ME C85 / CE C30 Final Exam

Final words of wisdom:

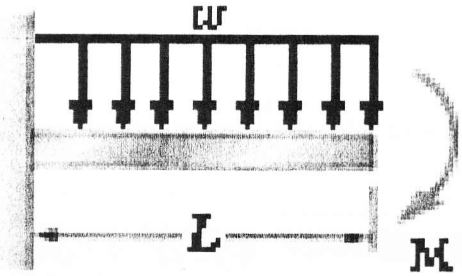
1. Read through the test before starting.
2. Be mindful of the time. Do not spend more than 45 minutes on any single problem.
3. At all costs, do not leave anything blank.
4. If you need more space, write on the back side of the page and make a note to that effect on the front side. *Do not detach or add any pages.*
5. Box your final answer and be wary of units.
6. Uphold academic integrity: don't cheat! The consequences are definitely not worth it.

Good luck!

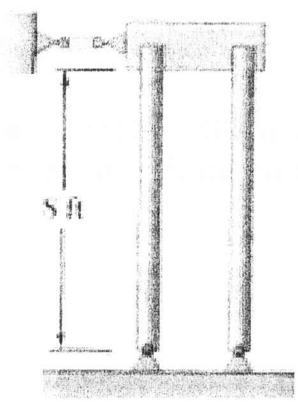
	Scored Points	Possible Points
Problem 1		25
Problem 2		32
Problem 3		23
Problem 4		20
Total		100

PROBLEM 1: 25pts (2.5pts each part)

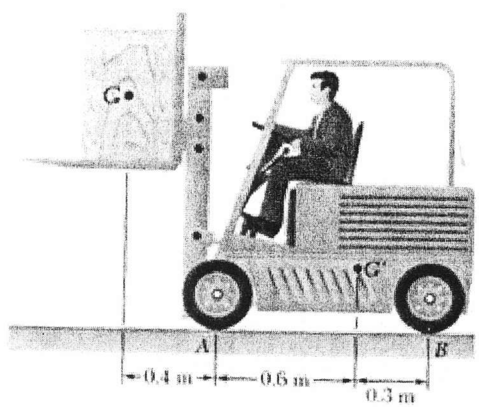
1. Sketch the shear force and bending moment diagrams for this beam problem.
   (Please provide your answer in the box under the sketch of the beam.)



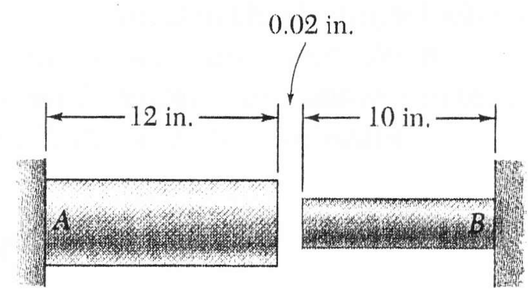
2. What effective length should be used in a buckling analysis of the columnar structure shown below?



3. The center of mass of this forklift is located at G'. Find the minimum weight of the crate that will cause the forklift to tip over given $W_{forklift}$ = 280 kN.



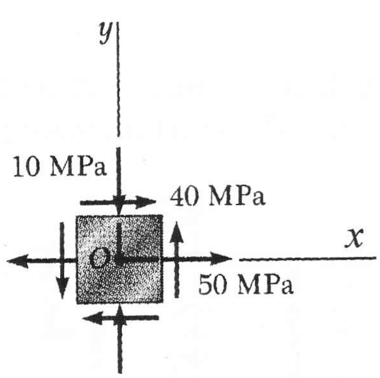
4. Find the change in temperature (same in each cylinder) that will cause the cylinders to expand just enough to fill the gap.



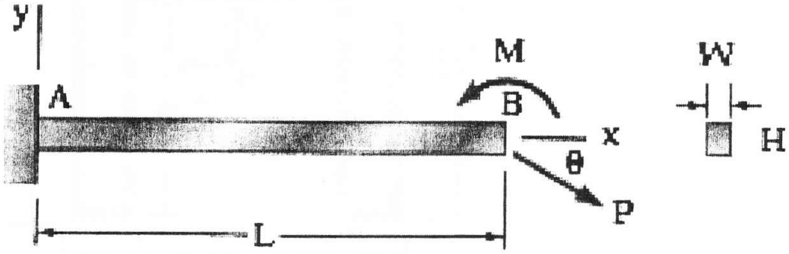
Aluminum
$A = 2.8 \text{ in}^2$
$E = 10.4 \times 10^6$ psi
$\alpha = 13.3 \times 10^{-6}/°C$

