Danelle

Rubric Ramesh problem 1: (Out of 20)

a) 2 pts Newton's Second Law (1 pt for using it at all) (2 pts using it correctly)
2 pts Correct FBD
2 pts proper use of the centripetal acceleration
1 pt ~~Clearly drawn (or at least indicated) axes~~ or something consistency
1 pt proper unit conversion
2 pts correct angle (solution method, based on what they have) (correct analyt. formula → half credit (1 pt))

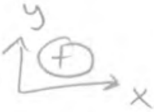
10 pts

b) 2pts correct FBD
2pts Newton's Second Law (half credit for writing it full for using correctly)
1 pt correct use of centripetal acceleration
1 pt For max speed, need max $F_s \Rightarrow F_s = \mu_s F_N$
2 pts correct dry road velocity
2 pts correct wet road velocity

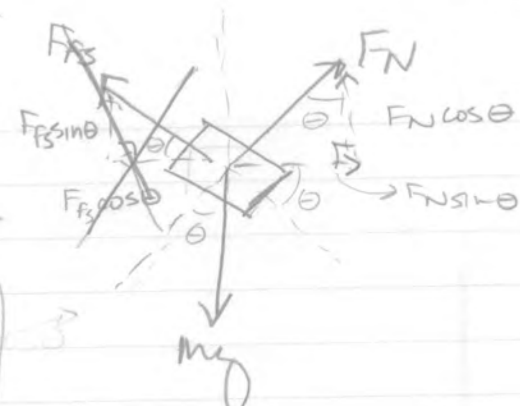
10 pts

* adjustment - if analytic formula is given and correct (for v and a calculational error results in inaccurate velocity, I'll only take of 1 point. (i.e. $v_{analytic}$ = 3 ... numbers = 1/2 each ...

Problem 1



a)



$$\boxed{\begin{aligned}\Sigma F_x &= ma_c = \frac{mv^2}{R} = F_N\sin\theta \\ \Sigma F_y &= 0 = F_N\cos\theta - mg \\ F_{Fs} &= 0\end{aligned}}$$

$(F_{Fs} = 0)$

$$\Rightarrow \quad \Sigma F_x = \frac{mv_c^2}{R} = F_N \sin\theta$$

$$\Sigma F_y = 0 = F_N\cos\theta - mg$$

$$F_N = \frac{mg}{\cos\theta}$$

$$\Rightarrow \frac{mv_c^2}{R} = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta$$

$$\boxed{v_c^2 = \frac{R\cancel{m}g\tan(\theta)}{\cancel{m}} = Rg\tan\theta}$$

$R = 200\,m$

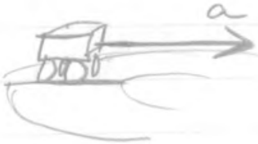
$$V = \frac{90\ km}{hr} \Rightarrow \frac{90\ \cancel{km}}{1\ \cancel{hr}}\left|\frac{1\ \cancel{hr}}{3600\,s}\right|\frac{1000\,m}{1\ \cancel{km}} = \sim 25\ m/s$$

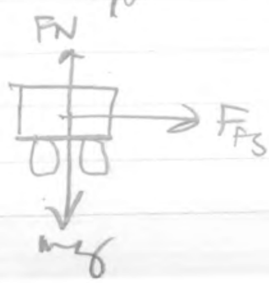
$$\boxed{\theta = \arctan\left(\frac{v_c^2}{Rg}\right) = \arctan\left(\frac{(25)^2}{(200)(9.8)}\right) = 17^\circ}$$

b) Top down view:

$R = 73$
$\theta = 90^\circ$

Front view





$(\longrightarrow \vec{a})$

Problem 1

Road dry: $\Sigma F_x = ma_c = F_{FS} = \frac{mv^2}{R}$

$F_S \le \mu_s F_N$
For the max, set $F_S = \mu_s F_N$.

$\Sigma F_y = 0 = F_N - mg$

$\Rightarrow F_N = mg$
$\Rightarrow F_{FS} = \mu_s \cancel{m} g = \frac{\cancel{m} v^2}{R} \Rightarrow R\mu_s g = v^2$

When the road is dry, $\mu_s = 0.88 \Rightarrow v = \sqrt{(73)(0.88)(9.8)} = 25.1$ m/s

When the road is snow covered, $\mu_s = 0.21 \Rightarrow v = \sqrt{(73)(0.21)(9.8)} = 12.3$ m/s

Problem 3

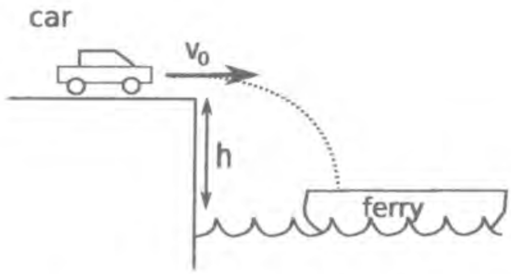


Figure 1: A car drives off a cliff of height h and lands without bouncing on a ferry.

