EECS 120, Fall 1992
Final

Carefully read the exam. Most parts of each question are independent. Good luck.

Problem #1

(15 points) in each case first sketch the signal x, carefully marking the time axis and magnitudes. Next determine if the Laplace transform X (s) of x exists, and if it does, evaluate it. Lastly determine if the Fourier transform X (**omega**) of x, and if it does, evaluate it.
(A) **x(t) = u(t)**
(B) **x(t) = t * (e^(-t)) * u(t)**
(C) **x(t) = Sum of ((e^(-t)) dell(t - n)) from n=0**
(D) **x(t) = cos t, -infinity < +infinity**
(E) **x(t) = e^t, -infinity < +infinity**

Problem #2

(15 points) Let f, g be the three pairs of signals specified blow and let h = f * g. For each case find (one) the values of C for which h **!=** 0, (two) the value of t where **ABS(h(t))**achieves its maximum value, and (three) **the integral from -infinity to +infinity of h(t)dt.**
(A) **f(t) = g(t) = PI(t)**
(B) **f(t) = g(t) = t*PI(t)**
(C) **f(t) = PI(t-1) and g(t) = ABS(t) * PI(t)**

Problem #3

(20 points)
(A) find a signal x whose energy is 1 and which is bend limited to 0.5 Hz, i.e. if X(f) is the FT of x, then **ABS(X(f)) = 0**, for **ABS(f) 0.5**. Prove that the signal you exhibit is bandlimited and its energy is 1.
(B) Find a signal x which is bandlimited to 1 Hz such that if it is sampled every 0.5 second, the resulting samples are given by: x(0) = 1 and x (0.5n) = 0 for n != 0.
(C) Is the signal which satisfies (B) unique or not? Explain why.
(D) Given a sequence of sample values x(n), **-infinity < +infinity**, find a signal z(t), **-infinity < +infinity**, explicitly in terms of the sample sequence such that (1) z is bandlimited to 1 Hz and (2) z(0.5n) = x(n) for all n.

Problem #4

The AM transmission band is between 550 KHz and 16 hundred KHz the Federal Communications Commission or F C C allocates 20 KHz to each licensed AM station. Thus the first station is assigned the band 550-570 KHz and carrier frequency 560 KHz. The next station is assigned 570-590 KHz and carrier frequency 580 KHz. And so on. Suppose all stations use DSB-LC modulation.
(A) what is the maximum bandwidth of the message signal of each signal? Sketch what you expect the spectrum between 550 KHz and 610 KHz to look like. Carefully mark the contribution of each station.
(B) give a block diagram design of a receiver which can be used to demodulate any station. Remember that the signal received by the antenna will contain all the stations. If your design contains filters, indicate their frequency response.
(C) Suppose we wish to receive the 590-610 kHz station using your receiver. Let m be the original message signal. Explain how your receiver recovers m. In your explanation provide sketches of the FT of key signals in your receiver.

Problem #5

(15 Points) Consider the difference equation:

$$y(k) + 2*y(k-1) + y(k-2) = x(k) + 2*x(k-1)$$

(A) Find the transfer functino H(z). Is the system BIBO stable?
(B) Suppose y(-1) = y(-2) = x(-1) = 1, x(k)=0 for k=0. Find y(k), k=0.
(C) Obtain a direct form realization of the difference equation using only two delay elements.

Problem #6

(20 points) In this problem we will consider the analog system

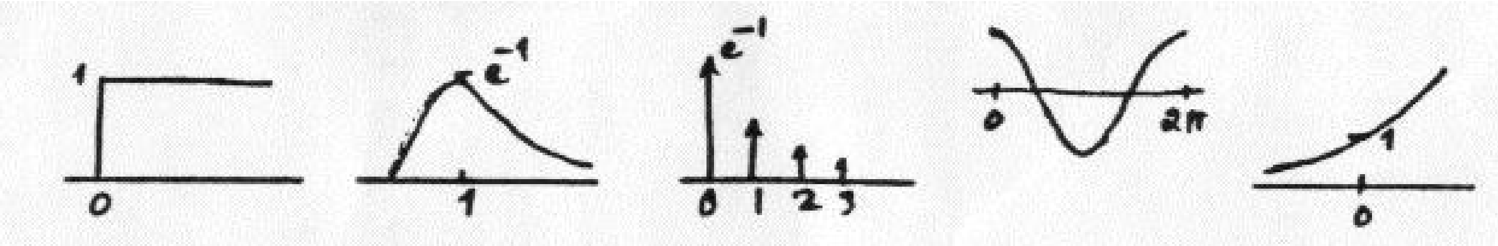
H(subscript a)(s) = 1 / ((1+s)^2)