Stainless steel
$A = 1.2 \text{ in}^2$
$E = 28.0 \times 10^6$ psi
$\alpha = 9.6 \times 10^{-6}/°C$

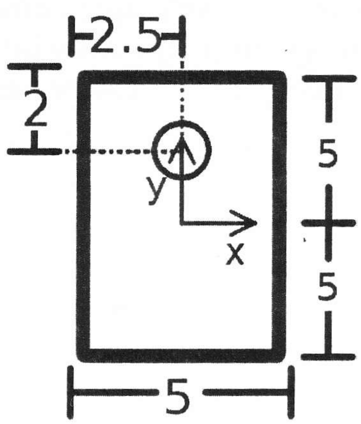
5. Sketch Mohr's Circle for the following stress state. From that, *estimate* the maximum principal stress, the minimum principal stress, and maximum shear stress.



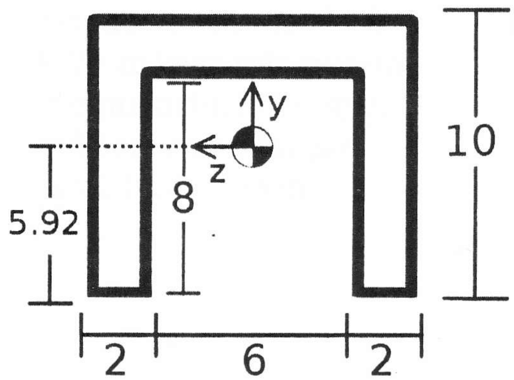
6. What is the total deflection in the x- and y-direction of point B of this beam, which has a Young's modulus E, rectangular cross-sectional geometry as shown, and which is subjected to the pure moment M and force P as shown? Express your answer in terms of the dimensions W, H, and L, and the loads M and P. and any other relevant parameters.



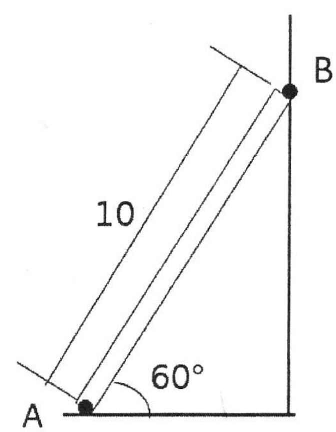
7. Find the centroid for this rectangular cross-section, with dimensions (in inches) as shown and hole of radius 1 inch.



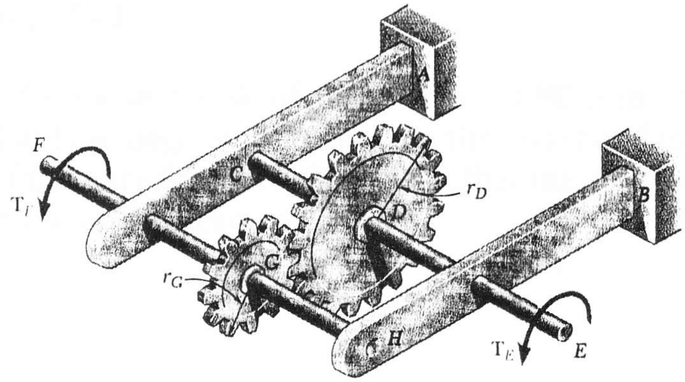
8. For this cross-section, calculate the moment of inertia I_z. The centroid is located 5.92 inches above the bottom edge, as shown.



9. A uniform rod of mass m is balanced between the ground and wall as shown, having coefficients of static friction $\mu_{s,} = 0.30$ for both ground and wall. We want to find the maximum mass m of the rod such that the rod won't slip. Toward that end, draw a free-body diagram and write out as many equations as there are unknowns for this problem. *Do not solve.*

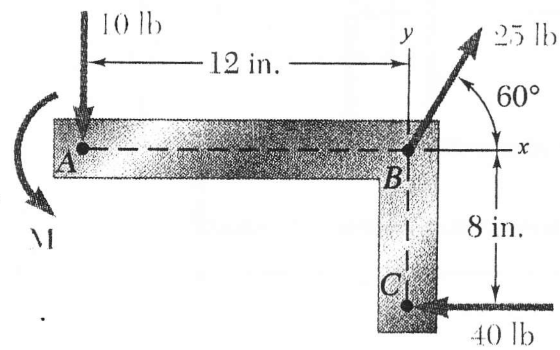


10. If an applied torque in shaft F is T_F = 1200 lb-in., as shown, find the reaction torque in shaft E, T_E, to keep this system in equilibrium. The gear radii are r_D = 8 in. and r_G = 3 in. as shown.

