chart, make sure that you indicate how you have got your numbers. For example, if reading off mobility, clearly write down what doping density that corresponds to.
2. **Clearly mark (underline or box) your answers.**
3. **Specify the units on answers whenever appropriate.**

SCORE: 1	________	**/ 16**
2	________	**/ 14**
3	________	**/ 10**
Total	________	**/ 40**

For all questions, if not explicitly mentioned, ignore the BJT capacitances.

Prob 1. [16 pts]

Consider the following amplifier:



(a) [5 pts] Explain how this amplifier is robust against variation in R_B and R_C.

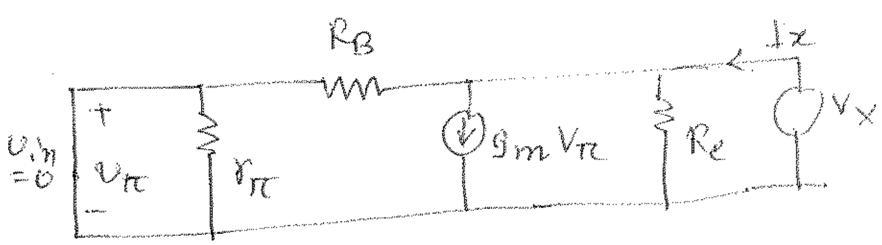
$$V_{BE} = V_{cc} - (I_e + I_B) R_e - I_B R_B$$

$$= V_{cc} - \frac{\beta+1}{\beta} I_e R_e - \frac{I_c R_B}{\beta}$$

$$V_{BE} = V_{cc} - I_c \left[\frac{(\beta+1) R_e - R_B}{\beta} \right]$$

Thus I_c is linearly dependent on V_{BE} rather than being exponentially dependent.

(c) [5 pts] Find out the small signal output resistance. You may ignore the early effect.

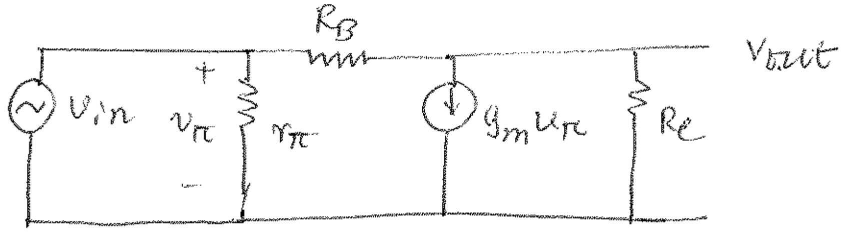


r_π is shorted out

$g_m v_\pi$ is open.

$\therefore$ $R_{out} = R_B \parallel R_e$

(d) [6 pts] Find out the small signal voltage gain. You may ignore the early effect.



$$v_\pi = v_{in}$$

$$v_{in} - v_{out} = R_B\left[g_m v_\pi + \frac{v_{out}}{R_E}\right]$$

$$\therefore \frac{v_{in} - v_{out}}{R_B} = g_m v_{in} + \frac{v_{out}}{R_E}$$

$$v_{in}\left[\frac{1 - g_m R_B}{R_B}\right] = v_{out}\left[\frac{1}{R_B} + \frac{1}{R_E}\right]$$

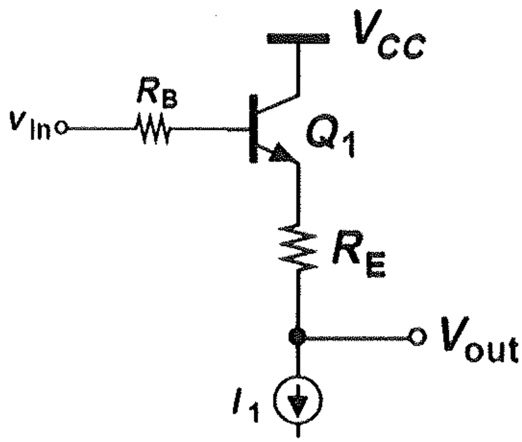
$$\boxed{\frac{v_{out}}{v_{in}} = -\left(\frac{g_m R_B - 1}{R_B}\right)\left(\frac{R_B R_E}{R_B + R_E}\right) = -(g_m R_B - 1)\frac{R_E}{R_B + R_E}}$$

