ME 132, Spring 2003, Quiz # 1

# 1	# 2	# 3	# 4	# 5	# 6	TOTAL
8	5	12	10	15	10	60

1. (a) What is the general form of the solution to the differential equation

$$\ddot{x}(t) + 5\dot{x}(t) + 6x(t) = 0$$

   (b) What is the general form of the solution to the differential equation

$$\ddot{x}(t) + 5\dot{x}(t) + 6x(t) = 12$$

   Your expressions both should have two free constants.

2. Consider the complex number

$$\gamma = \frac{3-4j}{8+6j}$$

   (a) Determine $|\gamma|$

   (b) Determine $\angle\gamma$

3. 12 different input(u)/output(y) systems are given below. The unit-step response, starting from <u>zero</u> initial conditions at $t = 0^-$, are shown. Match the system with the step response.

   (a) $\ddot{y}(t) + 8.4\dot{y}(t) + 36y(t) = -12\dot{u}(t) + 36u(t)$

   (b) $\ddot{y}(t) + 0.4\dot{y}(t) + y(t) = 5\dot{u}(t) + u(t)$

   (c) $\ddot{y}(t) + 0.4\dot{y}(t) + y(t) = 4\dot{u}(t) - u(t)$

   (d) $\ddot{y}(t) + 2\dot{y}(t) + 25y(t) = -8\dot{u}(t) + 25u(t)$

   (e) $\ddot{y}(t) + 8.4\dot{y}(t) + 36y(t) = -36u(t)$

   (f) $\ddot{y}(t) + 1.4\dot{y}(t) + y(t) = -5\dot{u}(t) - u(t)$

   (g) $\ddot{y}(t) + 0.4\dot{y}(t) + y(t) = -4\dot{u}(t)$

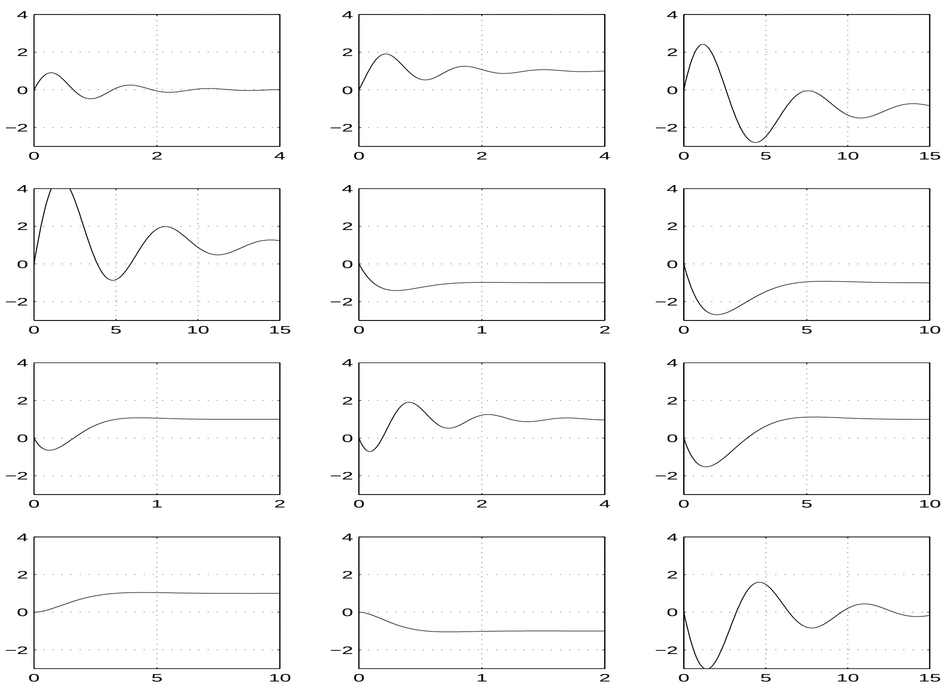
   (h) $\ddot{y}(t) + 8.4\dot{y}(t) + 36y(t) = -12\dot{u}(t) - 36u(t)$