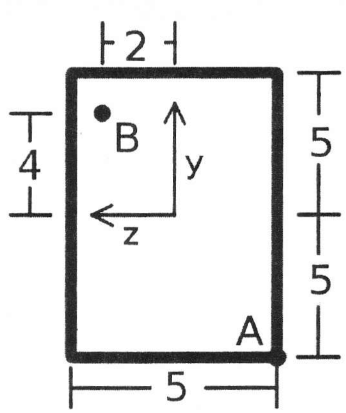


PROBLEM 2: 32 pts (8 pts each part)

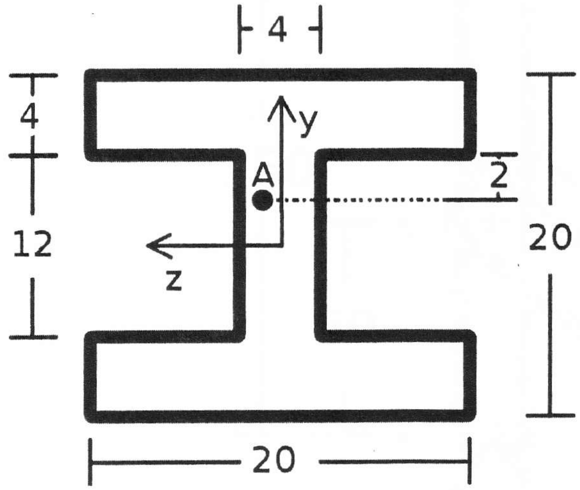
1. The three forces and a counter-clockwise couple of magnitude M = 80lb-in. are applied to an angle bracket as shown. Find the angle with respect to the x-axis and location on line AB (*i.e.*, the x-coordinate) of the pure force resultant such that the statically equivalent force-couple combination has a zero couple.



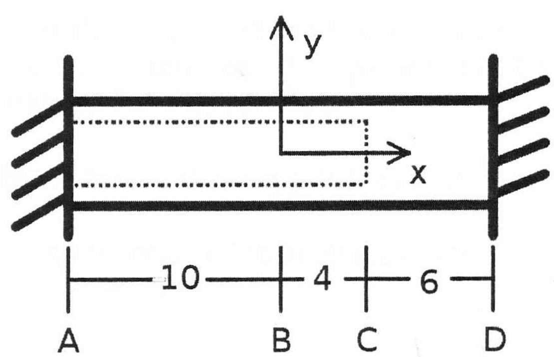
2. A compressive axial force P, acting into the page and of magnitude 1000 N, is applied at point B on the cross-section of a beam. Find the normal stress in the x-direction at point A, and indicate whether it is compressive or tensile. (All dimensions in mm.)



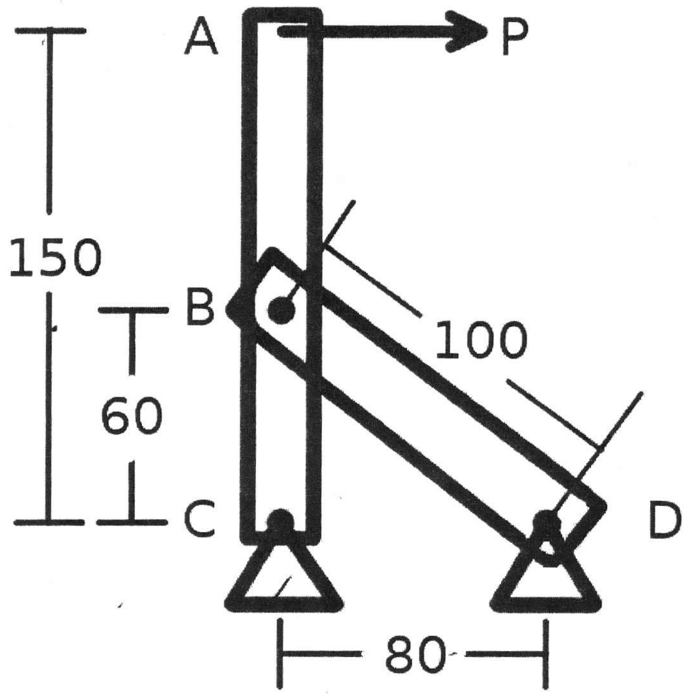
3. The cross-section of the beam shown here is subjected to a vertical shear force V of magnitude 1000 kN acting in the negative y-direction. For the cross-section, assume I_z = 11029 mm⁴. Find the transverse shear stress at point A.



