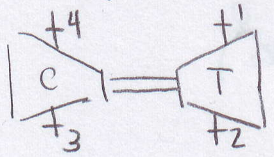
UNIVERSITY OF CALIFORNIA
Mechanical Engineering

ME105/Fall01 Combustion Processes
Fernandez-Pello

Name Solution

MIDTERM EXAMINATION #2 (11/13/00)

1. In an air turbine/compressor unit, the turbine extracts work from the working fluid and transfer it to the compressor. The air enters the turbine at 500 C, and 2 MPa and exits it at 100 KPa. The air enters the compressor at 20 C and 100 KPa. The kinetic and potential energy through the compressor and turbine can be neglected. Considering that the turbine has an isentropic efficiency of 90% and the compressor of 80%, calculate the pressure at the compressor exit.



$\eta_c = .8 \qquad \eta_t = .9$

State 1: $T_1 = 500°C = 773K$
$P_1 = 2MPa$

State 2: $P_2 = 100kPa$ (for 800K, $k = 1.354$)

$$T_{2,s} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 773K\left(\frac{2000kPa}{100kPa}\right)^{\frac{1.354-1}{1.354}} = 1692$$

iterate: $\frac{773K + 1692}{2} = 1232K \rightarrow$ use k @ 1000K

$$T_{2,s} = 773K\left(\frac{2000kPa}{1000}\right)^{\frac{1.336-1}{1.336}}$$

$T_{2,s} = 1642$

State 3: $T_3 = 20°C = 293K$
$P_3 = 100kPa$

Turbine

$$\dot{W}_{in}^{\,0} + \dot{Q}_{in}^{\,0} + \dot{m}h_1 = \dot{W}_{out} + \dot{Q}_{out}^{\,0} + \dot{m}h_2$$

$$\frac{\dot{W}_{out}}{\dot{m}} = \frac{\dot{m}(h_1 - h_2)}{\dot{m}}$$

$$w_{out} = h_1 - h_2 = C_p(T_1 - T_{2,s}) = (1.142)(773K - 1642) = -992\ kJ$$

(C_p for 1000K)

Compressor

$$\eta_t w_T = \eta_c w_C \rightarrow \frac{\eta_t}{\eta_c} w_T = w_C = \left(\frac{.9}{.8}\right)(992kJ) = 1116\ kJ$$

$$\dot{W}_{in} + \dot{Q}_{in}^{\,0} + \dot{m}h_3 = \dot{W}_{out}^{\,0} + \dot{Q}_{out}^{\,0} + \dot{m}h_4$$

$$\frac{\dot{W}_{in}}{\dot{m}} = \frac{\dot{m}(h_4 - h_3)}{\dot{m}}$$

$$w_{in} = h_4 - h_3 = C_p(T_{4,s} - T_3)$$

(Problem 1 cont.)

$$\frac{W_{in}}{C_p} - T_3 = T_{4,s} = \frac{1116\,kJ}{1.005} - 293K = 817K$$

C_p → C_p for 300K

iterate: $\frac{300K + 817K}{2} = 558.5$ → use c_p for 550K

$$T_{4,s} = \frac{1116\,kJ}{1.040} - 293K = \underline{780K}$$

$$\left(\tfrac{k}{k-1}\right) \frac{T_{4,s}}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} \left(\tfrac{k}{k-1}\right)$$

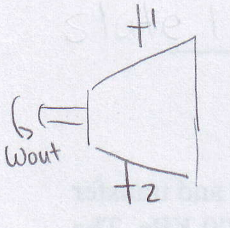
$$\left(\frac{T_{4,s}}{T_3}\right)^{\frac{k}{k-1}} = \frac{P_4}{P_3} \rightarrow P_4 = P_3 \left(\frac{T_{4,s}}{T_3}\right)^{\frac{k}{k-1}} = (100\,kPa)\left(\frac{780K}{293K}\right)^{\frac{1.040}{1.040-1}}$$

(k ← c_p for 550K)

$$\boxed{P_4 = 11373\ GPa}$$

2. Steam enters an adiabatic turbine at 7MPa, 600C and 80m/s and leaves at 50 kPa, 150C, and 140 m/s. If the power output of the turbine is 6 MW, find the following

a) the mass flow rate of the stem flowing through the turbine
b) the isentropic efficiency of the turbine



State 1: $V_1 = 80\ m/s$, $P_1 = 7MPa$, $T_1 = 600°C$ $\}$ $h_1 = 3650.6\ kJ/kg$, $s_1 = 7.0910\ kJ/kgK$ A-6

State 2: $P_2 = 50\ kPa$, $T_2 = 150°C$, $V_2 = 140\ m/s$ $\}$ $h_{2,a} = 2780.2\ kJ/kg$, $s_{2,a} = 7.9413\ kJ/kgK$ A-6

a) Find $\dot{m}$

$E_{in} = E_{out}$

$$\dot{Q}_{in}^{\,0} + \dot{W}_{in}^{\,0} + \dot{m}(h_1 + \frac{V_1^2}{2} + g z_1^{\,0}) = \dot{Q}_{out}^{\,0} + \dot{W}_{out} + \dot{m}(h_1 + \frac{V_2^2}{2} + g z_2^{\,0})$$

$$\dot{m}(h_1 + \frac{V_1^2}{2}) = \dot{W}_{out} + \dot{m}(h_2 + \frac{V_2^2}{2})$$

$$\dot{m}(h_1 + \frac{V_1^2}{2} - h_2 - \frac{V_2^2}{2}) = \dot{W}_{out}$$

$$\dot{m} = \frac{\dot{W}_{out}}{(h_1 - h_2 + \frac{V_1^2}{2} - \frac{V_2^2}{2})} = \frac{6000\ kW}{(3650.6\ kJ/kg - 2780.2\ kJ/kg + \frac{(80^2\ m/s - 140^2\ m/s)}{2}(\frac{1\ kJ/kg}{1000\ m^2/s^2}))}$$

$$\boxed{\dot{m} = 6.95\ kg/s}$$

b) Find η_T

$\eta_T = \frac{W_s}{W_a}$

State 2 (isentropic): $P_2 = 50\ kPa$, $s_1 = s_2$ $\}$ $x_{2s} = \frac{s_{2s} - s_f}{s_{fg}}$

$x_{2s} = \frac{7.0910 - 1.0912}{6.5019} = .9228$

$h_{2s} = h_f + x_{2s} h_{fg}$
$= 340.54 + (.9228)(2304.7)$
$h_{2,s} = 2467.3\ kJ/kg$

$$\dot{Q}_{in}^{\,0} + \dot{W}_{in}^{\,0} + \dot{m}(h_1 + \frac{V_1^2}{2} + g z_1^{\,0}) = \dot{Q}_{out}^{\,0} + \dot{W}_{out} + \dot{m}(h_2 + \frac{V_2^2}{2} + g z_2^{\,0})$$

$$\dot{W}_{out,s} = \dot{m}(h_1 - h_{2,s} + \frac{V_1^2 - V_2^2}{2})$$

$$\dot{W}_{out,s} = (6.95\ kg/s)(3650.6\ kJ/kg - 2467.3\ kJ/kg + \frac{80^2\ m/s - 140^2\ m/s}{2}(\frac{1\ kJ/kg}{1000\ m^2/s^2}))$$

$$\dot{W}_{out,s} = 8178\ kJ$$

$$\eta_T = \frac{\dot{W}_a}{\dot{W}_s} = \frac{6000\ kJ}{8178\ kJ} = \boxed{.73}$$