Name: ______________________________

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EECS 121 — MIDTERM

Problem 1 (10 points) — True or False

Scoring= +2 for correct answer with explanation
+1 for correct answer without explanation
–1 for incorrect answer
0 if left blank

a) $E(X+Y) = E(X) + E(Y)$ **only** if the random variables X and Y are independent.

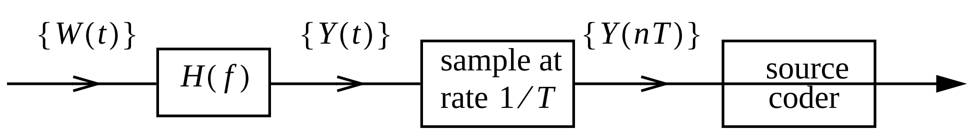
b) $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$ if X and Y are uncorrelated.

c) If X and Y are uncorrelated discrete-valued random variables, then $P(X = x | Y = y) = P(X = x)$ for all y and x.

d) Let X be a random variable and consider the process $Y(t) = Xt$ for all t. Then $\{Y(t)\}$ is strict sense stationary.

e) Let X, Y, Z be 3 random variables defined on the same probability space. Suppose X and Y are pairwise independent, Y and Z are pairwise independent and X and Z are pairwise independent. Then X, Y, Z are independent random variables.

Problem 2 (15 points)



Consider the system above. $\{W(t)\}$ is a zero-mean white Gaussian process with power spectrum density $\frac{N_0}{2}$. $H(f)$ is a stable LTI filter.

[2 pts.] **a)** What is the power of $\{W(t)\}$, $E[|W(t)|^2]$?

[2 pts.] **b)** What is the power spectrum density of the output $\{Y(t)\}$ in terms of $H(f)$?

[2 pts.] **c)** If $H(f) = \begin{cases} 1 & \text{if } |f| < W \\ 0 & \text{else} \end{cases}$, then what is the average power in the output $E[|Y(t)|^2]$?

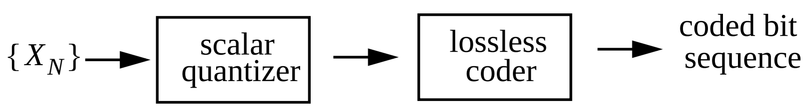
[2 pts.] **d)** Suppose we sample the output $\{Y(t)\}$ at rate $\frac{1}{T} = 2W$ samples per second. What is the auto-correlation function of the discrete-time process $\{Y(nT)\}$? Is the process Gaussian?

[2 pts.] **e)** Suppose we now sample the process at rate $\frac{1}{T} = 4W$. What is the auto-correlation function of the sampled process?

[5 pts.] **f)** We now want to quantize the samples $\{Y(nT)\}$ using DPCM coding based on linear prediction of the current sample from the previous sample. For the two cases above (i.e., sampling rates $2W$ and $4W$), find the optimal linear predictor. Is there any benefit of doing DPCM over PCM?

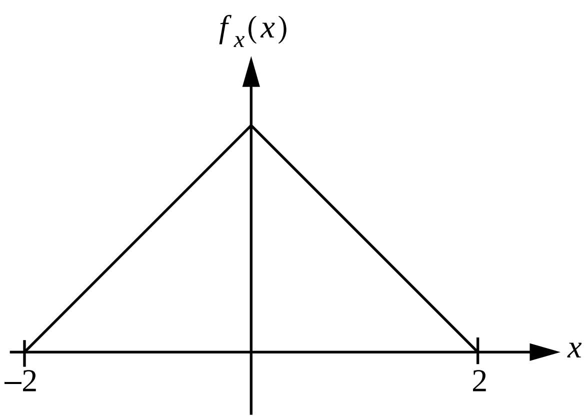
Problem 3 (13 points)

Consider a source coder with a scalar quantizer followed by a lossless coder.



Each sample X_n is scalar quantized, and the levels are then represented by a possibly variable length binary code. The codewords are then transmitted.

Suppose $\{X_n\}$ is stationary and each sample has the following density:

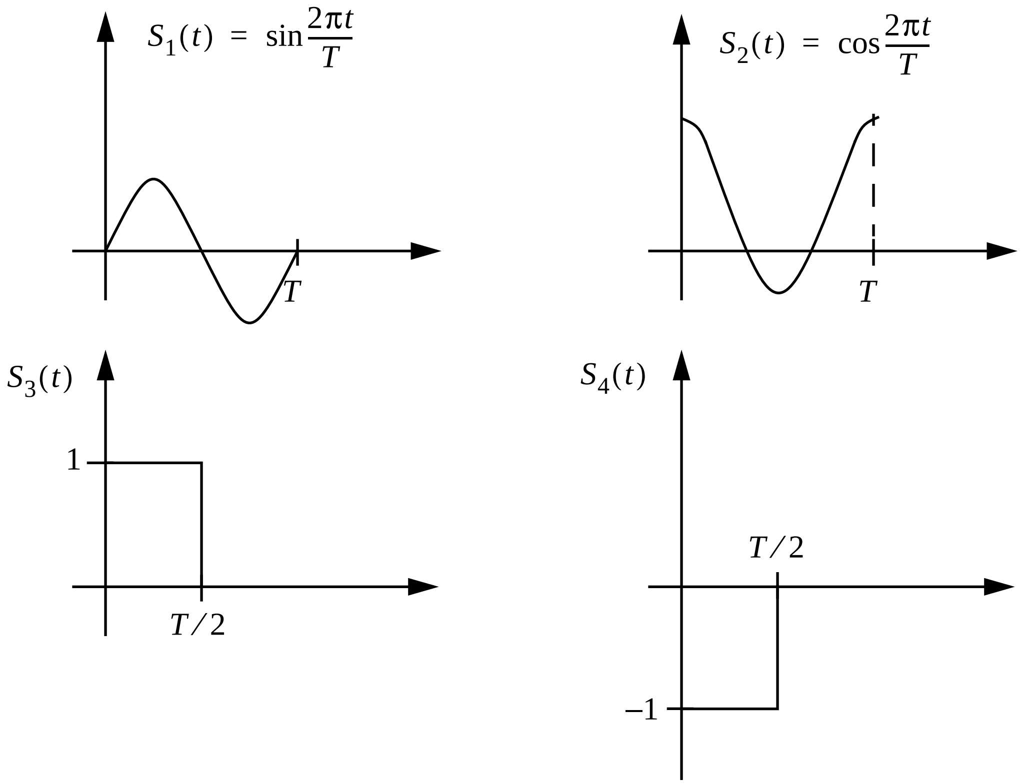


[7 pts.] **a)** The levels of the quantizers are chosen to be at $\pm\frac{1}{3}, \pm\frac{5}{3}$. Calculate the SQNR.

[6 pts.] **b)** Find the optimal lossless coder to minimize the average bit rate of the coded stream. What is the minimum average bit rate?

Problem 4 (12 points)

Consider a modulation system with four signal waveforms:



The signals are transmitted over an AWGN channel, $Y(t) = s_m(t) + W(t)$.

[7 pts.] **a)** Find an orthonormal basis for the signal waveforms and hence design a decorrelator to extract a set of sufficient statistics for optimal detection. How many sufficient statistics are needed?

[5 pts.] **b)** Suppose the received signal is projected onto the waveforms $s_1(t), s_2(t), s_3(t), s_4(t)$ instead. Do these four components provide sufficient information for optimal detection of the transmitted waveform? Why?