4. A circular shaft of outer radius 100 mm is fixed into the wall at each end. From A to C, it is hollow, with a wall thickness of 10 mm; from C to D, it is solid. The shaft is subjected to a torque T = 1000 N-m at B. Find the reaction torque at end A. The shear modulus G for the shaft is 30 x 10³ MPa.



PROBLEM 3: 23 pts



In this structure, rigid bar AC is connected to deformable bar BD via a hinge joint at B, and both bars are pinned at their bases C and D, which cannot move. Horizontal force *P* is applied at A. All dimensions are in mm. All bars have rectangular cross-sections. For this problem:

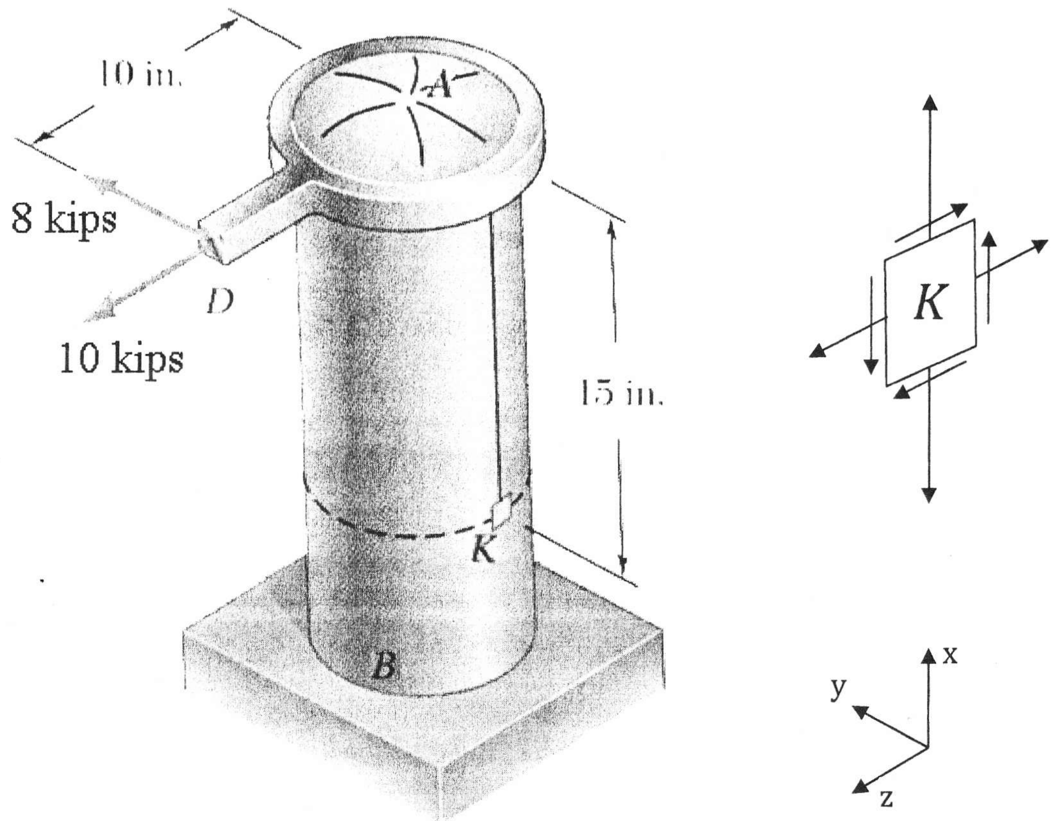
A) Find the compressive force acting on deformable bar BD in terms of the applied force P.

B) For P=1 kN, what is the necessary temperature change that is required to keep rigid bar ABC vertically upright? Use the material properties shown in the Table below.

C) The pins used in the hinge joints B and D have a diameter of 5 mm and fit snugly into corresponding drill holes in bar BD. Using the various properties shown in the Table below, what is the maximum allowable value of *P* such that failure of deformable bar BD is avoided? Hint: you need to consider more than one failure mode.

I (mm^4)	E (MPa)	σ_y (MPa)	α 1/(°C)	Thickness into the page (mm)	Cross-sectional area (mm^2)
833	70×10^3	50	22.2×10^{-6}	10	100

PROBLEM 4: 20 pts



The pressurized cylindrical tank AB has an 8-in. inner diameter and a 0.32-in. wall thickness. The internal pressure is 600psi. Two external forces of 8 and 10 kips are applied in the positive *y* and positive *z* directions, respectively, at the end of a rigid cantilever causing the pressure vessel to twist and bend.

a) Calculate the hoop and longitudinal stresses due only to the internal pressure loading of this pressure vessel.
b) At point *K* on the cylinder surface, the state of stress is planar since it is a free surface. Thus, the non-zero stresses are the normal stresses (σ_x and σ_z) and the shear stresses (τ_{xz} and τ_{zx}). Write out the equations for each of these stress components at point *K* when the external two external forces (see above) are applied in addition to the internal pressure. Hint: stresses can arise from internal pressure, and a combination of torsion, bending, and transverse shear loading.
c) For this combined loading condition, calculate the maximum principal (tensile) stress at point K.