A car with initial velocity v_0 drives off a cliff, lands on a ferry, and then skids to a stop (coefficient of kinetic friction is μ_k). Assume there is no friction between the ferry and the water and that the ferry starts at rest.

a) What is the speed of the car right before it lands?

b) After the car comes to a complete stop, what is the velocity of the car+ferry system?

c) How far (relative to the ferry) does the car skid before coming to a complete stop?

a.) Energy is conserved when the car flies off the cliff. (4)

$$E_0 = mgh + \tfrac{1}{2} m_c V_0^2$$
$$E_f = \tfrac{1}{2} m_c V_f^2$$
$$E_0 = E_f$$
$$m_c g h + \tfrac{1}{2} m_c V_0^2 = \tfrac{1}{2} m_c V_f^2$$
$$2gh + V_0^2 = V_f^2$$
$$\boxed{V_f = \sqrt{V_0^2 + 2gh}}$$

b.) We can consider this to be a perfectly inelastic collision after the car stops. (8)
Since there are no external forces in the x direction, the x momentum is conserved.

$$m_c V_0 = (m_f + m_c) V_{cf}$$
$$\boxed{V_{cf} = \frac{m_c V_0}{m_f + m_c}}$$

c.) We can use work kinetic energy theorem to solve this. The only work done (8)
after the collision is by kinetic friction.

$$-(\mu_k m_c g) d = \tfrac{1}{2} m_c V_{cf}^2 - \tfrac{1}{2} m_c V_0^2$$
$$-\mu_k g d = \frac{\left(\frac{m_c}{m_f+m_c}\right)^2 V_0^2 - V_0^2}{2}$$

$$\boxed{d = \frac{-\left(\frac{m_c}{m_f+m_c}\right)^2 V_0^2 + V_0^2}{2\mu_k g}}$$

Problem 4 Rubric:

<u>concepts</u>:

+5 → set forces equal in (a)

+5 → use cons. of energy correctly (b)

+5 → recognize where PE minimum (b)

+2 → algebra on (a)

+3 → algebra on (b)

(a)

equil.

$F = kx$; $x = 17cm$

FBD:

$$\Sigma F_y = 0 = -kx + mg$$

$$\Rightarrow \quad mg = kx$$

$$k = \frac{mg}{x} = \frac{(1kg)(9.8\ m/s^2)}{(.17m)}$$

$$k = \boxed{57.64\ \frac{N}{m}}$$

(b) what is max kinetic energy?

$$E = \frac{1}{2}mv^2 + \frac{1}{2}ky^2 + m_2gy$$

when potential energies minimum, KE is greatest ; energy is conserved

$$E_{y=-42cm} = \frac{1}{2}(2kg)(0 m/s)^2 + \frac{1}{2}(58 N/m)(-.42m)^2 + (2.0kg)(9.8m/s^2)(-.42m)$$
$$= -3.1 J$$

when force zero, potential energy is minimum (force related to derivative on potential energy)

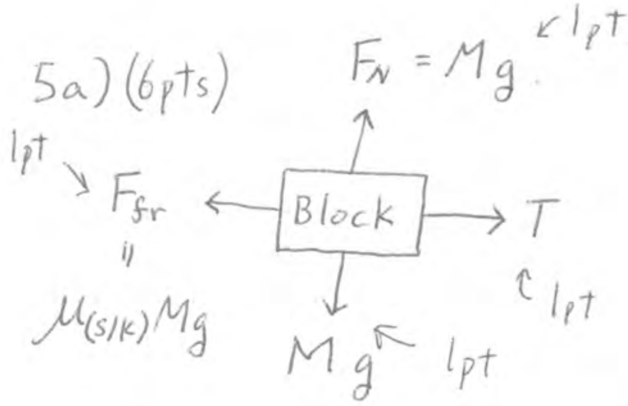
call this "y_3" height

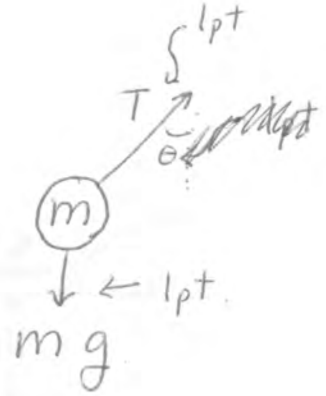
$$\Rightarrow \quad -ky_3 - m_2 g = 0$$
$$y_3 = -\frac{m_2 g}{k} = \boxed{-.34m}$$

$$K_{max} = -\frac{1}{2}ky_3^2 - m_2 g y_3 + E$$

$$\boxed{K_{max} = .21 J}$$

5a) (6pts)





5b) (10pts)

$$\mu_s Mg \leq T \quad (3pts)$$

$$T\sin\theta = \frac{mV^2}{l\sin\theta} \quad (3pts)$$

$$T = \frac{mV^2}{l\sin^2\theta}$$

$$T\cos\theta = mg \quad (3pts)$$

$$* \quad \cos\theta = \frac{mg}{T}$$

$$\cos^2\theta = \left(\frac{mg}{T}\right)^2$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - \left(\frac{mg}{T}\right)^2$$

$$T = \frac{mV^2}{l\left(1-\left(\frac{mg}{T}\right)^2\right)}$$

$$\boxed{\text{min } T \text{ required} = \mu_s Mg}$$

$$T = \frac{mV^2}{l\left(1-\left(\frac{mg}{\mu_s Mg}\right)^2\right)}$$

$$T = \frac{mV^2}{l\left(1-\left(\frac{m}{\mu_s M}\right)^2\right)}$$

$$\mu_s Mg = \frac{mV^2}{l\left(1-\left(\frac{m}{\mu_s M}\right)^2\right)}$$

$$l\left(\mu_s Mg - \frac{m^2 g}{\mu_s M}\right) = mV^2$$

$$\boxed{\sqrt{\frac{l\mu_s Mg}{m} - \frac{lmg}{\mu_s M}} = V} \quad (1pts)$$

$$* \quad \theta = \cos^{-1}\left(\frac{mg}{\mu_s Mg}\right)$$

$$\mu_s Mg = \frac{mv^2}{l\sin^2\left(\cos^{-1}\left(\frac{m}{\mu_s M}\right)\right)}$$

$$\boxed{\sqrt{\frac{l\mu_s Mg}{m}\sin^2\left(\cos^{-1}\left(\frac{m}{\mu_s M}\right)\right)} = V}$$

(1pts)