UNIVERSITY OF CALIFORNIA **ME105/Fall 2002**

Mechanical Engineering **Prof.C.Fernandez-Pello**

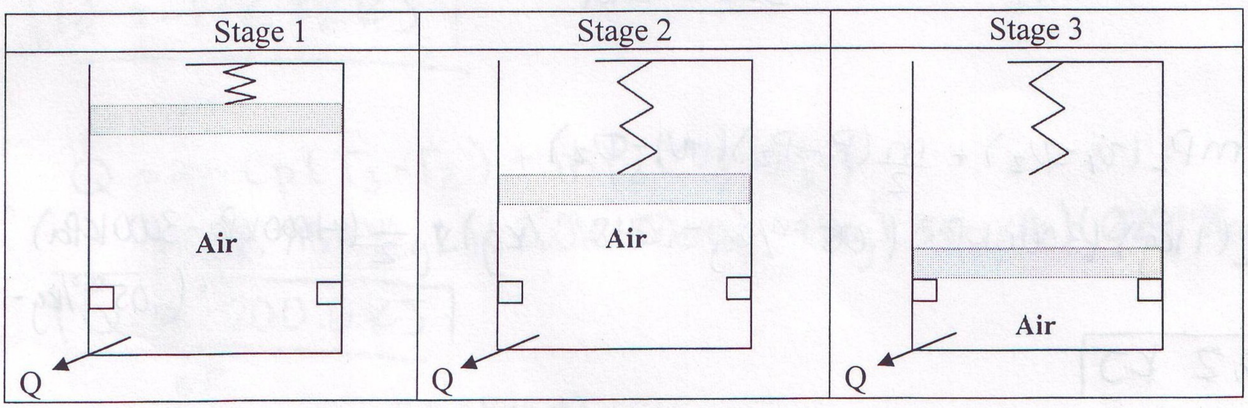
NAME Solution

This examination is open book and open notes. Both problems are equally weighted. Please write your name in the space provided above.

MIDTERM EXAMINATION 1

Question 1 (50 pts)

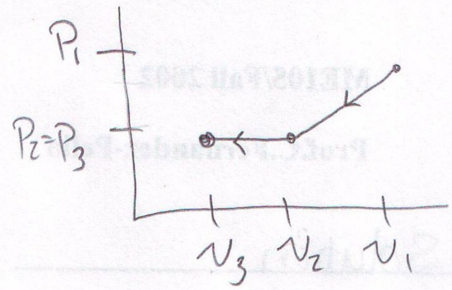
A piston-cylinder device as shown in the figure contains 1 kg of air. At stage 1, the piston is compressing a linear spring. At this point, the pressure is 4 MPa, the temperature is 700 K. Heat is removed from the system and the spring relaxes. Since the spring is not permanently attached to the piston, the piston looses contact with the spring. At the point where the spring is fully relaxed and has just lost contact with the piston, the pressure is 3 MPa and the temperature is 500 K (stage 2). Heat continues to be removed from the system until the piston has hit a set of stops (stage 3). At this point the temperature is 300 K.



a) Sketch the process in a Pv diagram (4 pts)

b) Find the work from stage 1 to 2 (23 pts)

c) Find the heat lost from stage 2 to 3 (23 pts) a)



b)

State 1

$P_1 = 4\ \text{MPa}$

$P_2 = 700\text{K}$

State 2

$P_2 = 3\ \text{MPa}$

$T_2 = 500\ \text{K}$

$R_{air} = .287\ \text{kJ/kgK}$

$$v_1 = \frac{RT_1}{P_1} = \frac{(.287\ \text{kJ/kgK})(700\text{K})}{4000\ \text{kPa}} = .05\ \text{m}^3/\text{kg}$$

$$v_2 = \frac{RT_2}{P_2} = \frac{(.287\ \text{kJ/kgK})(500\text{K})}{3000\ \text{kPa}} = .048\ \text{m}^3/\text{kg}$$

$$W = mP_2(v_1 - v_2) + \frac{m}{2}(P_1 - P_2)(v_1 - v_2)$$

$$= (1\text{kg})(3000\text{kPa})(.05\ \text{m}^3/\text{kg} - .048\ \text{m}^3/\text{kg}) + \frac{1}{2}(4000\text{kPa} - 3000\text{kPa}) \cdot (.05\ \text{m}^3/\text{kg} - .048\ \text{m}^3/\text{kg})$$

$$\boxed{W = -7.2\ \text{kJ}}$$

c)

state 2

$P_2 = 3\,MPa$ $\quad v_2 = .048\ m^3/kg$

$T_2 = 500K$

$u_2 = 359.49\ kJ/kg$ $\Big\}$ Table A-17

$h_2 = 503.02\ kJ/kg$

state 3

$T_3 = 300K$

$P_3 = P_2$

$u_3 = 217.67\ kJ/kg$

$h_3 = 300\ kJ/kg$

$v_3 = \dfrac{RT_3}{P_3} = .029\ m^3/kg$

$C_p = .718\ kJ/kgK$

$C_v = 1.005\ kJ/kgK$

$$Q - W = \Delta U + \Delta KE^{\to 0} + \Delta PE^{\to 0}$$

$$Q - W_b = \Delta U$$

$$Q = \Delta U + W_b$$

$$Q = m(u_3 - u_2) + mP_2(v_3 - v_2)$$

$$Q = (1kg)(217.67\ kJ/kg - 359.49\ kJ/kg) + (1kg)(3000\ kPa)(.029\ m^3/kg - .048\ m^3/kg)$$

$$\boxed{Q = -198.82\ kJ}$$

or

$$Q = mC_p(T_3 - T_2) + mP_2(v_3 - v_2)$$

$$= (1kg)(.718\ kJ/kgK)(300K - 500K) + (1kg)(3000\ kPa)(.029\ m^3/kg - .048\ m^3/kg)$$

