UNIVERSITY OF CALIFORNIA, BERKELEY
College of Engineering
Department of Electrical Engineering and Computer Sciences

EE 105: Microelectronic Devices and Circuits Fall 2010

MIDTERM EXAMINATION #1

Time allotted: 60 minutes

NAME: Solutions ____________________

STUDENT ID#: ____________________

INSTRUCTIONS:

1. **SHOW YOUR WORK. (Make your methods clear to the grader!)**
   Specially, while using chart, make sure that you indicate how you have got your numbers. For example, if reading off mobility, clearly write down what doping density that corresponds to.
2. **Clearly mark (underline or box) your answers.**
3. **Specify the units on answers whenever appropriate.**

SCORE: 1	________	**/ 14**
2	________	**/ 16**
3	________	**/ 15**
Total	________	**/ 45**

PHYSICAL CONSTANTS

Description	Symbol	Value
Electronic charge	q	1.6×10^{-19} C
Boltzmann's constant	k	8.62×10^{-5} eV/K
Thermal voltage at 300K	$V_T = kT/q$	0.026 V

PROPERTIES OF SILICON AT 300K

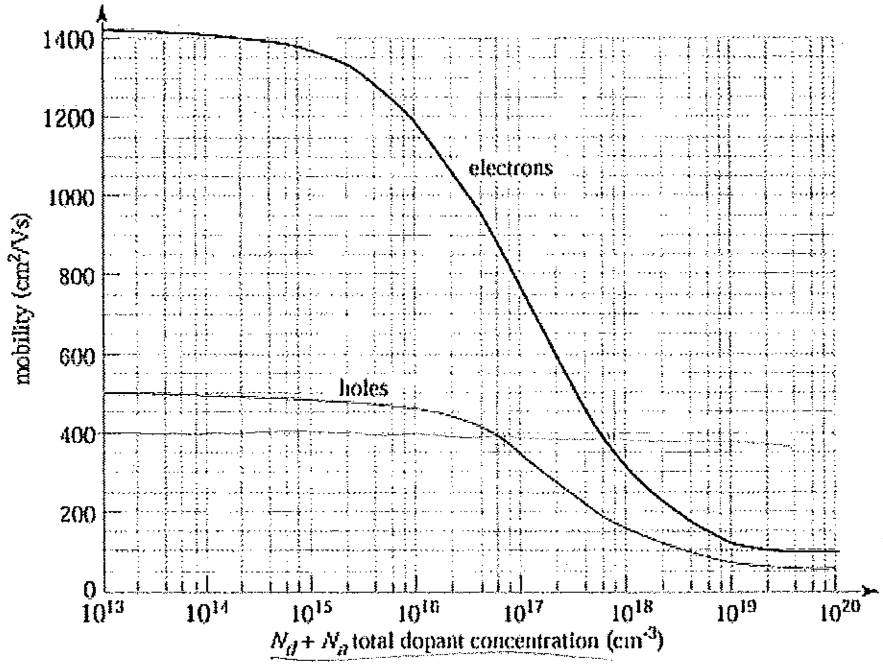
Description	Symbol	Value
Band gap energy	E_G	1.12 eV
Intrinsic carrier concentration	n_i	10^{10} cm^{-3}
Dielectric permittivity	ε_{Si}	1.0×10^{-12} F/cm

USEFUL NUMBERS

$V_T \ln(10) = 0.060$ V at T=300K
$\exp(30) \sim 10^{13}$

Depletion region Width: $W = \sqrt{\frac{2\varepsilon}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\left(V_{bi}-V_{Applied}\right)}$

Electron and Hole Mobilities in Silicon at 300K



Prob 1. [14 pts]

(a) Assume that a Si sample has been doped with Ga (which is a group III element).

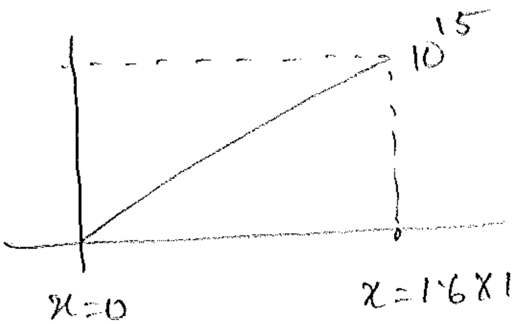
- (i) Will the electron density increase or decrease if one increases temperature? Why? [2pt]
- (ii) Will the hole density increase or decrease if one increases temperature? Why? [2pt]

Ga → $\underline{III}$ element → acceptor

(i) $n = \frac{n_i^2}{N_{Ga}}$ will increase as n_i increases with increasing temperature

(ii) $p = N_{Ga}$ will not depend on temperature.