   (i) $\ddot{y}(t) + 1.4\dot{y}(t) + y(t) = -4\dot{u}(t) + u(t)$

   (j) $\ddot{y}(t) + 2\dot{y}(t) + 25y(t) = 6\dot{u}(t) + 25u(t)$

   (k) $\ddot{y}(t) + 1.4\dot{y}(t) + y(t) = u(t)$

(l) $\ddot{y}(t) + 2\dot{y}(t) + 25y(t) = 6\dot{u}(t)$



4. A process, with input u, and output y, is governed by the equation

$$\ddot{y}(t) + \dot{y}(t) + y(t) = u(t)$$

A PI (Proportional plus Integral) controller is proposed

$$\begin{aligned} u(t) &= K_P\left[r(t) - y(t)\right] + K_I z(t) \\ \dot{z}(t) &= r(t) - y(t) \end{aligned}$$

Here r is a reference input.

(a) Using the controller equations, express $\dot{u}(t)$ in terms of $r, \dot{r}, y$ and $\dot{y}$.

(b) By differentiating the process equation, and substituting, derive the closed-loop differential equation relating r and y (there should be no u in the equation).

(c) Using the 3rd order test for stability, determine the conditions on K_P and K_I such that the closed-loop system is stable.

5. A process, with input u, disturbance d and output y is governed by

$$\dot{y}(t) = 2y(t) + 3u(t) + d(t)$$

(a) Is the process stable?

(b) Suppose $y(0) = 1$, and $u(t) = d(t) \equiv 0$ for all $t \geq 0$. What is the solution $y(t)$ for $t \geq 0$.

(c) Consider a proportional-control strategy, $u(t) = K_1 r(t) + K_2 [r(t) - y(t)]$. Determine the closed-loop differential equation relating the variables (y, r, d).

(d) For what values of K_1 and K_2 is the closed-loop system stable?

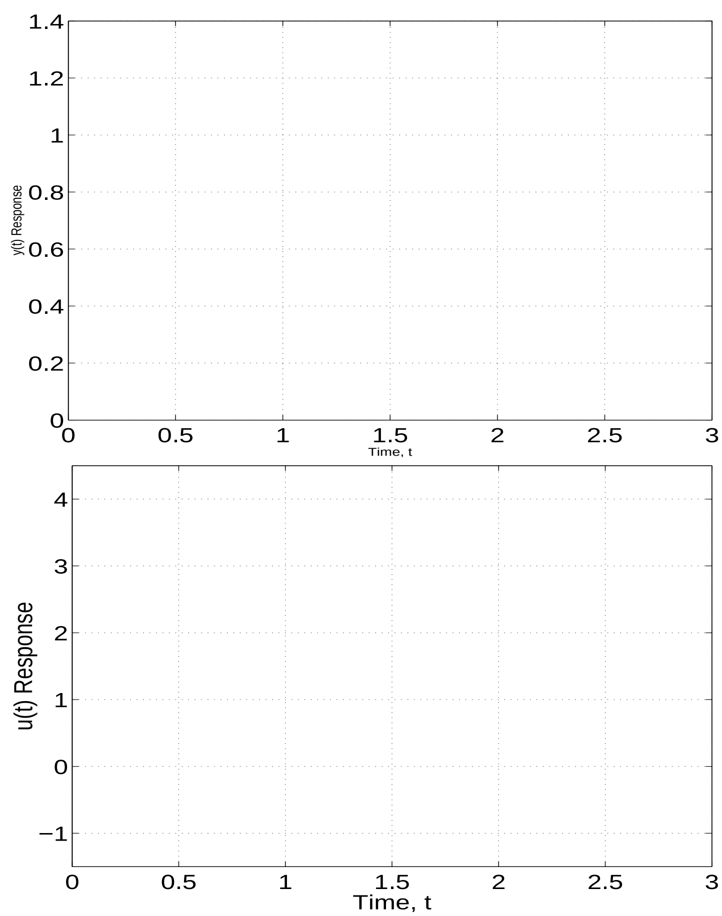
(e) As a function of K_2, what is the steady-state gain from $d \rightarrow y$ in the closed-loop system?