as a low pass filter with a pass band from 0 to 1 Hz. Note that the amplitude response at 1 Hz is 0.5 or -6 db.
(A) Draw the Bode plots of this filter (both amplitude and phase). Carefully mark the axes. In the amplitude response plot also indicate the magnitude of the slope.
(B) Suppose you have to design a low pass filter with the following specification: sampling frequency = 10 kHz, and pass band from 0 to 2.5 kHz with 6 db attenuation at 2.5kHz. Using the filter H(subscript a) find a second order digital filter H(subscript d)(z) with this specification.
(C) Sketch the amplitude and phase response of your filter H(subscript d). Carefully mark the axes.

Solutions

Problem #1

The three signals are sketched below:



(A) The LT and FT both exist and are given by:
X(s) = 1/s, X(omega) = (1 / j*omega) + (1/pi)*(dell(omega))
(B) The LT and FT both exist and are given by: **X(s) = 1/((s+1)^2), X(omega) = / ((j*omega+1)^2)**

Problem #2

Note that if h = f * g then

INTEGRAL from -infin to infin of h(t)dt = INTEGRAL from -infin to +infin of f(t)dt X INTEGRAL from -infin to +infin of g(t)dt.

(A) h(t) **!=** 0, for **-**1 < 1. **ABS(h(t))** reaches its maximum value at t = 0. **INTEGRAL from -infin to +infin of h(t) dt = 1**.
(B) h(t) **!=** 0, for **-**1 < 1. **ABS(h(t))** reaches its maximum value at t = 0. **INTEGRAL from -infin to +infin of h(t) dt = 0**.
(C) h(t) **!=** 0, for 0 < 2. **ABS(h(t))** reaches its maximum value at t = 1. **INTEGRAL from -infin to +infin of h(t)dt = 0.5**.

