University of California, Berkeley Physics

137A Spring 2011
MidTerm Exam I
Maximum score: 100 points

1. (50 points)

Considering the one-dimensional problem of a particle of mass m in a potential

$$V = 0 \qquad x < -a$$
$$V = -\frac{40\hbar^2}{ma^2}, \quad -a < x < 0$$
$$V = \infty \qquad x > 0$$

(1) Sketch the potential and the first 3 energy eigenfunctions.

(2) Solve symbolically the energy eigenvalues and eigenfunctions.

(3) How many bound states exist in this potential?

(4) Now considering a scattering state coming from $-\infty$ with energy of $\frac{\hbar^2}{ma^2}$. What will the reflectivity?

2. (50 points)

A Schrodinger equation in one dimension reads $-\partial^2\psi/\partial x^2 - 2\sec h^2 x \cdot \psi = E\psi$. (We have set $\hbar$=1 and m=1/2.)

(1) Show that exp(ikx)(tanhx+cons) is a solution of the Schrodinger equation for a particular value of constant. Calculate the transmission and reflection coefficients.

(2) Using analytic extension of the transmission function of (1) to find the eigenenergy and eigenfunction of one bound state in this potential.

(3) Sketch the wavefunction you found in (2). Give a simple argument that it must be the ground state of the potential.

Definition: $\sec h(x) \equiv \frac{2}{e^x + e^{-x}}, \tanh(x) \equiv \frac{e^{2x} - 1}{e^{2x} + 1}$.