(f) As a function of K_1 and K_2, what is the steady-state gain from $r \rightarrow y$ in the closed-loop system?

(g) Choose K_1 and K_2 so that the steady-state gain from $r \rightarrow y$ equals 1, and the steady-state gain from $d \rightarrow y$ equals 0.1.

(h) With those gains chosen, sketch (try to be accurate) the two responses $y(t)$ and $u(t)$ for the following situation:

$$y(0) = 0, \quad r(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq 1 \\ 1 & \text{for } 1 < t \end{cases}, \quad d(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq 2 \\ 1 & \text{for } 2 < t \end{cases}$$



6. A 1st order process

$$\dot{y}(t) = u(t) + d(t)$$

is controlled by a proportional control

$$u(t) = K_P\left[r(t) - y_m(t)\right]$$

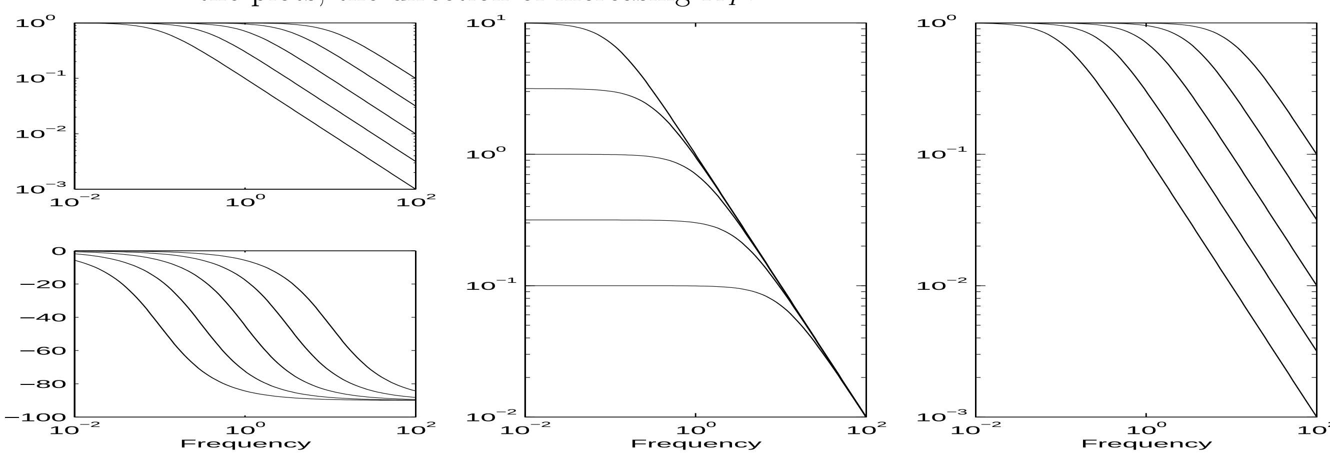
where $y_m(t) = y(t) + n(t)$. The interpretation of signals is: u is control input; y is process output; d is external disturbance on process; r is a reference input, representing a desired value of y; n is measurement noise.

(a) Eliminate u from the equations, and get the closed-loop differential equation relating (r, d, n) to y.

(b) Under what conditions on K_P is the closed-loop system stable?

(c) How is the time-constant of the closed-loop system related to K_P?

(d) Shown below are the closed-loop frequency responses from $(r, d, n) \rightarrow y$, as K_P increases from 0.1 to 10. Indicate on each graph with an arrow "cutting" across the plots, the direction of increasing K_P.



(e) Shown below are the closed-loop frequency responses from $(r, d, n) \rightarrow u$, as K_P increases from 0.1 to 10. Indicate on each graph with an arrow "cutting" across the plots, the direction of increasing K_P.