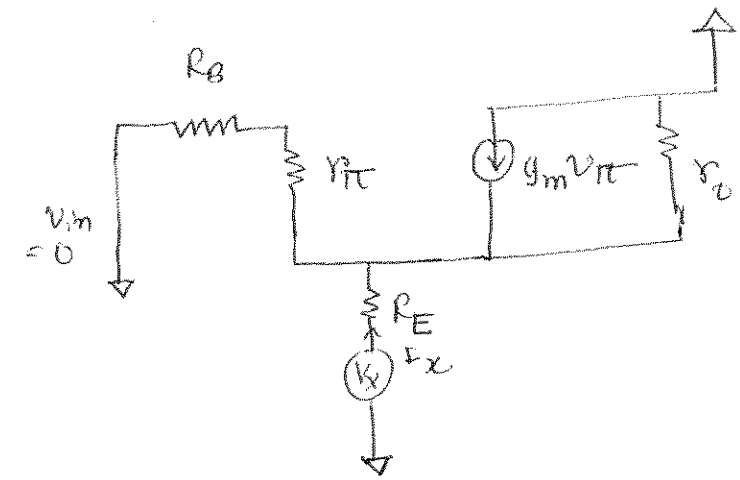
if, $g_m R_B \gg 1$

$$\boxed{A_v = -g_m (R_B \parallel R_E)}$$

Prob 2 [14 pts].

(a) [6 pts] For the following amplifier, derive an expression for small signal input resistance starting from the small signal model including the early effect.





<u>KCL</u>

$$i_x + \frac{v_\pi}{r_\pi} + g_m v_\pi = \frac{V_x - i_x R_E}{r_o} \quad \text{--- (1)}$$

<u>KVL</u>

$$V_x - i_x R_E + v_\pi + \frac{v_\pi}{r_\pi} R_B = 0 \quad \text{--- (2)}$$

$$\therefore V_x = i_x R_E - \frac{v_\pi}{r_\pi}(r_\pi + R_B)$$

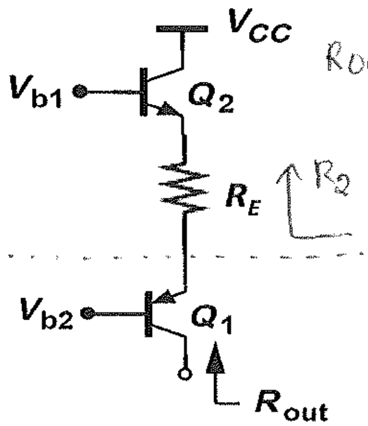
$$= i_x R_E - \frac{v_x/r_o - i_x(1 + R_E/r_o)}{\beta+1}(r_\pi + R_B)$$

$$v_x\left[1 + \frac{r_\pi + R_B}{\beta+1} \cdot \frac{1}{r_o}\right] = i_x\left[R_E + \frac{R_E + r_o}{r_o} \cdot \frac{r_\pi + R_B}{\beta+1}\right]$$

$$= i_x R_E\left[1 + \frac{r_\pi + R_B}{\beta+1} \cdot \frac{1}{r_o}\right] + i_x \frac{r_\pi + R_B}{\beta+1}$$

$$\therefore \boxed{Z_{out} = \frac{v_x}{i_x} = R_E + \frac{\frac{r_\pi + R_B}{\beta+1} \times r_o}{r_o + \frac{r_\pi + R_B}{\beta+1}} = R_E + \left(r_o \parallel \frac{r_\pi + R_B}{\beta+1}\right)}$$

(b) [4 pts] Is the following a good cascode? Why or Why not? Assume that $g_m R_E >> 1$.



$$R_{out} = r_{o1} + (1 + g_{m1} r_{o1})(R_2 \parallel r_{\pi 1})$$

From prob (a)

$$R_2 = R_E + \left(r_o \parallel \frac{r_\pi}{\beta + 1}\right)$$

$$\cong R_E + \frac{1}{g_{m2}} \cong R_E \quad [\because g_m R_E >> 1]$$

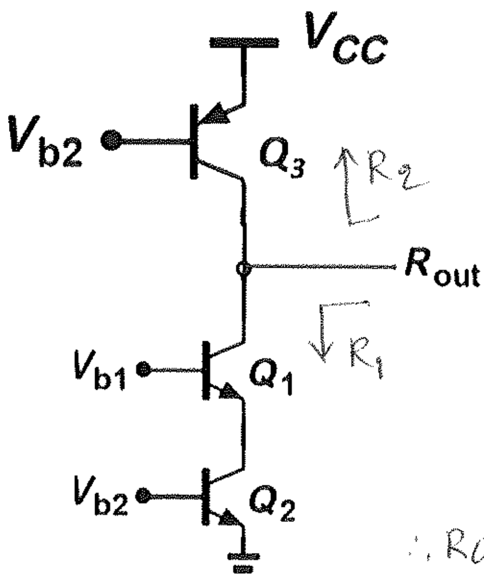
Assuming both transistors have similar

$$g_m = g_{m1} = g_{m2}$$

$$\therefore R_{out} \cong g_{m1} r_{o1} (R_E \parallel r_{\pi 1})$$

Hence it is a good cascode.