(b) Assume that a Si sample has a hole concentration which increases linearly over a distance of 1.6 µm from an *initial value* at x=0 to a final value of 10^{15} cm^{-3} at x=1.6 µm. It is known that the absolute value of maximum current density in this sample is 10.4 A/cm^2 and the average mobility of holes throughout this sample can be approximated to be 400 cm^2/V-sec. Find out the hole concentration at x=0. Can you think of a situation (e.g. for a practical device) where such a profile of hole concentration may appear? [4 pt]



$$J = q D_p \frac{dp}{dx}$$

$$= q \times \mu_p \times \frac{KT}{q} \times \frac{p(x=1.6\mu m) - p(x=0)}{1.6\times10^{-4}\ cm}$$

$$10.4 = \frac{1.6\times10^{-19} \times 400 \times 0.026 \times (10^{15} - p(x=0))}{1.6\times10^{-4}}$$

$$\Rightarrow 10^{15} - p(x=0) = \frac{\cancel{10.4}\times\cancel{1.6}\times10^{-4}}{\cancel{1.6}\times10^{-19}\times \cancel{4\times26}\times\cancel{10^2}\times\cancel{10^{-3}}} = 10^{15}$$

$\therefore \boxed{p(x=0) = 0} \Rightarrow$ no carrier concentration at $x=0$.

This could happen at the collector-base junction of a p-n-p transistor.

(c) Assume that a Si sample of length 500μm at *T=300°K* is uniformly doped with acceptors at a concentration of $2.5 \times 10^{\cancel{18}\,17}$ cm^{-3}. Following that, the semiconductor is *compensated* such that the carrier mobility comes out to be 400 cm^2/v-sec. If 5V is applied across this sample, find out the current density.

[6pt]

$$400\ \mathrm{cm^2/V\text{-}sec} \longrightarrow 6 \times 10^{17}\ \mathrm{cm^{-3}} = N_a + N_d$$

$$\therefore N_d = (6 - 2.5) \times 10^{17} = 3.5 \times 10^{17}\ \mathrm{cm^{-3}}$$

$$\therefore n = N_d - N_a = (3.5 - 2.5) \times 10^{17} = 10^{17}\ \mathrm{cm^{-3}}$$

$$J = \sigma E$$

$$= q n \mu_n E$$

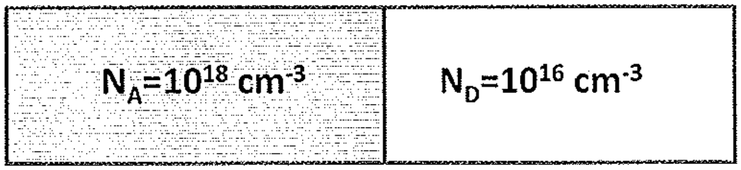
$$= 1.6 \times 10^{-19} \times 10^{17} \times 400 \times \frac{5}{500 \times 10^{-4}}$$

$$= \frac{1.6 \times 4 \times \cancel{5}}{\cancel{5}} \times \frac{10^{-19} \times 10^{17} \times \cancel{10^2}}{\cancel{10^2} \times 10^{-4}}$$

$$\boxed{J = 6.4 \times 10^{2}\ \mathrm{A/cm^2}}$$

Prob 2. [16 pts]

(a) [10 pts] Consider that a Si sample has been doped as shown below:



(i) Briefly state how depletion region is created at the junction. [2 pt]

holes diffuse from p to n leaving behind immobile negative charges, electrons diffuse from n to p leaving behind immobile positive charges.

(ii) Find out the built in potential at T=300K. [2 pt]

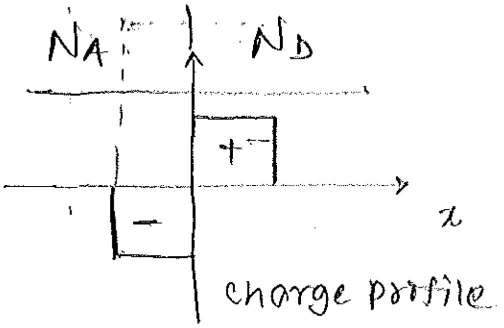
$$V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = V_T \ln \frac{10^{34}}{10^{20}} = 14 V_T \ln 10$$

$$\boxed{V_0 = 14 \times 0{\cdot}06 = 0{\cdot}84\ V}$$

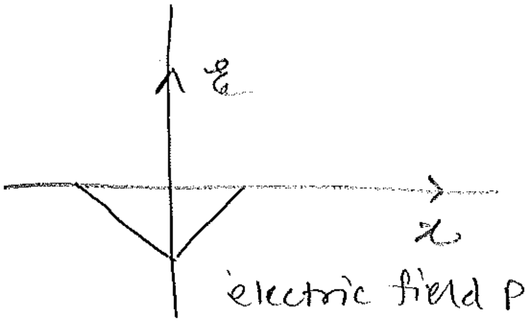
(iii) If the two sides of the sample is now shorted, do you expect a current to flow due to the built in potential? Justify your answer. [2 pt]