$$\boxed{Q = -200.6\ kJ}$$

or

(when P=const)

$$Q = \Delta U + W_b = m(h_3 - h_2)$$

$$Q = (1kg)(300.19\ kJ/kg - 503.02\ kJ/kg)$$

$$\boxed{Q = -199.83\ kJ}$$

or

$$Q = mC_p(T_3 - T_2)$$

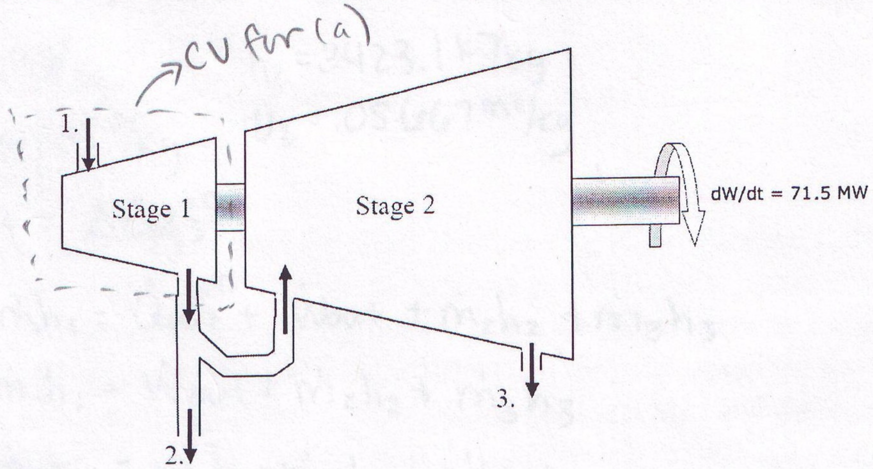
$$Q = (1kg)(1.005\ kJ/kgK)(300K - 500K)$$

$$\boxed{Q = -201\ kJ}$$

NAME Solution

Question 2 (50pts)

A steam turbine with two stages is fed with 50 kg/s of steam at T1=800 C and P1=10 MPa. The preheat stream leaves the turbine at point 2, with T2=500 C and P2=6 MPa. The exit of the second stage has a flow rate of 40 kg/s at P3=1 MPa. Changes in potential/kinetic energy are negligible. The system is adiabatic. The turbine produces 71.5 MW of power.



a) Calculate the work in the first stage. (25 pts)

b) Provide five properties at state 3. (25pts)

a) State 1

$T_1 = 800°C$

$P_1 = 10\ MPa$

$\dot{m}_1 = 50\ kg/s$

table A-6: $h_1 = 4114.5\ kJ/kg$, $v_1 = .048629\ m^3/kg$

State 2

$T_2 = 500°C$

$P_2 = 6\ MPa$

$\dot{m}_2 = \dot{m}_1$

$h_2 = 3423.1\ kJ/kg$, $v_2 = .05667\ m^3/kg$ (table A-6)

$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{sys} \rightarrow 0$

$\dot{Q}_{in}^{\,0} + \dot{W}_{in}^{\,0} + \dot{m}h_1 = \dot{Q}_{out}^{\,0} + W_{out} + \dot{m}h_2$

$W_{out} = \dot{m}h_1 - \dot{m}h_2 = \dot{m}(h_1 - h_2)$

$W_{out} = 50\ kg/s\ (4114.5\ kJ/kg - 3423.1\ kJ/kg)$

$\boxed{W_{out} = 34.57\ MW}$

b)

State 1	State 2	State 3
$T_1 = 800°C$	$T_2 = 500°C$	$P_3 = 1MPa$
$P_1 = 10MPa$	$P_2 = 6MPa$	$\dot{m}_3 = 40\ kg/s$
$\dot{m} = 50\ kg/s$		

$h_1 = 4114.5\ kJ/kg$ $\quad h_2 = 3423.1\ kJ/kg$

$v_1 = .0486629\ m^3/kg$ $\quad v_2 = .05667\ m^3/kg$

$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{sys} \nearrow 0$

$\dot{Q}_{in} \nearrow 0 + \dot{W}_{in} \nearrow 0 + \dot{m}_1 h_1 = \dot{Q}_{out} \nearrow 0 + \dot{W}_{out} + \dot{m}_2 h_2 + \dot{m}_3 h_3$

$\dot{m}_1 h_1 = \dot{W}_{out} + \dot{m}_2 h_2 + \dot{m}_3 h_3$

$\dot{m}_3 h_3 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{W}_{out}$ $\qquad \dot{m}_2 = \dot{m}_1 - \dot{m}_2 = 10\ kg/s$

$$h_3 = \frac{\dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{W}_{out}}{\dot{m}_3}$$

$$h_3 = \frac{(50\ kg/s)(4114.5\ kJ/kg) - (10\ kg/s)(3423.1\ kJ/kg) - 71.5\ MW}{40\ kJ/kg}$$

$h_3 = 2500\ kJ/kg$

$$x_3 = \frac{h_3 - h_f}{h_{fg}} = .8624$$

Table A-5 $\left\{ \begin{array}{l} u_3 = u_f - x_3 u_{fg} = 2332\ kJ/kg \\ v_3 = v_f - x_3 v_{fg} = .16777\ m^3/kg \end{array} \right.$

Properties of State

$P_3 = 1MPa$

$T_3 = 179.88°C$ (A-5)

$h_3 = 2500\ kJ/kg$

$u_3 = 2332\ kJ/kg$

$v_3 = .16777\ m^3/kg$

$x_3 = .8627$

(any 5 of these 6 answer the question)