(c) [4 pts]Find out the output resistance of the following cascode. Is it a good cascode? Why or why not? Assume that parameters for all transistors are identical.



$$R_{out} = R_1 \parallel R_2$$

$$R_1 = r_{o1} + (1 + g_{m1} r_{o1})(r_{o2} \parallel r_{\pi 1})$$

$$\cong g_{m1} r_{o1} (r_{o2} \parallel r_{\pi 1})$$

$$R_2 = r_{o3}$$

$$\therefore R_{out} = \left(g_{m1} r_{o1} (r_{o2} \parallel r_{\pi 1}) \right) \parallel r_{o3}$$

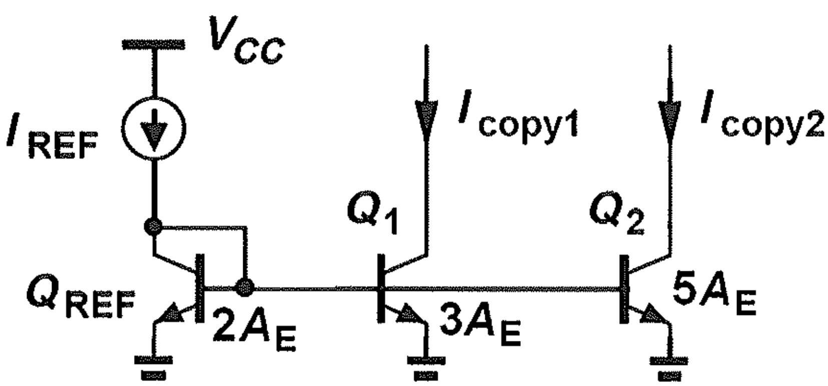
since all transistors are identical

$$R_o = \{ g_m r_o (r_o \parallel \beta / g_m) \} \parallel r_o \quad \therefore R_{o\,max} = r_o$$

hence not a good cascode

Prob 3 [10 pts]:

(a) [6 pts] For the following configuration, what is I_{copy1} accounting for the base currents?



$$I_{ref} = \frac{I_{copy_1}}{\beta} + \frac{I_{copy_2}}{\beta} + \frac{I_{cref}}{\beta} + I_{cref} \quad \text{— (1)}$$

$$I_{cref} = I_{copy_1} \times \frac{2}{3} \; ; \; I_{cref} = \frac{2}{5} I_{copy_2}$$

$$\therefore I_{copy_2} = 5/2 \times 2/3 \, I_{copy_1}$$

$$\therefore \text{From (1)} \rightarrow I_{ref} = \frac{I_{copy_1}}{\beta} + \frac{5}{3\beta} I_{copy_1} + \frac{2}{3\beta} I_{copy_1} + 2/3 \, I_{copy_1}$$

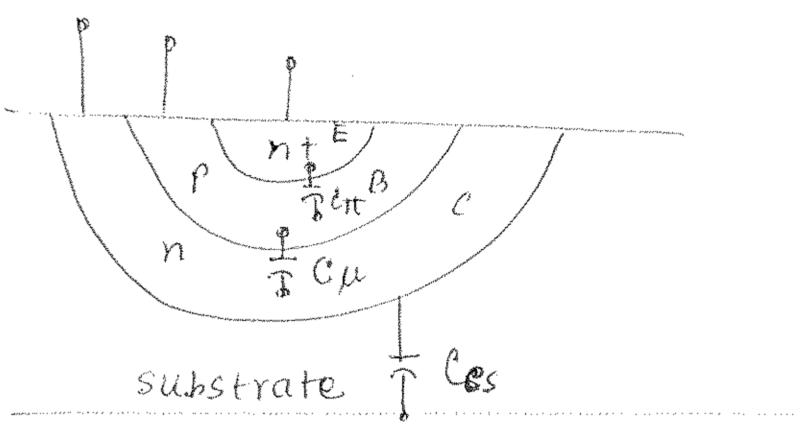
$$\therefore I_{ref} = I_{copy_1}\left[\frac{1}{\beta} + \frac{5}{3\beta} + \frac{2}{3\beta} + \frac{2}{3}\right]$$

$$= I_{copy_1}\left(\frac{10 + 2\beta}{3\beta}\right)$$

$$\therefore I_{copy_1} = I_{ref}\left(\frac{3\beta}{10 + 2\beta}\right)$$

$$\boxed{I_{copy_1} = I_{ref}\left(\frac{3}{10/\beta + 2}\right)}$$

(b)[4 pts] For a bipolar junction transistor, draw all the relevance capacitances. Also mention their physical origin.



Emitter junction: Depletion capacitance + Diffusion capacitance $= C_\pi$

collector junction: Depletion capacitance $= C_\mu$

Due to substrate ground plane $= C_{cs}$