NO; because without an applied voltage diffusion current cancels out the drift current. Notably, the built-in potential build up to oppose the diffusion current that would otherwise flow due to differences in carrier concentration across the junction.

(iV) Draw the charge density and electric profile for this junction. [2 pt]



charge profile



electric field profile.

(V) Qualitatively explain what happens to charge density and electric field when a reverse bias is applied as opposed to equilibrium condition. Justify your answer. [2 pt]

The charge profile will be widened. This is because a reverse bias uncovers additional immobile charges at the junction.

Due to widened charge profile the peak electric field will be enhanced. Note that electric field is found by integrating the charge density.

(b) A p-n junction diode will be designed such that: (i) the p side is much more heavily doped than the n side (ii) it has a built in potential of 0.72 V and (iii) at $V_{applied}$=-0.88V (reverse bias), it gives a capacitance of 50fF/(μm)^2 (1 femto= 10^{-15}). Find out the doping concentration for acceptors and donors. Clearly state all the approximations. [6 pt]

$$C = \frac{\varepsilon_{si}}{W} ; \quad W = \sqrt{\frac{2\varepsilon}{q}\left(\frac{1}{Na} + \frac{1}{Nd}\right)(V_{bi} - V_{app})}$$

$$\therefore \frac{1}{C^2} = \frac{W^2}{\varepsilon_{si}^2} = \frac{2\varepsilon_{si}\left(1/Na + \frac{1}{Nd}\right)(V_{bi} - V_{app})}{q\,\varepsilon_{si}^2} = \frac{2\frac{(Na+Nd)}{NaNd}(V_{bi} - V_{app})}{q\,\varepsilon_{si}}$$

$$\therefore \boxed{\frac{Na+Nd}{NaNd} = \frac{q\,\varepsilon_{si}}{2C^2(V_{bi} - V_{app})}} \text{ ; } -①$$

$$V_{bi} = \frac{kT}{q} \ln \frac{NaNd}{n_i^2}$$

$$\therefore \boxed{N_aN_D = n_i^2\, e^{\left(\frac{V_{bi}}{V_T}\right)}} -②$$

$$\therefore C = \frac{50 \times 10^{-15}\ \text{F}}{10^{-8}\ \text{cm}^2} = 5\times10^{-6}\ \text{F/cm}^2$$

$$\therefore \text{From } ① \quad \frac{N_a+N_d}{N_aN_d} = \frac{1{\cdot}6\times10^{-19}\times10^{-12}}{2\times1{\cdot}6\times25\times10^{-12}} \approx 10^{-20}$$

if $N_a \gg N_d$;

$$\frac{1}{N_d} \approx 10^{-20} \quad \therefore \boxed{N_d = 10^{22}\ \text{cm}^{-3}}$$

$$\text{But } N_aN_d = n_i^2 e^{\frac{0.72}{0.026}} \cong 10^{20}\times10^{+12} = 10^{32}$$

$$\therefore \boxed{N_a \cong 10^{12}\ \text{cm}^{-3}}$$

But this clearly violates the condition $N_a \gg N_d$.

$\boxed{\text{Hence such a diode cannot be designed.}}$

From practical considrations, we may still want a diode with p side heavily doped and a capacitance of 50 fF/μm² at Vapp = −0·88 V. In that case we can design the diode with a slightly diffrent built in potential. For example with $V_0 = 1{\cdot}26$ V,

$$\frac{N_a+N_d}{N_aN_D} \approx 10^{-20} \quad ; \quad N_aN_D = 10^{41}$$

so that $N_D \cong 10^{20}\ \text{cm}^{-3}$ and $N_A = 10^{21}\ \text{cm}^{-3}$

Prob 3. [14 pts]

(a) [8 pt]

(i) For a BJT biased in the active mode, what are the mechanisms of carrier transport at the Base-Emitter and Base-Collector junction? [2 pt]

Base-emitter → Diffusion

Base-collector → Drift.