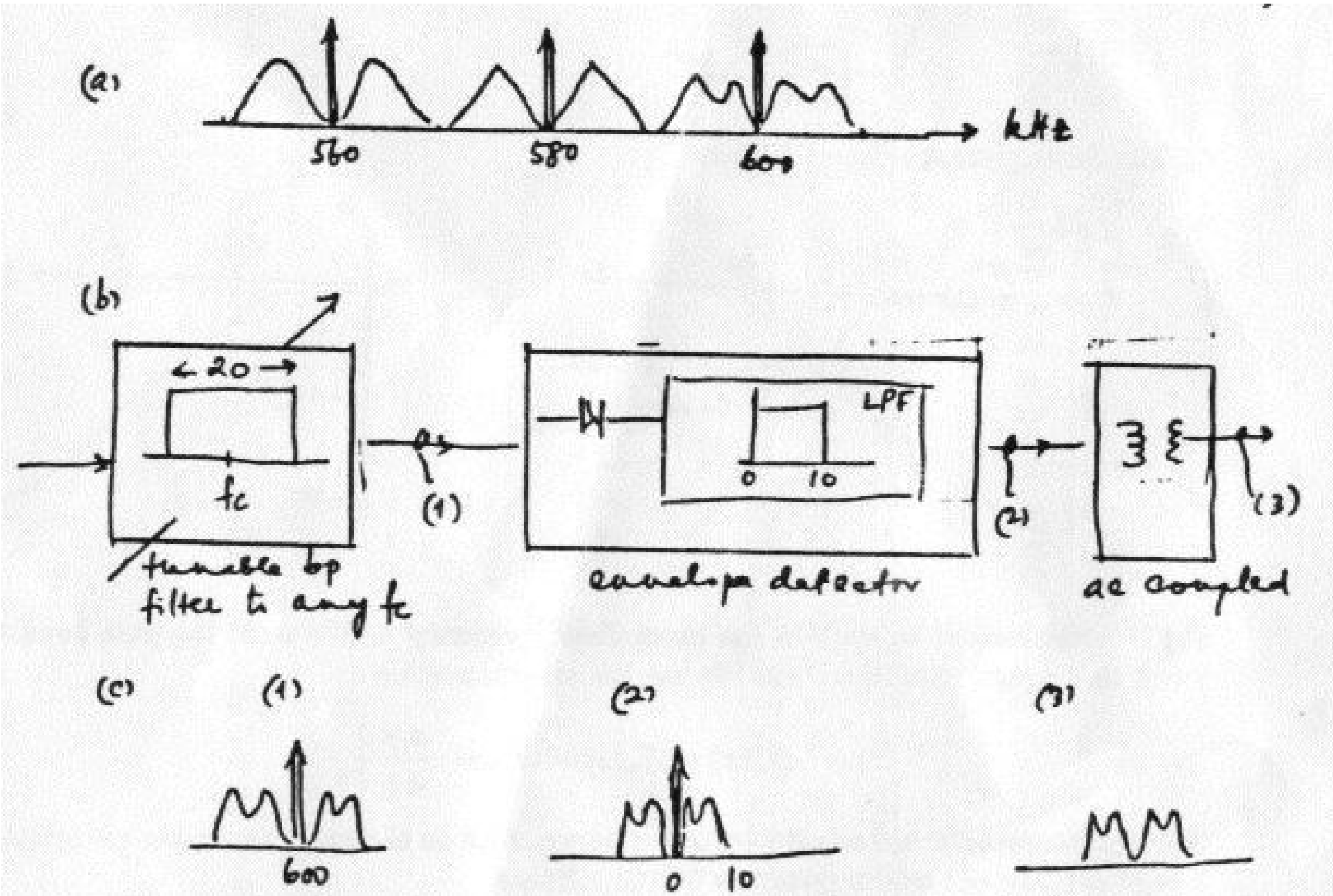
Problem #3

(A) The signal x whose FT is **X(f) = PI(f)** will do the job because it is bandlimited to 0.5 Hz and because, by Parseval's theorem, its energy is given by **INTEGRAL from -infin to +infin of ABS(X(f))^2 df = 1**. Thus x(t) = sinc t is the requred signal. (Of course there are many such signals).
(B) The reuqired signals is x(t) = sinc 2t. We have x(0) = 1, and x(0.5n) = sinc n = 0 for **n!=0**. Moreover **X(f) = 0.5*PI(0.5f)**. so that **ABS(X(f)) = 0 for ABS(f) 1**.
(C) By the Sampling Theorem, there is a unique analog signal which is bandlimited to one**-**half the sampling frequency which matches a specified sequence of samples. Hence the signal in (B) is unique.
(D) By the Sampling Theorem, there is a unique signal z and it is given by

z(t) = SIGMA from n=-infin to +infin of x(n)*sinc 2(t-0.5n).

Problem #4

(A) The maximum bandwidth is 10kHz. The total transmitted signal between 550-610kHz has a spectrum shown below.
(B) The simplest block diagram is a cascade connection of 3 blocks. Block A is a tunable bandpass filter with a pass band that is 20kHz wide and whose center is tunable to any frequency between 550k and 1,600k. The function of A is to suppress all stations except the one you wish to hear. Block B is a simple envelope detector. It may consist of a rectifier AC coupling to suppress the DC component in the envelope due to the carrier.
(C) The sketch below gives the spectrum at the end of each block above.



Problem #5

(A) From the difference equation we get

(1 + 2z^-1 + z^-2)Y(z) = (1 +2z^-1)X(z)

so the transfer function is

H(z) = (1+2z^-1) / (1+z^-1)^2

This has a double pole at z = -1 so it is not BIBO stable. (B) Take ZT of the difference equation and use the initial conditions:

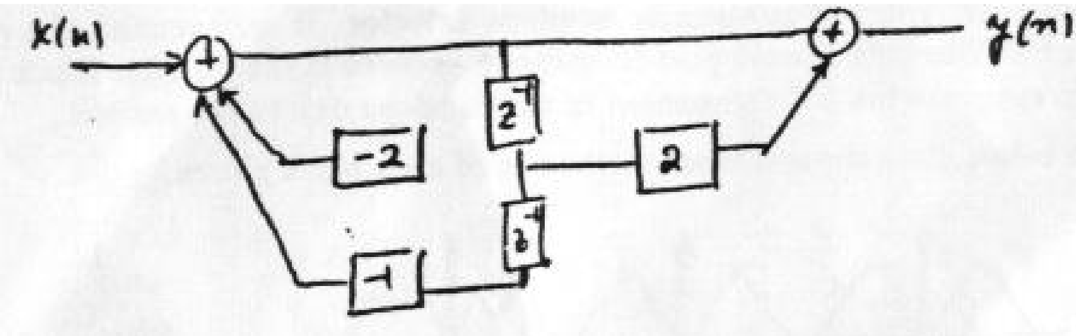
y(k) **LEFT-RIGHT**ARROW** Y(z), y(k-1) **LEFT-RIGHT**ARROW** y(-1) + (z^-1)Y(z), y(k-2) **LEFT-RIGHT**ARROW** y(-2) + (z^-1)y(-1) + Y(z)

x(k) **LEFT-RIGHT**ARROW** X(z), x(k-1) **LEFT-RIGHT**ARROW x(-1) + (z^-1)X(z)