MEC85 / CEC30 Final Exam Equation Sheet

Chapter 9: Stress and Strain - Axial Loading

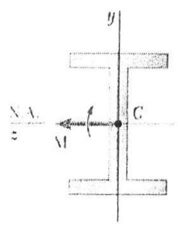
$$\sigma = E\epsilon \qquad \delta_L = \frac{PL}{AE} \qquad \delta_T = \alpha \Delta T L$$

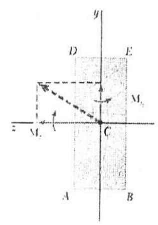
Chapter 10: Torsion

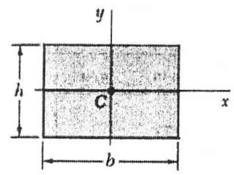
$$\phi = \frac{TL}{JG} \qquad \gamma = \frac{\rho\phi}{L} \qquad \tau = \frac{T\rho}{J}$$

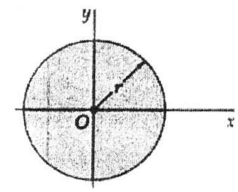
Chapter 11: Pure Bending

Parallel Axis Theorem: $I = I_{centroid} + A\bar{y}^2$









$$\sigma = \frac{My}{I} \qquad \sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} \qquad I = \frac{1}{12}bh^3 \qquad I = \frac{1}{4}\pi r^4,\ J_O = \frac{1}{2}\pi r^4$$

Chapter 12: Analysis and Design of Beams for Bending

$$-w = \frac{dV}{dx} \qquad V = \frac{dM}{dx}$$

Chapter 13: Shearing Stresses in Beams

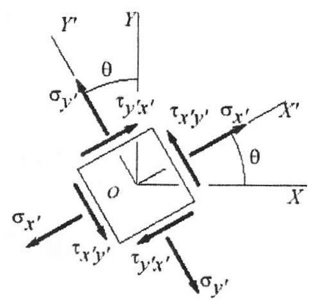
$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} \qquad \tau = \frac{VQ}{It} \qquad Q = A\bar{y}$$

Chapter 14: Transformation of Stress

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \tau_{max} \qquad \tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Thin-Walled Pressure Vessels: $\sigma_{hoop} = \frac{pr}{t}$ $\qquad \sigma_{longitudinal} = \frac{pr}{2t}$ $\qquad Q_{semi-circle} = \frac{2}{3}r^3$

Chapter 15: Deflection of Beams

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

APPENDIX C Beam Deflections and Slopes

Beam and Loading	Elastic Curve	Maximum Deflection	Slope at End	Equation of Elastic Curve
1		$-\frac{PL^3}{3EI}$	$-\frac{PL^2}{2EI}$	$y = \frac{P}{6EI}(x^3 - 3Lx^2)$
2		$-\frac{wL^4}{8EI}$	$-\frac{wL^3}{6EI}$	$y = -\frac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
3		$-\frac{ML^2}{2EI}$	$-\frac{ML}{EI}$	$y = -\frac{M}{2EI}x^2$
4		$-\frac{PL^3}{48EI}$	$\pm\frac{PL^2}{16EI}$	For $x \leq \frac{1}{2}L$: $y = \frac{P}{48EI}(4x^3 - 3L^2x)$
5		For $a > b$: $-\frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL}$ at $x_m = \sqrt{\frac{L^2 - b^2}{3}}$	$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL}$ $\theta_B = +\frac{Pa(L^2 - a^2)}{6EIL}$	For $x < a$: $y = \frac{Pb}{6EIL}[x^3 - (L^2 - b^2)x]$ For $x = a$: $y = -\frac{Pa^2b^2}{3EIL}$
6		$-\frac{5wL^4}{384EI}$	$\pm\frac{wL^3}{24EI}$	$y = -\frac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$
7		$\frac{ML^2}{9\sqrt{3}EI}$	$\theta_A = +\frac{ML}{6EI}$ $\theta_B = -\frac{ML}{3EI}$	$y = -\frac{M}{6EIL}(x^3 - L^2x)$

Chapter 16: Columns

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2}$$