(ii) What are the two major mechanisms that can contribute to the base current? What steps are taken to minimize their effects? [2 pt]

(i) Diffusion into emitter

(ii) recombination in the base.

To minimize diffusion, emitter is much more heavily doped than the base.

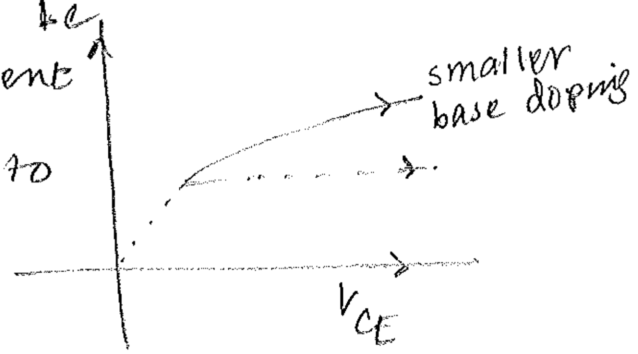
To minimize recombination, base region is made short compared to that determined by minority carrier lifetime.

(iii) How is collector doped with respect to the base (more heavily or more lightly)? Why? [2 pt]

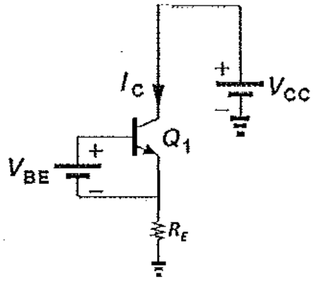
Collector is more lightly doped to minimize early effect.

(iv) What happens to the early voltage if base doping is reduced? Why? [2 pt]

smaller base doping means now there will be additional encroachment of collector-base depletion region into base. This means I_C will become a stronger function of V_{CE}. This means r_0 will decrease which in turn means that early voltage will decrease.



(b) [7 pt]



(i) Assume that for the transistor shown above, V_{BE}=0.8 V, β=2 and R_E=1 kΩ. If to make sure I_c=1 mA, V_{CB} must be at least 0.4 V, Find out the minimum V_{cc} that is required to get I_c=1 mA. [5 pt]

$V_{CB} \geqslant 0{\cdot}4V$

$V_C - V_B \geqslant 0{\cdot}4V$; $V_C = V_{CC}$

$\therefore V_{CC} \geqslant V_B + 0{\cdot}4V$

$V_B = V_{BE} + V_{RE}$

$= 0{\cdot}8 + (I_C + I_B) R_E$

$V_B = 0{\cdot}8 + I_C\left(1+\frac{1}{2}\right)R_E$

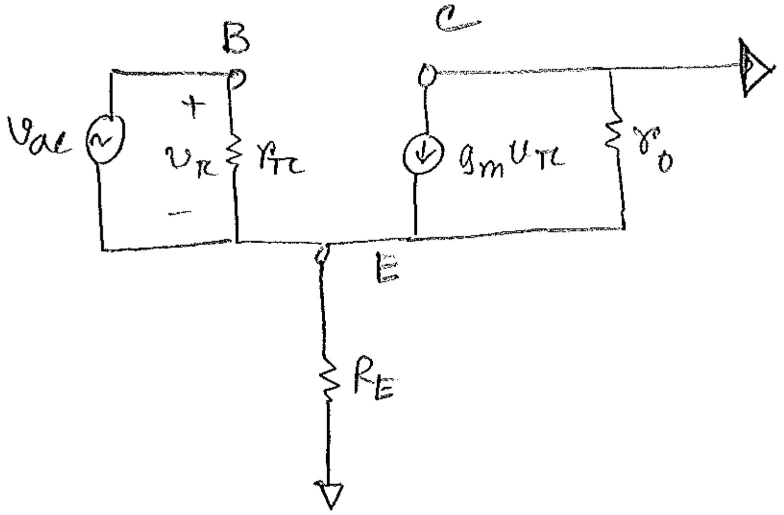
$= 0{\cdot}8 + 10^{-3}\times10^{3}\times 3/2$

$= 0{\cdot}8 + 1{\cdot}5 = 2{\cdot}3V$

$\therefore V_{CC} \geqslant 2{\cdot}3 + 0{\cdot}4$

$\therefore \boxed{V_{CC} \geqslant 2{\cdot}7V}$

(ii) Draw the small signal model for the above transistor including R_E and assuming that there is an independent small signal source in series with the DC source V_{BE}. [2 pt]



$$g_m = \frac{I_c}{V_T} = \frac{10^{-3}}{26 \times 10^{-3}\ \Omega} = \frac{1}{26\ \Omega}$$

$$r_\pi = \beta / g_m = 52\ \Omega$$