Using the initial conditions and X(z) = 0, and substituting gives

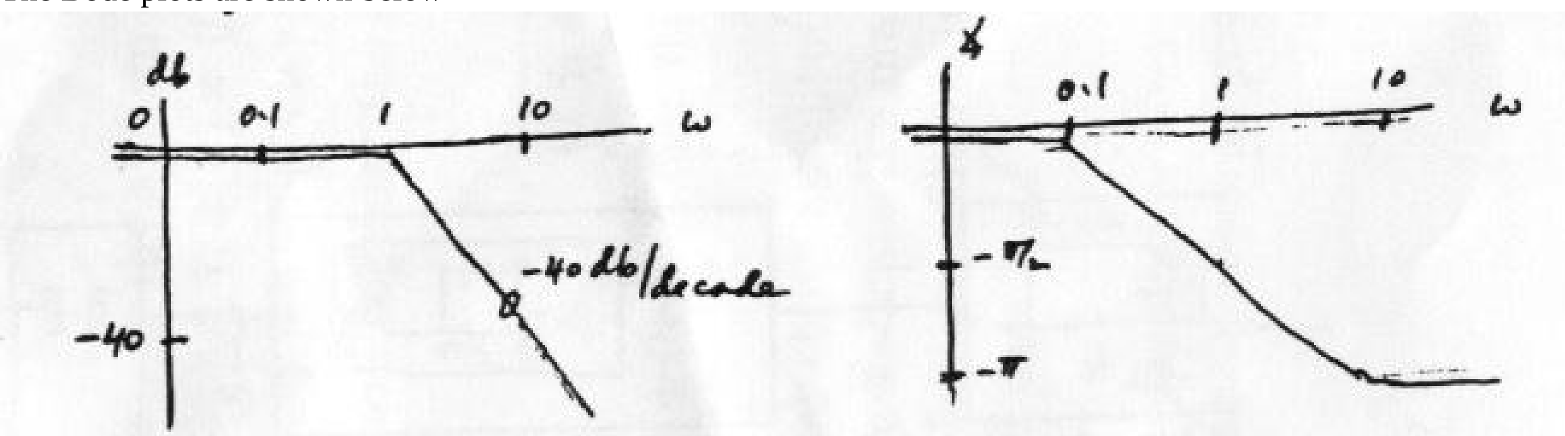
Y(z) = -1 / (1 + z^-1) **LEFT-RIGHT**ARROW** y(k) = (-1)^(k+1), k=0

(C) Here is a direct form realization with two delay elements:



Problem #6

The Bode plots are shown below



(B) It is convenient to work in the normalized frequency r. We want the pass band from r = 0 to **r(subscript c)** = 2.5/10 = 0.25. We use the transformation

H(subscript d)(z) = H(subscript a)(s) with s = c* (z - 1) / (z + 1). [1].

The two frequencies are related by **omega = c tan PI r**. And so the two pass bands are related by **1 = c tan 0.25PI = 1** which gives c = 1 in [1]. Hence

H(subscript d)(z) = 1 / ((z-1)/(z+1) + 1)^2 = (z+1)^2 / (4z^2) = (1/4)(1 + z^-1)^2.

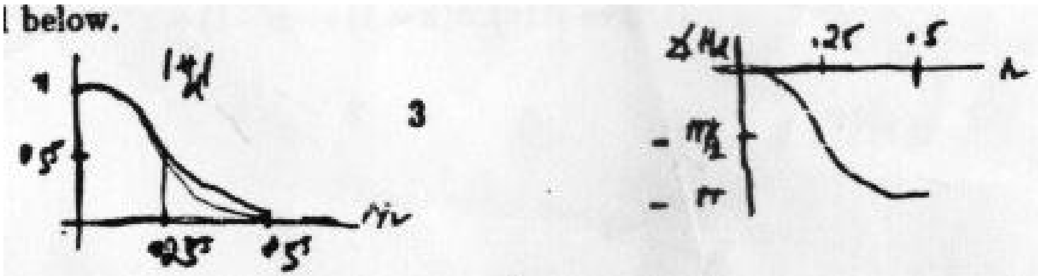
(C) The frequency response is

H(subscript d)(e^(2PIr)) = (1/4)(1 + e^(-j2PIr))^2

which in polar coordinates gives

ABS(H(subscript d)(e^2PIr) = (1/4)(2 + 2cos(2PIr)), ANGLE H(subscript d)(e^2PIr) = -2 ARCTAN ((sin 2PIr) / (1 + cos(2PIr))).

These are sketched below.



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