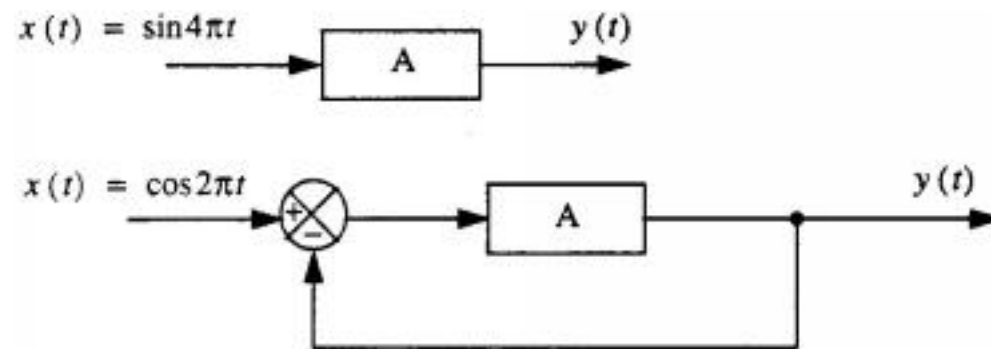


## EECS 121 -- MIDTERM 1 (Closed book and notes)

1. Consider a linear and time-invariant system (denoted by  $A$ ) for which an input  $x(t) = \cos 2(\pi)ft$  produces an output

$$y(t) = 1 / (1 + (2(\pi)f)^2) * (\cos 2(\pi)ft + (2(\pi)f)\sin 2(\pi)ft)$$

Find the output  $y(t)$  in each of the following situations.

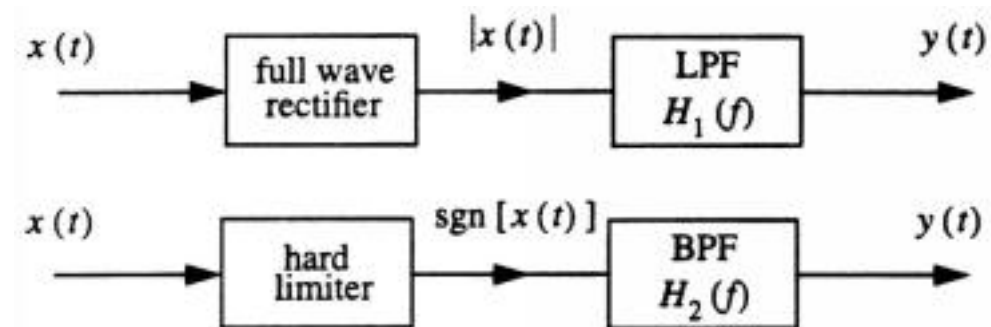


2. Consider a narrowband signal

$$x(t) = A(t) \cos (200(\pi)t + (\theta)(t))$$

where  $A(t) = ( (\sin(\pi)t / ((\pi)t) )^2$  and  $(\theta)(t) = 2(\pi)\sin 2(\pi)t$ .

Find the output  $y(t)$  of each of the following systems where  $x(t)$  is the input:



The filter transfer functions are given by

$$H_1(f) = 1, |f| \leq 1; = 0 \text{ otherwise} \quad H_2(f) = 1, \text{ if } |f - 100| \leq 1 \text{ or } |f + 100| \leq 1; = 0 \text{ otherwise}$$

3. Let  $X$  be a random variable with a probability density function given by

$$P_X = 1/2 |x| \leq 1; = 0 |x| > 1$$

a. Find  $E[X]$

b. Let  $Y$  be another random variable whose conditional density given  $X$  is

$$p(y|x) = 1 / \sqrt{2\pi} * e^{-1/2 * (y - x)^2}$$

Find  $EXY$ .

4. For each of the following functions  $R(t, s)$ , determine whether it can be an autocorrelation function. Explain.

a.  $R(t, s) = e^{-|t - s|} * \cos 20(\pi)(t - s)$

b.  $R(t, s) = \cos^2(t + s)$

c.  $R(t, s) = 1 - t - s + ts, 0 \leq s, t \leq 1$

5. Let  $X_t$  have a power spectral density function

$$P_x(f) = 1, |f| \leq 1; = 0 \text{ otherwise}$$

a. Find the autocorrelation function  $R_x(\tau)$ .

b. Let  $X_t$  be the input and  $Y_t$  the output to a linear and time invariant system with transfer function

$$H(f) = e^{-|f|}, -\infty < f < \infty$$

Find the average power of the output  $E|Y_t|^2$ .